Nuclear Spin-Dependent Parity Violation in Diatomic Molecules

Sidney Cahn
Yale University

Funding: Yale University, NSF
• The mechanisms for PNC in atoms & molecules
• Nuclear spin-dependent PNC & the nuclear anapole moment
• The most sensitive atomic PNC experiment (Berkeley, Dy)
• The (great) advantages of diatomic molecules
• A Signal Strategy
• Sim. Signals

• Exp. Details: **Source, Interaction Region, Detection Region**

• Real Signals for $^{138}\text{Ba F}$

• Prospects: “anapole moment table”
  + better measurements of $V_e A_N$ couplings $C_{2N}, C_{2P}$
The primary mechanism for PNC in atoms

\[ C_{1(N,P)}: A_e V_N \text{ term leads to term in atomic Hamiltonian} \]

simplest PNC invariant

\[ H_W \propto G_F \left( \vec{\sigma} \cdot \vec{p} \right) \delta^3(\vec{r}) \]

\[ 1 / m_Z^2 \]

short-range Yukawa potential

axial vector associated with electron
With an energy difference $\Delta = 3.29 \cdot 10^{15} \text{ Hz}$

$$\frac{H_W}{\Delta} \equiv g = \frac{G_F m_e^2 \alpha^2}{\sqrt{2\pi}} = 3.65 \cdot 10^{-17}$$

“This value is, of course, fantastically small.”

-I. B. Khriplovich

“If your experiment needs statistics, you ought to have done a better experiment.”
**Effect of Z⁰-exchange on atomic structure**

\[ H \propto G_F (\vec{\sigma} \cdot \vec{p}) \delta^3(\vec{r}) \] mixes \( s_{1/2} \) and \( p_{1/2} \) states:

\[
\left| s_{1/2} \right\rangle \rightarrow \left| s_{1/2} \right\rangle + \frac{\left\langle s_{1/2} \left| H_{\text{Weak}} \right| p_{1/2} \right\rangle}{(E_{p_{1/2}} - E_{s_{1/2}})} \cdot \left| p_{1/2} \right\rangle = \left| s_{1/2} \right\rangle + i\eta \cdot \left| p_{1/2} \right\rangle
\]

(*Mixing coefficient \( \eta \) pure imaginary because of \( T \) invariance*)

**PNC effects grow rapidly with \( Z \):**

\[
\left\langle \left| \vec{p} \right| \right\rangle \propto Z; \quad \left\langle \delta^3(\vec{r}) \right\rangle = |\psi(0)|^2 \propto Z;
\]

\[
A_e V_N : Q_W = NC_{1n} + ZC_{1p} = -N + (1 - 4\sin^2 \theta_W)Z \approx -N \propto Z
\]

\[
\left\langle s_{1/2} \left| H_{\text{Weak}} \right| p_{1/2} \right\rangle \propto i \cdot R(Z) \cdot Z^3
\]

\[
\eta = gZ^3R = 2 \cdot 10^{-10} \quad R \approx 10 \quad Z \approx 50
\]
$VeAN$ term depends on nuclear spin: NSD-PNC

$VeAN$ term gives Hamiltonian:

$$H \propto GF (\vec{I} \cdot \vec{\sigma})(\vec{\sigma} \cdot \vec{p}) \delta^{3}(\vec{r})$$

Why is it smaller than $AeVN$?

1. $Ve/Ae = (1-4\sin^{2}\theta_{W}) \sim 0.08$
2. Only one unpaired nuclear spin:
   $\sim 1/Z$ compared to $Q_{W}$

$$VeAN/Q_{W} \sim (1-4\sin^{2}\theta_{W})/Z \sim 10^{-3}$$
   for heavy atoms
Suppression of tree-level NSD-PNC diagram makes radiative corrections non-negligible!

Tree-level NSD-PNC from suppressed $V_e A_N$ term

PNC weak interactions inside nucleus induce nuclear “anapole moment” that couples to electron magnetically

Coherent sum of weak charge ($A_e V_N$) and electromagnetic hyperfine interaction

PANIC/MIT July 26, 2011
**NSD-PNC vs. Z/A**

**NSD-PNC Hamiltonian**

\[ H_{NSD} \propto (\kappa_Z + \kappa_a + \kappa_Q) (\vec{I} \cdot \vec{\sigma})(\vec{\sigma} \cdot \vec{p}) \delta^3(\vec{r}) \]

**Differing dependences on Z/A**

\[ \kappa_{ZP} = -\kappa_{ZN} = -(1-4\sin^2\theta_W)g_A/2 \approx -0.05 \text{ (indep. of } A) \]

\[ \kappa_a \approx \frac{9}{10} \frac{\alpha \mu}{m r_0} A^{2/3} g_{\text{eff}} \approx 0.08 g_{\text{eff}} \left( \frac{A}{100} \right)^{2/3} \quad (g_{\text{eff},P} \approx 4, \ g_{\text{eff},N} \lesssim 1) \]

\[ \kappa_Q \propto A^{2/3} \text{ is both small (}< \kappa_a/4) \text{ & well understood--ignore} \]

*In heavy atoms, anapole dominates: } |\kappa_a| > |\kappa_Z|!

(Collective enhancement causes radiative correction > tree level!)

\[ |\kappa_a| \approx |\kappa_Z| \text{ for } A \approx 10 \text{ (odd proton)} \]

\[ A \approx 100 \text{ (odd neutron)} \]

PANIC/MIT July 26, 2011
A closer look at the nuclear anapole moment

PNC weak interactions between nucleons perturb nuclear structure:

\[
\begin{align*}
N_1 & \quad \pi, \rho, \omega \\
\downarrow & \quad Z^0, W^\pm \\
N_2 & \quad \sim (\vec{\sigma} \cdot \vec{p}) \sum_i g_{\text{eff},i} \cdot F_i(\vec{r}, \vec{\tau})
\end{align*}
\]

Hamiltonian for valence nucleon gives spin-momentum correlation

6 terms in principle;

\textit{only 2 linear combinations of terms important in practice}
Status of hadronic PNC measurements & effect of future anapole measurements

Odd-p isotopes (×4)

Odd-n isotopes (×3)

Assuming ~30% uncertainty in nuclear structure calculations
Determination of $C_2$'s (?)

Measurements with a few heavy nuclei would determine intranuclear PNC couplings to $\sim 30\%$ (nuclear structure unc.)

Additional measurements could be interpreted as measurements of $\kappa_Z$ with uncertainty

$$\delta \kappa_Z / \kappa_Z \sim 0.2 \times (\kappa_a / \kappa_Z) \propto A^{2/3}$$

The goal: 20% measurement of NSD-PNC in nuclei with $\kappa_a \sim \kappa_Z$

i.e., $Al$ ($Z=13$, $A=27$); $Sr$ ($Z=38$, $A=87$)

This requires a technique with greatly improved sensitivity!
Determination of $C_2$'s (?)

PV-DIS (JLAB) projected

light odd-n isotope ($^{87}$Sr) (proj.)

Bates SAMPLE e-p vs. e-D elastic scattering 2000+2003

Current allowed region

Standard Model prediction

SLAC e-D deep-inelastic scattering ('79)
A Yale Collaboration Named

Zeeman-tuned optically prepared and detected molecular beams for the investigation of electroweak effects using stark interference.
Using molecules to get at NSD-PNC

- Diatomic molecules systematically have close rotation+hyperfine levels of opposite parity--B-field tuning can give $\Delta E \sim 10^{-11} \text{ eV}$! [Sushkov, Flambaum, Sov. Phys. JETP 48, 608 (1978), Flambaum, Khriplovich, Phys. Lett. A 110, 121 (1985) Kozlov, Labzowsky, & Mitruschenkov, JETP 73, 415 (1991)]

- Ground state levels have long lifetimes, narrow lines ⇒ high sensitivity to PNC energy shifts (from AC Stark) [Fortson]

- Can use proven technique for nearly-degenerate levels (atomic Dy) [Nguyen, Budker, DeMille, & Zolotorev PRA 56, 3453 (1997)]

- Versatile, well-characterized beam source for all desired species [widely used; charaterized by Tarbutt et al., {Hinds}, J. Phys. B 35, 5013 (2002).]

- Wide range of molecular species with required spectroscopic data already known [Herzberg, etc.]

- Estimated sensitivity sufficient for desired low-Z nuclei (odd $p$ and odd $n$), plus wide range of heavier nuclei

- Simple molecules (1 valence $e^-$, $^2\Sigma_{1/2}$ state) amenable to interpretation (via calculated valence electron wavefunctions) at $\sim 20\%$ level [Kozlov & Labzowsky, J. Phys B 28, 1933 (1995)]
The most sensitive atomic PNC experiment: Dy

Search for parity nonconservation in atomic dysprosium

A. T. Nguyen, D. Budker, D. DeMille, and M. Zolotorev

\[ |H_w| = |2.3 \pm 2.9 \text{ (statistical)} \pm 0.7 \text{ (systematic)}| \text{ Hz} \]

“Close enough” energy levels
\[ \Delta = 3.1 \text{ MHz} \]

B-field tuning to near degeneracy gives small energy denominator
\[ \sim 10^{-11} \text{ eV(!)} \]

and maximal sensitivity
c.f. Cs (Boulder) \( \delta H_w \sim 14 \text{ Hz} \)

FIG. 3. Partial Zeeman structure of \(^{163}\text{Dy}\) \( F = 10.5 \) sublevels of \( A \) and \( B \). Zero energy is chosen arbitrarily.
Free radical rotation + hyperfine Zeeman

$^{137}$BaF rotation/hfs Zeeman shifts (I=3/2)

Magnetic field B (Gauss)

Energy (MHz)
Experimental schematic
Near Crossing
Apply Oscillating E Field

Center of Magnet
**Strategy to detect PNC in near-degenerate levels**

\[ H = \begin{pmatrix} \langle A \rangle & 0 & iH_W + dE \\ \langle B \rangle & -iH_W + dE & \Delta \end{pmatrix} \]

\[
E = E_0 \sin(\omega t); \quad dE_0 \ll \omega; \quad \Delta \ll \omega
\]

\[
\left| \langle A | \psi(T) \rangle \right|^2 = 4 \sin^2 \left( \frac{\Delta T}{2} \right) \left[ (1 \text{or } 2) \frac{H_W}{\Delta} \frac{dE_0}{\omega} + \left( \frac{dE_0}{\omega} \right)^2 \right]
\]

D. DeMille, S.B. Cahn, D. Murphree, D.A. Rahmlow, and M.G. Kozlov
Using molecules to measure nuclear spin-dependent parity violation
Signal and Asymmetry

\[ \text{Signal} \approx S(E+) = 4 \sin^2 \left( \frac{\Delta T}{2} \right) \left[ \left( \frac{dE_0}{\omega} \right)^2 \right] \]

\[ \text{Asymmetry} (A) = \frac{S(E+) - S(E-)}{S(E+) + S(E-)} \approx \frac{H_W}{\Delta} \frac{\omega}{dE_0} \]
Simulated/Calculated signals for $^{137}\text{BaF}$

$$A = \frac{H_w}{\Delta} \frac{\omega}{dE_0}$$

$^{137}\text{BaF}; \Delta \sim 2\pi \times 1 \text{ kHz}$

$$\omega = 2\pi \times 10 \text{ kHz}$$

$$dE_0/\omega = 0.1$$

$$H_w = 2\pi \times 4 \text{ Hz}$$
Viable nuclei for anapole measurement

- Everything OK (one dot/isotope)
- Only molecular spectral data needed
  - Only isotopically enriched sample needed
- Maybe possible with cryogenic beam source

“All science is either physics or stamp collecting”
**Sensitivity estimates**

\[ \tau \sim 100 \mu\text{s} \ (6 \text{ cm @ typ. Velocity}) \]
\[ \Gamma \sim 1 \text{ kHz} \approx 1/(2\pi\tau) \ (0.1 \text{ ppm B-field homogeneity}) \]
\[ \frac{\text{d}N}{\text{d}t} \sim 10^4/\text{s} \ (\text{Hinds pulsed beam}) \text{ for time } T \]

With

\[ \delta H_W = \frac{1}{\tau \sqrt{T \cdot \left(\frac{\text{d}N}{\text{d}t}\right)}} \]

\[ \delta H_W \sim 10 \text{ Hz}/\sqrt{T(\text{sec})} \]

<table>
<thead>
<tr>
<th>molecule</th>
<th>Z</th>
<th>(k_a/k_2)</th>
<th>(H_W(\text{Hz}))</th>
<th>T (10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{199}\text{HgF})</td>
<td>80</td>
<td>1/1</td>
<td>60</td>
<td>3 s</td>
</tr>
<tr>
<td>(^{137}\text{BaF})</td>
<td>56</td>
<td>0.7/1</td>
<td>10</td>
<td>100 s</td>
</tr>
<tr>
<td>(^{87}\text{SrF})</td>
<td>38</td>
<td>0.5/1</td>
<td>4</td>
<td>20 m</td>
</tr>
<tr>
<td>(^{113}\text{InO})</td>
<td>49</td>
<td>3/1</td>
<td>20</td>
<td>30 s</td>
</tr>
<tr>
<td>(^{27}\text{AlS})</td>
<td>13</td>
<td>1/1</td>
<td>1.5</td>
<td>100 m</td>
</tr>
</tbody>
</table>
The $\gamma^u\gamma_5$ Project

Nucleon “Axe”-ial Weak Couplings

D. DeMille, S.B. Cahn, D. Murphree, D.A. Rahmlow, and M.G. Kozlov

Using molecules to measure nuclear spin-dependent parity violation


Near Crossing
Apply Single Pulse E Field
(step potential)

\[ E = E_0 e^{-\left(\frac{t}{\tau}\right)^2} \]

Center of Magnet
Simple Stark mixing at a level crossing

\[ |A\rangle \rightarrow |A\rangle + c_B(t)|B\rangle e^{-i\Delta t}; \quad c_B(0) = 1 \]

\[ c_A(\infty) = \int_{-\infty}^{\infty} dE_0 e^{-t^2/\tau^2} e^{-i\Delta t} dt \]

Fourier Transform of Electric Field

for weak excitation \((dE_0 \tau \ll 1)\), Gaussian lineshape:

\[ S(\Delta; t \rightarrow \infty) = N_0 \left| c_A(\infty) \right|^2 = \pi d^2 E_0^2 \tau^2 e^{-\frac{\Delta^2 \tau^2}{2}} \]

On resonance, for arbitrary excitation strength: "Rabi flopping"

\[ S(\Delta = 0; t \rightarrow \infty; E_0) = \sin^2 \left( \sqrt{\pi dE_0 \tau} \right) \]
Typical level-crossing data

\( \Delta m = 0 \) crossings

B-field at crossing from position of peak

Interaction time \( \tau \) from width of peak agrees with calc. within 5%

E-field from step potential on cylindrical boundary

Optical Pumping Ratio \( R \)

\[
e^{-\frac{\Delta^2 \tau^2}{2}}
\]

Detuning from level crossing \( \Delta \) (kHz)
E-field time dependence & lineshape similar to that for PV measurement

Typical level-crossing data
\[ \Delta m = +/- 1 \text{ crossings} \]

B-field at crossing from position of central dip

Radial E-field
Calculated Level Crossings in $^{138}$BaF from
spin-rotational Hamiltonian: Energy vs. Magnetic Field

\[ |m_s, m_I, m_N> \]

- $m_F=-1$
- $m_F=0$
- $m_F=+1$

Magnetic Field (Gauss)

Energy (MHz)
Recently Extracted $^{138}$BaF X-ings

<table>
<thead>
<tr>
<th>Type $\Delta m$</th>
<th>Initial</th>
<th>Calc. X (Gauss)</th>
<th>Obs. X (Gauss)</th>
<th>Calc. d (Hz/V/cm)</th>
<th>Obs. d (Hz/V/cm)</th>
<th>Unc.obs. (Hz/V/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m=0$</td>
<td>1</td>
<td>4604.7</td>
<td>4604.8402</td>
<td>-3422</td>
<td>3456</td>
<td>39</td>
</tr>
<tr>
<td>$\Delta m=1$</td>
<td>0</td>
<td>4609.91</td>
<td>0</td>
<td>-16.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta m=-1$</td>
<td>1</td>
<td>4616.06</td>
<td>4616.138</td>
<td>-4700.</td>
<td>4150</td>
<td>2000</td>
</tr>
<tr>
<td>$\Delta m=0$</td>
<td>0</td>
<td>4621.25</td>
<td>4621.265</td>
<td>-150.</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>$\Delta m=-1$</td>
<td>1</td>
<td>4623.01</td>
<td>0</td>
<td>-65.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta m=0$</td>
<td>0</td>
<td>4628.2</td>
<td>4628.212</td>
<td>-3500.</td>
<td>3574</td>
<td>6.8</td>
</tr>
<tr>
<td>$\Delta m=-1$</td>
<td>0</td>
<td>4638.83</td>
<td>4638.674</td>
<td>-960.</td>
<td>800</td>
<td>400</td>
</tr>
<tr>
<td>$\Delta m=-1$</td>
<td>0</td>
<td>4640.2</td>
<td>4640.068</td>
<td>-4500.</td>
<td>4000</td>
<td>2000</td>
</tr>
</tbody>
</table>
Rabi-Flopping In $^{138}$BaF

(Dipole Matrix Element at Crossing $d = 3574. +/− 6.8$ Hz/V/cm)
Conclusions & Outlook

• Molecule PNC experiments look very promising for study of NSD-PNC: excellent S/N & systematics, leverage from developed techniques

• Multiple nuclei (odd p, odd n) immediately available for anapole moment determination at ~20% level

• Measurements of fundamental $Z^0$ couplings ($C_2$’s) at 20-50% level possible after several iterations

• Likely extensions with new molecular data, new sources of cold molecules: an “anapole factory”?!?
He at 4K Buffer Gas Cooling  Magnetic Hexapole Lens

We haven’t got the money, so we’ve got to think!
Once measurements on heavy and light nuclei are completed in diatomic molecules, we will add one atom, a proton, and measure the anapole moment of the nucleon.
Yale ZOMBIEs

ZOMBIE Bokor* ZOMBIE Patient Zero: Sid Cahn
Dave DeMille

ZOMBIE Grad. Stud. ZOMBIE Post-Doc
Jeff Ammon Emil Kirilov

Misha Kozlov PNPI
Foreign ZOMBIE

ZOMBIE Bokor*
Dave DeMille

ZOMBIE Patient Zero: Sid Cahn

ZOMBIE Grad. Stud. ZOMBIE Post-Doc
Jeff Ammon Emil Kirilov

Misha Kozlov PNPI
Foreign ZOMBIE

ZOMBIE Bokor*
Dave DeMille

ZOMBIE Patient Zero: Sid Cahn

ZOMBIE Grad. Stud. ZOMBIE Post-Doc
Jeff Ammon Emil Kirilov

Misha Kozlov PNPI
Foreign ZOMBIE

Yale ZOMBIEs


Dave Rahmlow U Conn. Ven. Financial Former ZOMBIEs

Dennis Murphree Ven. Financial

Rising ZOMBIE Corey Adams

Ed Deveney Bridgewater State Non-local ZOMBIEs

Rich Paolino USCG Academy
PNC in the nucleus induces a nuclear spin helix

\[ = \text{magnetic dipole} + \text{anapole} \]

Simple model for nuclear anapole
(valence nucleon + constant-density core):

\[ \vec{a} \sim G_F g_{\text{eff}} \frac{e\mu}{m_p r_0} A^{2/3} \hat{I} \]

PANIC/MIT July
CAD ZOMBIE-land?

Prod  Prep  Stark  Detect
ZOMBIE-land in the flesh!
Can these measurements be interpreted?

- Semi-empirical method determines electron wavefunctions from experimental data on hfs & g-factors
- Broad expectation of ~20% uncertainty from molecular theory
- Same method used to calculate electron EDM enhancement in YbF
- EDM enhancement for YbF also calculated by 3 different \textit{ab initio} methods--agree with each other & semi-empirical within <20%
- Should be checked explicitly for anapole (vs. EDM) & for other molecules (already done for BaF w/20% agreement)
- Possibility to cross-check vs. anticipated \textit{atomic} results for Yb, Ba!

\[ \therefore \text{~20\% accuracy in interpretation seems very likely AND can be checked explicitly vs. other systems} \]
What about systematics?

- multiple reversals \((dE/dt, \Delta, B, |A\rangle \leftrightarrow |B\rangle)\) ⇒ systematics require product of two imperfections

- main residuals from stray dc \(E\)-field + \(B\)-field gradient BUT effect of stray \(E\) strongly suppressed at crossing point (requires spin flip); \(H_W/d \sim 1 \text{ mV/cm} \) typical

- multiple crossing points with wildly different PNC/Stark ratios because of angular factors ⇒ can check for systematic deviations in non-trivial way

- cross-check vs. atomic experiments (Ba, Yb)?