Transport Coefficients of Deconfined Strongly Interacting Matter: From Weak to Strong Coupling

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Motivation – Bulk and Shear Viscosities

\[ \partial_\mu T^{\mu\nu} = 0 \]

\[ T^{\mu\nu} = T^{\mu\nu}_{(0)}(\epsilon, P; u^\mu) + T^{\mu\nu}_{(1)}(\zeta, \eta; u^\mu, \partial_\nu u^\mu) \]

\( \zeta \): bulk (volume) viscosity

\( \eta \): shear viscosity

Application of ideal hydrodynamics modelling heavy-ion collisions at RHIC and LHC suggests at most small dissipative effects; viscous calculations confirm this.
Quasiparticle Modell (QPM)

QPM based on $\Phi$- functional approach to QCD:

$$\frac{\Omega[D, S]}{T} = \frac{1}{2} \text{Tr} \left[ \ln D^{-1} - \Pi D \right] - \text{Tr} \left[ \ln S^{-1} - \Sigma S \right] + \Phi[D, S]$$

$$\Phi = \frac{1}{12} \begin{array}{c} \includegraphics{phi1} \\ \includegraphics{phi2} \\ \includegraphics{phi3} \end{array} + \frac{1}{8} \begin{array}{c} \includegraphics{phi4} \\ \includegraphics{phi5} \end{array} - \frac{1}{2} \begin{array}{c} \includegraphics{phi6} \end{array}$$

$$\Pi = \frac{1}{2} \begin{array}{c} \includegraphics{pi1} \\ \includegraphics{pi2} \end{array} + \frac{1}{2} \begin{array}{c} \includegraphics{pi3} \end{array} - \begin{array}{c} \includegraphics{pi4} \end{array}, \quad \Pi = 2 \frac{\delta \Phi}{\delta D}$$

$$\Sigma = \begin{array}{c} \includegraphics{sigma1} \end{array}, \quad \Sigma = -\frac{\delta \Phi}{\delta S}$$

→ modell for equilibrium thermodynamics → corresponding energy-momentum tensor:

$$T^{\mu\nu}_{(0)}(T) = \sum_i \int \frac{d^3 \vec{p}}{(2\pi)^3 E_i(T)} p^\mu p^\nu f_i^{(0)} + g^{\mu\nu} B[\{\Pi_j(T)\}]$$

for excitations with medium-modified dispersion relations (thermal mass) $E_i^2(T) = \vec{p}^2 + \Pi_i(T)$
Effective Kinetic Theory

- self-consistent generalization of \( T_{(0)}^{\mu\nu} \) to non-equilibrium systems:

- space-time dependence of \( T(x) \) implies \( E = E(x) \)
  
  is a functional of the distribution function \( f(x, p) \)

- to assure **basic relations**:
  
  - \( \partial_\mu T^{\mu\nu}(x) = 0 \)
  
  - \( \delta \langle T^{00} \rangle / \delta f(x, p) = E \) (Fermi liquids)
  
  - in thermal equilibrium: \( \epsilon + P = T \frac{\partial P}{\partial T} \)

  one generalizes (in case of a one-component system) to

\[
T^{\mu\nu}(x) = \int \frac{d^3 \vec{p}}{(2\pi)^3 E(x)} p^\mu p^\nu f(x, p) + g^{\mu\nu} B[\Pi(x)]
\]

\( T^{\mu\nu} \) closely related to **effective kinetic equation of Boltzmann-Vlasov type** for the single-particle distribution function \( f(x, p) \):

\[
(\mathcal{L} + \mathcal{V}) f = \mathcal{C}[f]
\]

- above conditions satisfied if

\[
\frac{\partial B}{\partial \Pi} = -\frac{1}{2} \int \frac{d^3 \vec{p}}{(2\pi)^3 E(x)} f(x, p)
\]

  related to form of Vlasov-term
Bulk and Shear Viscosity Coefficients for quasiparticle systems
Bulk and Shear Viscosities

→ decompose $T^{\mu\nu}$ and compare w/ definition: $T^{(1)}_{\mu\nu} = \zeta \Delta^{\mu\nu} \partial_\alpha u^\alpha + \eta S^{\mu\nu}_{\alpha\beta} \partial^\alpha u^\beta$

bulk viscosity:

$$\zeta = \frac{1}{T} \int \frac{d^3 \vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E}$$

$$\times \left\{ \left[ \left( \frac{p u}{T} \right)^2 - \frac{1}{2T} \frac{\partial \Pi}{\partial T} \right] T^2 v_s^2 + \frac{1}{3} [p^2 - (pu)^2] \right\}^2$$

shear viscosity:

$$\eta = \frac{1}{15T} \int \frac{d^3 \vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} [p^2 - (pu)^2]^2$$

Bulk and Shear Viscosities

differences: Excitations with constant vs. thermal mass

bulk viscosity:

\[
\zeta = \frac{1}{T} \int \frac{d^3 \vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} \\
\times \left\{ \left[ \left( \frac{pu}{T} \right)^2 - \frac{1}{2T} \frac{\partial \Pi}{\partial T} \right] T^2 v_s^2 + \frac{1}{3} [p^2 - (pu)^2] \right\}^2
\]

shear viscosity:

\[
\eta = \frac{1}{15T} \int \frac{d^3 \vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} [p^2 - (pu)^2]^2
\]

cf. Gavin (1985)
Relaxation Time

collision processes relevant for **shear** and **bulk viscosities** different;

**assumption**: same $\tau$, independent of $|\vec{p}|$

center on SU(3): 2 ↔ 2 gluon-gluon scatterings

parametrically

$$\tau^{-1} \sim TG^4(T) \ln(a/G^2(T))$$

based on perturbative considerations

cross section depends crucially on ratio of maximum to minimum momentum transfer $\sim a$


parametric dependencies of pQCD results for $\zeta$ and $\eta$ on coupling and temperature reproduced at large $T$
Adjust Parameters in Thermal Equilibrium – SU$_c$(3)

\[ \Pi_g(T) = \frac{1}{2} T^2 G^2(T), \quad \text{where} \quad G^2(T) = \frac{48\pi^2}{11N_c \log \left( \frac{\lambda(T-T_s)}{T_c} \right)^2} \]

Boyd et al., NPB 469 (1996)
Okamoto et al., PRD 60 (1999)

\( G^2(T) \) adjusted

\[ (\epsilon-3P)/T^4 \]

\( SU_c(3) \)

\( T/T_c \sim 1.15 \)

maximum around \( T/T_c \sim 1.15 \)
→ behaviour close to $T_c$ driven by $\tau$

Quantitative Results – Specific Shear Viscosity

$SU_c(3)$

Nakamura, Sakai, PRL 94 (2005)
Meyer, PRD 76 (2007)

Quantitative Results – Specific Bulk Viscosity

$h_{12}\bar{\chi}/s$

combined sound channel

$h_{holo}$

cf. Gürsoy et al. (2009)

cf. MB, Kämpfer & Redlich (2011)
Ratio of Bulk to Shear Viscosities
Big Theoretical Motivation: Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

- Kovtun-Son-Starinets bound: $\frac{\eta}{s} \geq \frac{1}{4\pi}$
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- similar *universal* bounds for other transport coefficients are unknown

**BUT** in some special classes of theories with holographically dual supergravity description there exists a lower bound for the ratio

Buchel bound: $\left( \frac{\zeta}{\eta} \right)_B \geq 2 \left( \frac{1}{k} - \nu_s^2 \right)$
**Big Theoretical Motivation:** Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

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**BUT** in some special classes of theories with holographically dual supergravity description there exists a lower bound for the ratio

**Buchel bound:** \( (\zeta/\eta)_B \geq 2 \left( \frac{1}{k} - v_s^2 \right) \)

- specific strongly coupled but nearly conformal theories (AdS/CFT)
- for scalar theory or photons in hot fluid \( \zeta/\eta \sim \Delta v_s^2 \equiv \left( \frac{1}{3} - v_s^2 \right) \)
- might expect that there is a gradual change from one behaviour to the other as a function of temperature
Bulk to Shear Viscosity Ratio

\[
\frac{\zeta}{\eta} = 15 \left( \Delta v_s^2 \right)^2 - 30 \Delta v_s^2 (\Pi - \tilde{a}) v_s^2 \frac{\mathcal{I}_0}{\mathcal{I}_{-2}} + 15(\Pi - \tilde{a})^2 \left( v_s^2 \right)^2 \frac{\mathcal{I}_2}{\mathcal{I}_{-2}},
\]

\[
\tilde{a} = T^2 \frac{\partial \Pi}{\partial T^2}, \; \mathcal{I}_k \ldots \text{momentum integrals}
\]

- at asymptotically large $T$: \( v_s^2 = \frac{1}{3} + \frac{5}{36} bT \frac{dG^2}{dT} + \mathcal{O} \left( G^2 T \frac{dG^2}{dT} \right) \)

- for $T \to T_c$: \( \Delta v_s^2 \to 1/3 \)

linear dependence on $\Delta v_s^2$ and Buchel's bound satisfied for $T \leq 1.15 T_c$
inclusion of quark degrees of freedom by assuming that relations between gluon and quark sector known from perturbative regime hold close to $T_c$.

leading-order estimate:

$$
\eta = \eta_g + \eta_q \text{ (additive)}
$$

$$
\eta_q \simeq 2.2 \frac{1 + 11 N_f / 48}{1 + 7 N_f / 33} N_f \eta_g
$$

- mild overall increase with $T$; still small at $3T_c$
Conclusions

- knowing QCD transport coefficients important for understanding the behavior of strongly interacting matter observed in high-energy nuclear collisions

  picture: excitations with effective thermal mass

  - inclusion of mean field term in energy-momentum tensor necessary for self-consistency of the approach
  - follows from kinetic equation of Boltzmann-Vlasov type

  - influence of a medium-dependent effective mass in dispersion relation minor on $\eta$ but prominent in $\zeta$

  - fairly nice agreement w/ available lQCD data ($\text{SU}_c(3)$); specific shear viscosity as small as $1/4\pi$

  - ratio of bulk to shear viscosities exhibits both quadratic and linear dependence on conformality measure; turning point located at the maximum in the scaled interaction measure
relation between $\eta/s$ and averaged transverse momentum transfer squared per unit distance of an energetic parton $\hat{q}$


$$\eta \sim \frac{1}{3} \rho \langle p \rangle \lambda$$

$$\hat{q} \simeq \frac{1}{12} \frac{\rho}{s} \langle p \rangle \langle \hat{s} \rangle \left( \frac{\eta}{s} \right)^{-1}$$

underlying assumption:
interaction between energetic parton and medium is of same structure and strength as interaction among thermal excitations