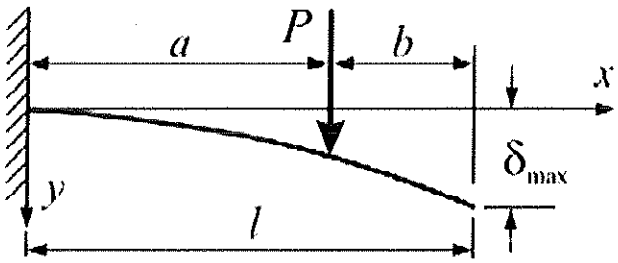
Results Cont.



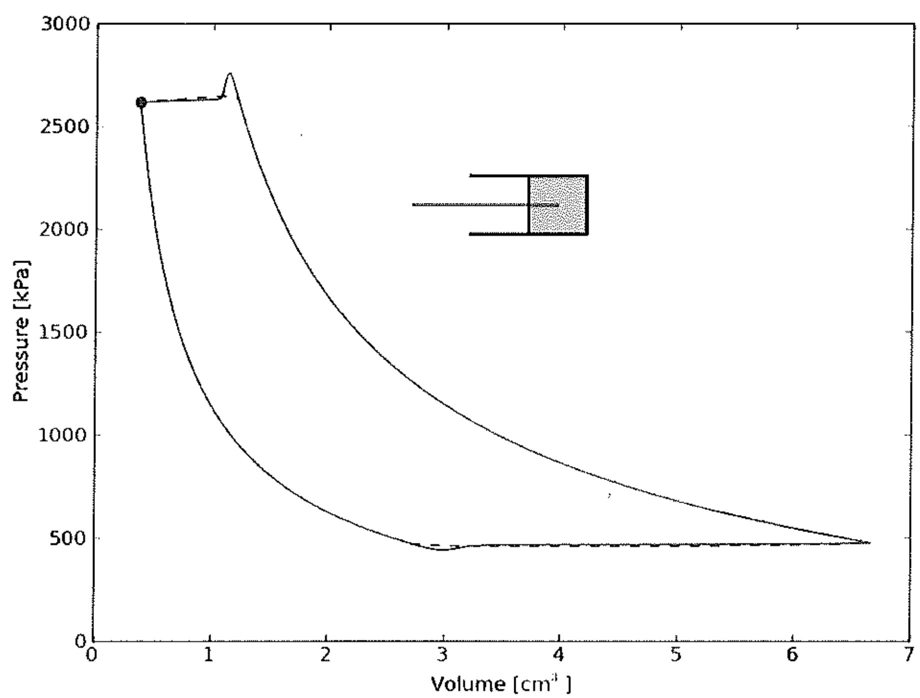
Material properties of spring steel

ρ	E
kg/m^3	GPa
8000	193

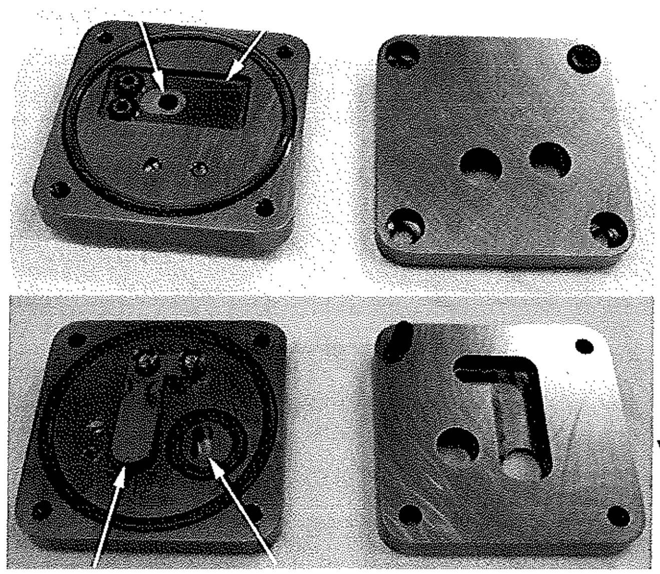
Key Valve Geometric Information

D_{port}	D_{valve}	a	l	thickness	stopper	x_{tr}
mm	mm	mm	mm	mm	mm	mm
5.9	7	14	18	.15	1.8	1.24

Valve Influence



Appendix



Nomenclature

ρ - Density, kg/m^3

θ - Crank Angle, rad

ω - Rotational Speed, rad/sec

V - Volume, m^3

$\dot{m}$ - Mass flow rate, kg/sec

h - Enthalpy, kJ/kg

u - Internal energy, kJ/kg

$\dot{Q}$ - Heat transfer, kW

T - Temperature, K

A - Area, m^2

x - Displacement, m

γ - Specific Heat Ratio, -

V - Gas Velocity, m/sec

μ - Viscosity, $Pa-\text{sec}$

U - Piston Velocity, m/sec

g - Leakage gap, m

M - Moving Mass, kg

C_D - Drag Coefficient, $-$

k - Stiffness, N/m

E - Modulus of elasticity, N/m^2

I - Bending moment of inertia, m^4

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Heat Transfer

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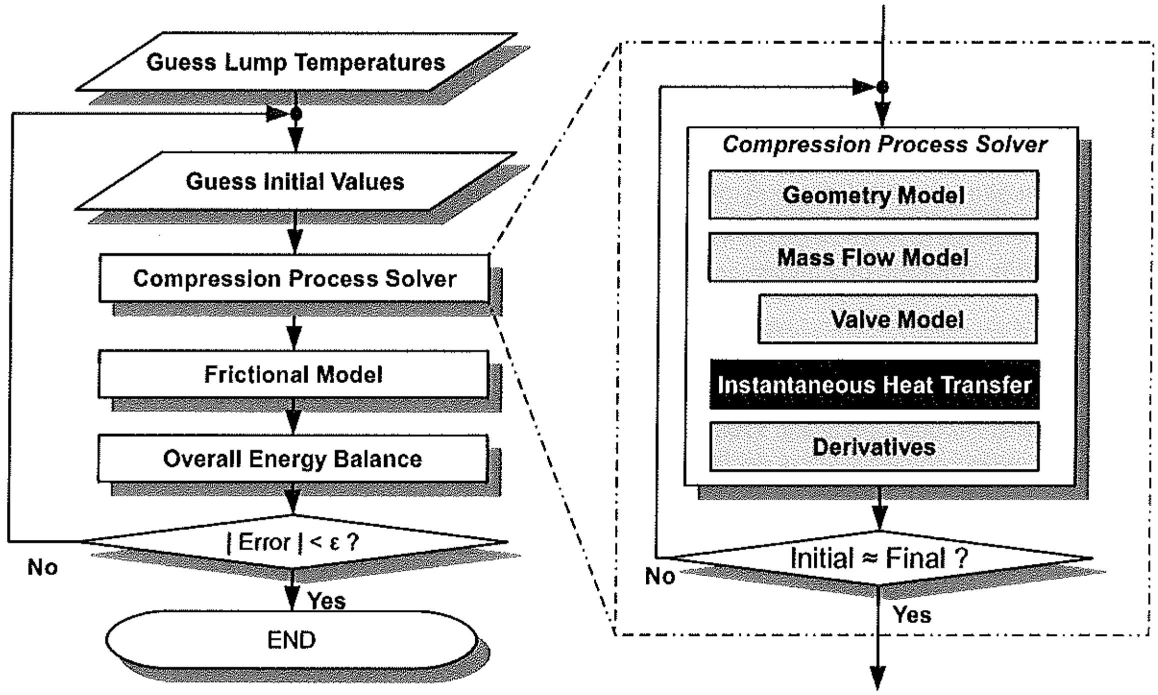
Heat Transfer



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Need for Heat Transfer

- Looking at the compression process equations

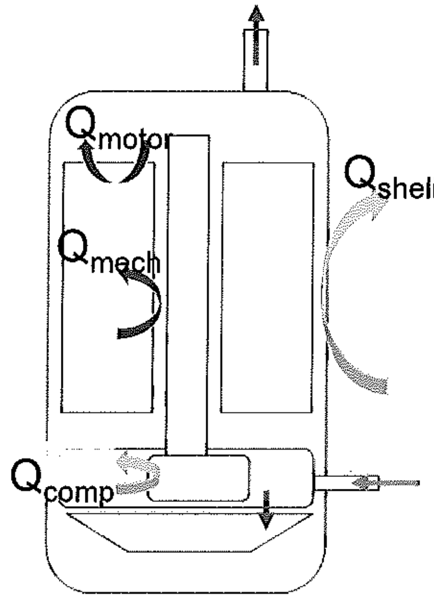
$$\frac{d\rho}{d\theta}=\frac{1}{V}\left[-\rho\frac{dV}{d\theta}+\frac{1}{\omega}(\dot{m}_{in}-\dot{m}_{out})\right]$$

$$\frac{dT}{d\theta}=\frac{-\rho h\frac{dV}{d\theta}-\left(uV+\rho V\frac{\partial u}{\partial \rho}\right)\frac{\partial \rho}{\partial \theta}+\frac{1}{\omega}\left(\dot{Q}+\dot{m}_{in}h_{in}-\dot{m}_{out}h_{out}\right)}{\rho V\frac{\partial u}{\partial T}}$$

- Need to calculate heat transfer at each instant throughout the compression process

Compressor Heat Transfer Models

- In general, the most significant heat transfer paths to the refrigerant involve convection
  - » Forced convection between refrigerant and interior compressor walls, Q_{comp} cylinder walls
  - » Free convection between shell and surroundings, Q_{shell}
- Also need to consider heat addition to refrigerant in shell due to motor and mechanical losses, Q_{motor} and Q_{mech}

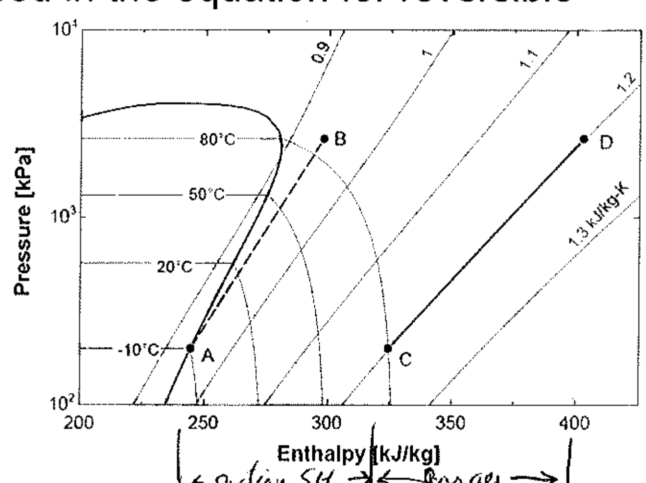


Compressor Heat Transfer Models

- Heat transfer increases the gas temperature and specific volume (reduces density) thus reducing the compressor efficiency
- This relationship is evidenced in the equation for reversible flow work,

$$W_{rev} = \int v dP$$

- The P-h diagram also illustrates that increased superheat increases compression work



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Compressor Heat Transfer Models

- Convective heat transfer is modeled by Newton's law of cooling:

$$\dot{Q} = h_c A\left(T_w - T_g\right)$$

Where positive Q indicates heat transfer to the gas.

A = Surface area, m^2

h_c = Heat transfer coefficient, W/m^2-K

T_g = Temperature of refrigerant, K

T_w = Temperature of wall, K   from overall energy balance (initial guess)

$\dot{Q}$ = Heat transfer rate, W

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Compressor Heat Transfer Models

- The heat transfer coefficient determines the resistance to heat transfer based on characteristics of the flow and geometry
- In general, for forced convection,

$Nu = a Re^b Pr^c$

$$h_c = a\frac{k}{D_h}\mathrm{Re}^b\,\mathrm{Pr}^c$$

» k = Thermal conductivity of the fluid, W/m-k

» D_h = Hydraulic diameter of flow path, m

» Re = Reynolds number

» Pr = Prandtl number

Compressor Heat Transfer Models

- The Reynolds number is based on fluid properties, the flow speed, and the channel diameter (ratio of inertial and viscous forces):

$$\mathrm{Re} = \frac{\rho u D_h}{\mu}$$

D_h = Average hydraulic diameter of chamber, m
u = Average velocity of gas in chamber, m/s
μ = Dynamic viscosity, Pa-s
ρ = Density, kg/m^3

- The Prandtl number only depends on fluid properties (ratio of momentum and thermal diffusivity):

$$\mathrm{Pr} = \frac{\mu C_p}{k}$$

C_p = Constant pressure specific heat, J/kg-K
k = Thermal conductivity, W/m-K
μ = Dynamic viscosity, Pa-s

Compressor Heat Transfer Models

- The exact correlation for heat transfer coefficient depends on the compressor type. Examples:
  - » Reciprocating

$$h_c = 0.053 \frac{k}{D_h} \mathrm{Re}_L^{0.8} \mathrm{Pr}^{0.6}$$

(Adair et al. 1972)

  - » Scroll (spiral HX correlation)

$$h_c = 0.023 \frac{k}{D_h} \mathrm{Re}^{0.8} \mathrm{Pr}^{0.4} \left[1.0 + 1.77 \left(\frac{D_h}{r_{aver}} \right) \right]$$

(Chen et al. 2002) ← radius

Compressor Heat Transfer Models

- For a reciprocating compressor, the heat transfer model involves the following steps:
  - » Use the known density and temperature to calculate the required fluid properties (Pr, k, μ)
  - » Calculate the hydraulic diameter of the flow path,

$$D_h = \frac{4A_c}{P}$$

  - » Estimate the velocity of the gas in the chamber, for example as one-half of the piston speed
  - » Calculate the Reynolds number, $\mathrm{Re} = \frac{\rho u D_h}{\mu}$

estimated or effective values

Compressor Heat Transfer Models

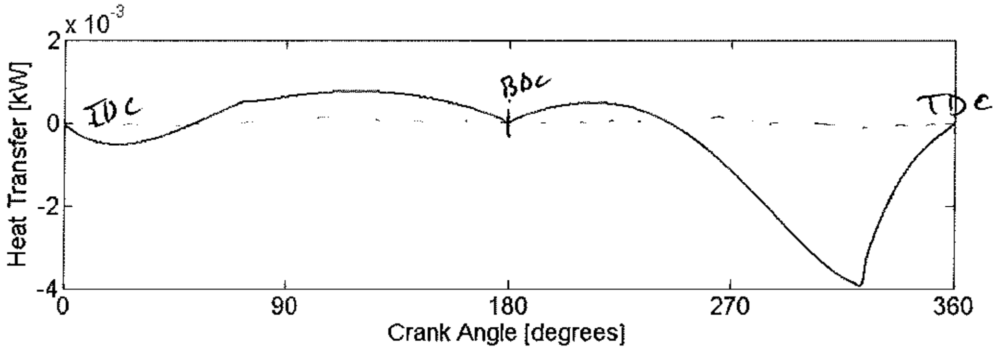
- Reciprocating compressor example continued:
  - » Calculate the convection coefficient using the correlation from Adair et al. (1972)

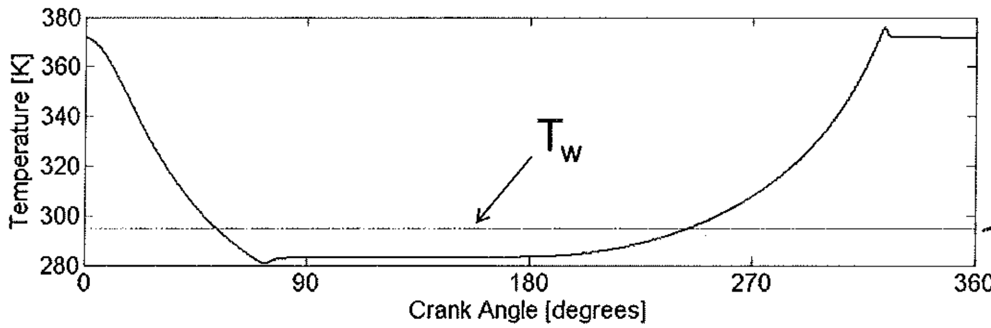
$$h_c = 0.053 \frac{k}{D_h} \mathrm{Re}_L^{\ 0.8} \mathrm{Pr}^{0.6}$$

  - » Use the wall temperature calculated from the overall, lumped heat transfer model (discussed later) or an initial guess for wall temperature to calculate the instantaneous heat transfer rate:

$$\dot{Q} = h_c A \left(T_w - T_g\right)$$

Heat Transfer Results





what is penetration depth + surface amplitude

Compressor Heat Transfer Models

- Reciprocating compressor example continued:
  - » The average heat transfer over one crank rotation can be calculated by numerically integrating the instantaneous heat transfer:

$$Q_{total} = \sum_{\theta=0^\circ}^{360^\circ} \frac{\dot{Q}}{\omega} \Delta\theta$$

  - » This total heat transfer rate will be used in the overall, lumped energy balance to update the temperatures of the lumped elements such as the cylinder walls

Nomenclature

A = Surface area, m^2

C_p = Constant pressure specific heat, J/kg-K

D_h = Hydraulic diameter of flow path, m

h = Specific enthalpy, J/kg

h_c = Heat transfer coefficient, W/m^2-K

k = Thermal conductivity of the fluid, W/m-K

$\dot{m}$ = Mass flow rate, kg/s

m_{cv} = Mass in control volume, kg

Nu = Nusselt number

P = Pressure, Pa

Pr = Prandtl number

$\dot{Q}$ = Heat transfer rate, W

r_{aver} = Average radius of curvature of chamber, m

Nomenclature

Re = Reynolds number

t = Time, seconds

T_g = Temperature of refrigerant, K

T_w = Temperature of wall, K

u = Average velocity of gas in chamber (for Reynolds number), m/s
OR specific internal energy (for energy balance), J/kg

V = Volume, m^3

v = Specific volume, m^3/kg

Greek symbols

θ = Crankshaft angle, degrees

μ = Dynamic viscosity, Pa-s

ω = Rotational speed of crank, degree/second

ρ = Density, kg/m^3

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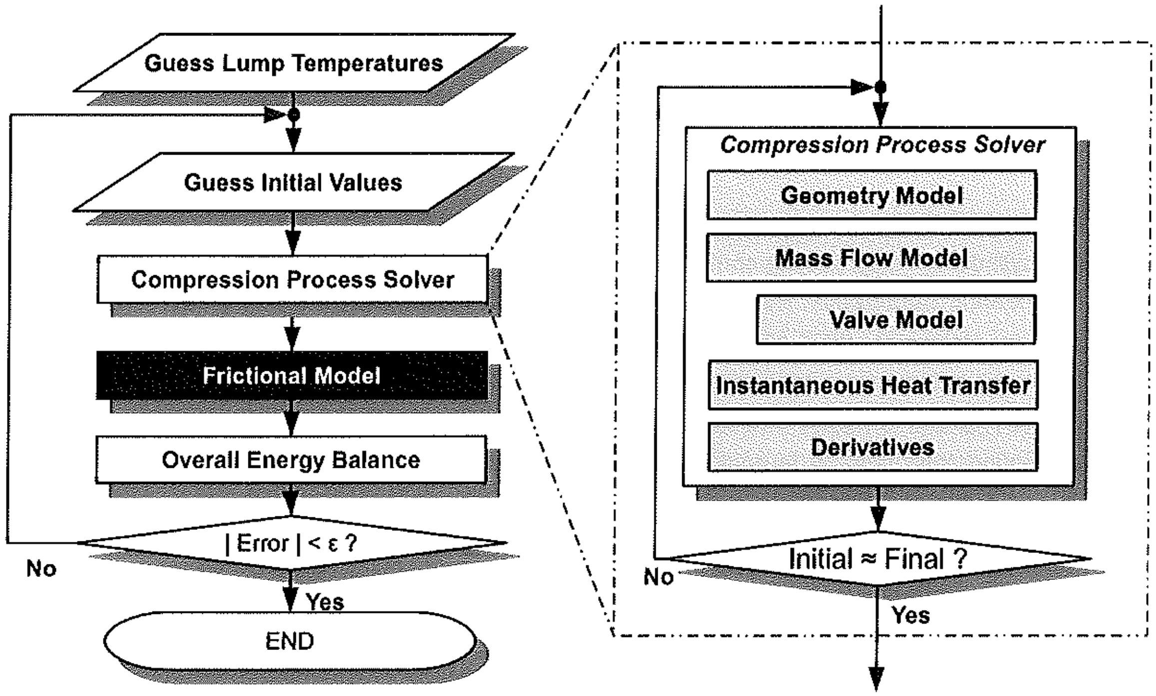
Friction & Mechanical Losses

Frictional Model



Frictional Losses

- Numerous models available, with varying levels of empiricism

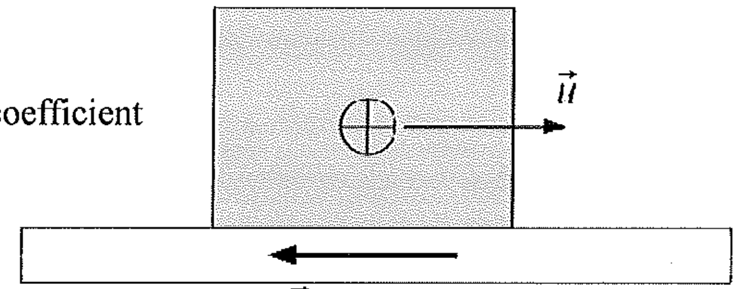
Mechanistic → **Empirical**

Detailed Dynamics and Frictional Model	Correlation of semi-empirical parameters	Correlation Of Mechanical Losses
	$\dot{W}_{ML} = f\left(\tau, \dot{W}_{gas}, \ldots\right)$	$\dot{W}_{ML} = f\left(T_s, T_d, \ldots\right)$

Frictional Losses

- Mechanistic Simple Example

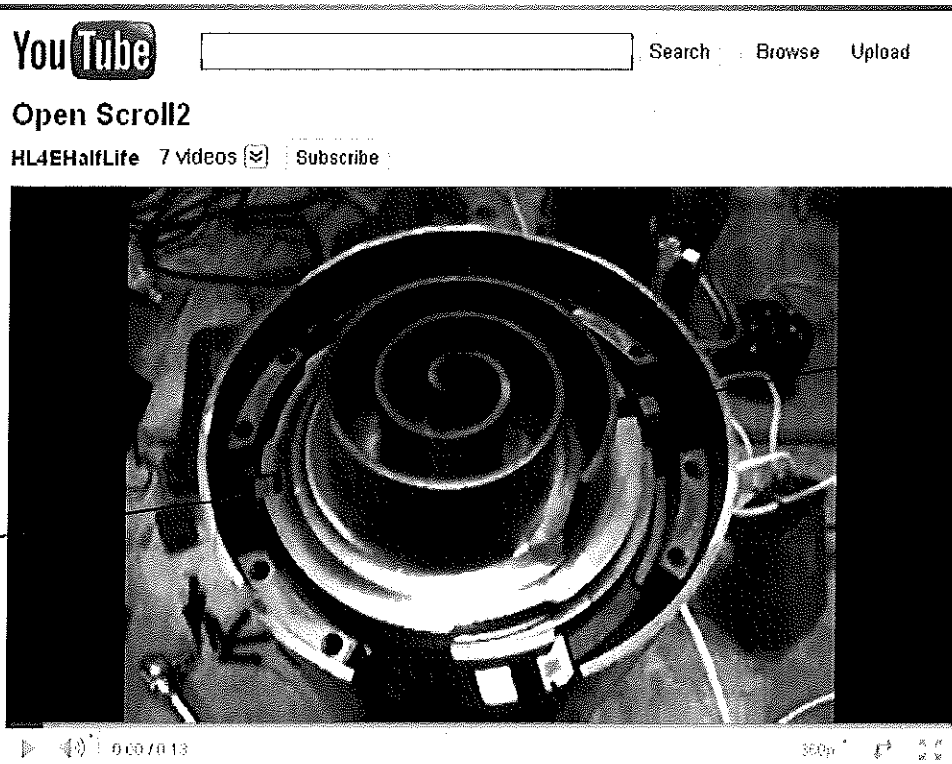
μ - Dry friction coefficient



$$\vec{F} = \mu m g$$

$$\dot{W} = \left|\vec{u} \cdot \vec{F}\right| = \mu u m g$$

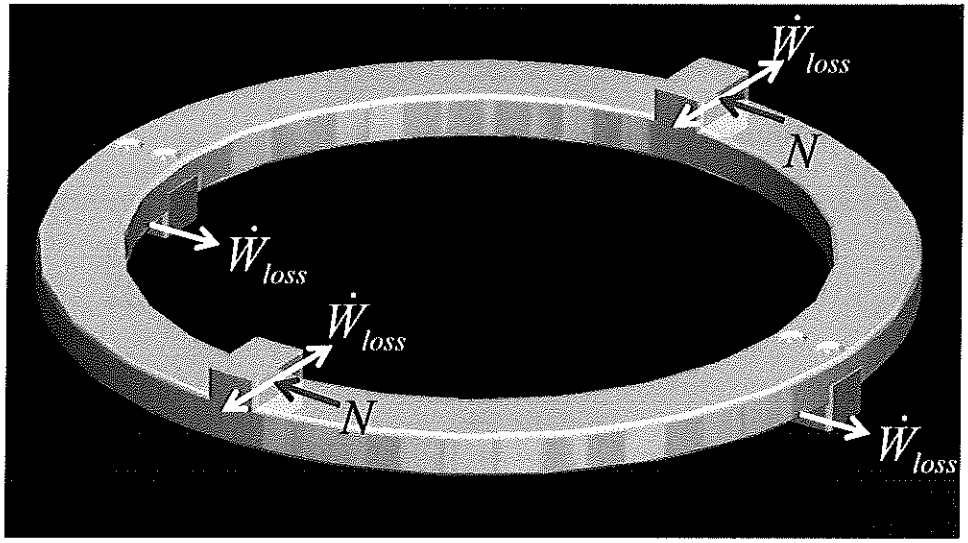
Frictional Losses



Oldham ring key

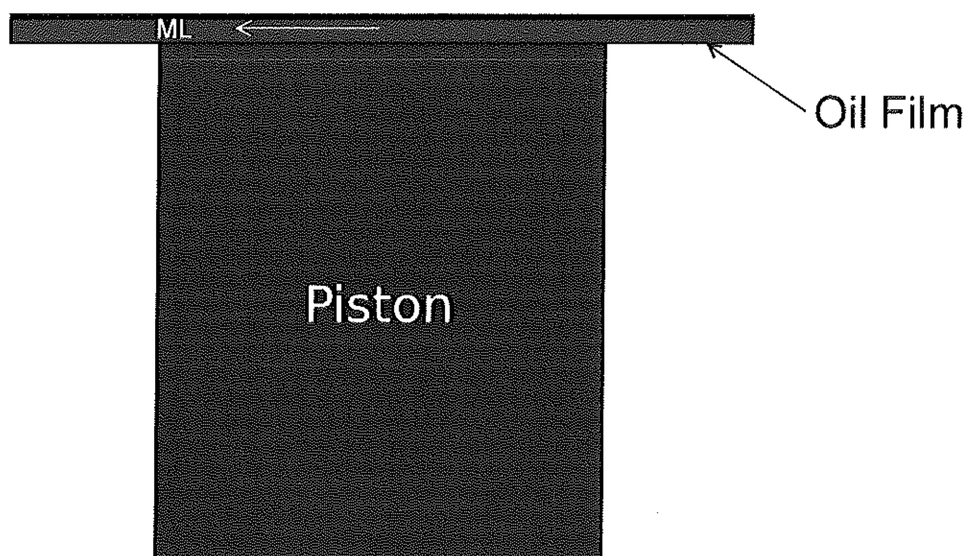
Frictional Losses

- Mechanistic ML Scroll Compressor Oldham Ring



Frictional Losses

Example: Piston – Cylinder Frictional Losses



Frictional Losses

- Example:
  - » Frictional shear between piston and cylinder wall
  - » d = 2 cm (diameter piston)
  - » L = 2 cm (length piston)
  - » x_{max} = 2 cm (piston stroke)
  - » δ = 20 μm (gap width)
  - » η = 0.0086 Pa-s (oil viscosity)
  - » ω = 377 rad/s (60 Hz)
  - » u_{max} = 7.54 m/s ($\omega * x_{max}$)
  - » u_{avg} = 4.8 m/s ($2/\pi * u_{max}$)
  - » Assume Couette flow and "unwrapped" bearing

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Frictional Losses

- Area:
  - »
$$A = \pi DL$$
$$= \pi(0.02\text{ m})(0.02\text{ m}) = 0.001257\text{ m}^2$$
- Shear Force
  - »
$$F = \frac{\eta A u_{avg}}{\delta}$$
$$= \frac{(0.0086\text{ Pa}-\text{s})(0.00126\text{ m}^2)(4.8\text{ m/s})}{20\text{ x }10^{-6}\text{m}} = 2.6\text{ N}$$
- Frictional Power
  - »
$$W = Fu_{avg}$$
$$= (2.6\text{ N})(4.8\text{ m/s}) = 12.48\text{ W}$$

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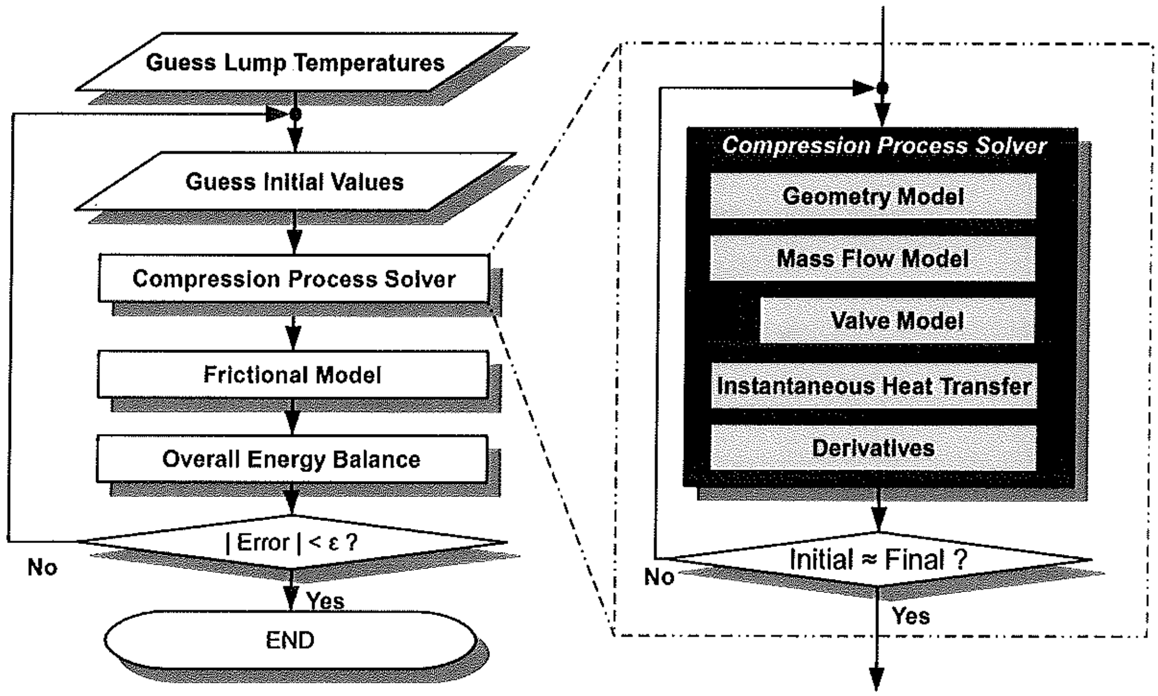
Numerical Methods / Solver

Numerical Methods



Numerical Methods

- Basic Problem:
  - » At step *i* of the revolution we know temperature, pressure, density, etc. (initially guessed)
  - » The derivatives of the two independent properties are then calculated with respect to the crank angle using the information from the sub-models
  - » Using derivatives, find temperature, pressure, density, etc. at the next step
  - » Repeat for the entire revolution
  - » Same method applies to multiple control volumes

Valve submodel can be solve simultaneously

Numerical Methods

- Vector of partial differential equations to integrate for the control volume

$$\mathbf{x} = \begin{bmatrix} T \\ \rho \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} \frac{dT}{d\theta} \\ \frac{d\rho}{d\theta} \end{bmatrix}$$

Numerical Methods

- Simple Euler

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta\theta \mathbf{y}_i$$

step size

or

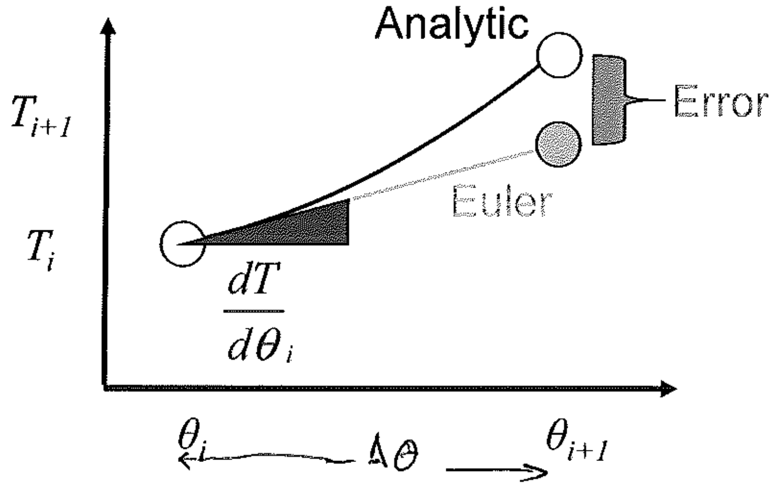
$$T_{i+1} = T_i + \Delta\theta \left.\frac{dT}{d\theta}\right|_i$$

$$\rho_{i+1} = \rho_i + \Delta\theta \left.\frac{d\rho}{d\theta}\right|_i$$

Vector Form **Variable Form**

Numerical Methods

- Simple Euler



Numerical Methods

- Other Potential Solver Methods (Chapra, 2006)
  - » Heun 2nd order predictor-corrector
  - » Runge-Kutta (RK4)
  - » Adaptive ~~Runge~~-Kutta (RK4/RK5)
  - » Implicit Euler
  - » Semi-Implicit Backward Euler
  - » …
  - » Among many others

Numerical Methods

- Runge-Kutta (RK4)

$$\mathbf{k}_1 = \mathbf{f}\left(\theta_n, \mathbf{y}_n\right)$$

$$\mathbf{k}_2 = \mathbf{f}\left(\theta_n + \frac{1}{2}\Delta\theta, \mathbf{y}_n + \frac{1}{2}\mathbf{k}_1\Delta\theta\right)$$

$$\mathbf{k}_3 = \mathbf{f}\left(\theta_n + \frac{1}{2}\Delta\theta, \mathbf{y}_n + \frac{1}{2}\mathbf{k}_2\Delta\theta\right)$$

$$\mathbf{k}_4 = \mathbf{f}\left(\theta_n + \Delta\theta, \mathbf{y}_n + \mathbf{k}_3\Delta\theta\right)$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n + \frac{1}{6}\Delta\theta\left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4\right)$$

Numerical Methods

- Simple Euler v. RK4
  - Simple Euler is 1st order O(h)
  - RK4 is 4th order $O(h^4)$
  - To maintain same local accuracy, need ~40 times fewer steps, but need to call derivative function 6 times per step, overall speedup can be significant
  - Both struggle with numerical stability

is one of big challenge in compressor modeling

Numerical Methods

Recip - stiff region at beginning and end.

- Adaptive Runge-Kutta
  - RK4 with embedded error estimate
  - Take big steps when you can, take baby steps when necessary
  - Chapra (2006) has good coverage and analysis

References

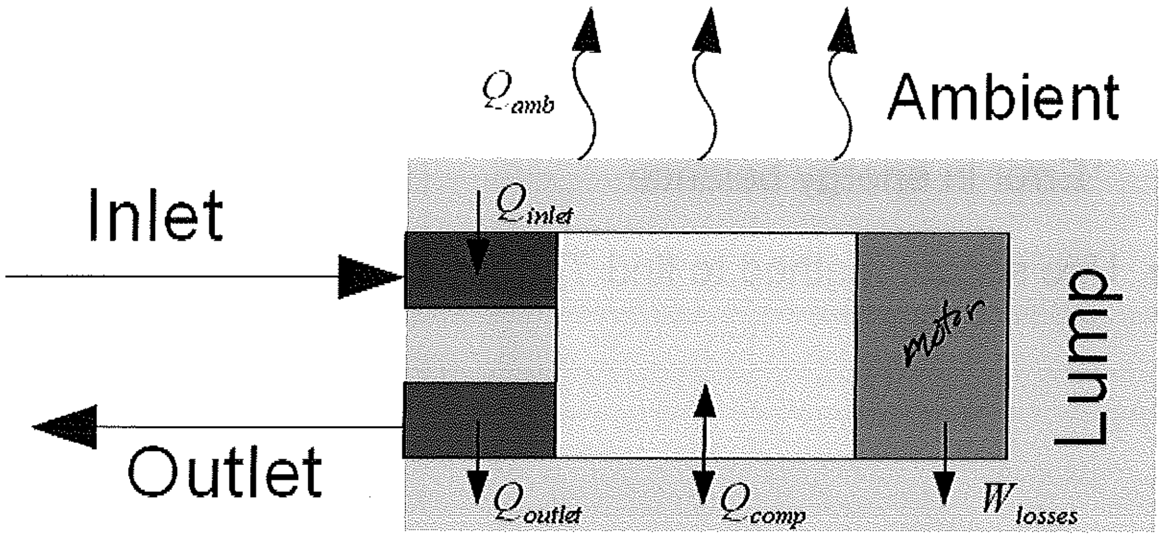
- Chapra, S.C., Canale, R.P., 2006, *Numerical Methods for Engineers – 5th Edition*, McGraw-Hill
  - » Excellent resource for a wide range of numerical methods, including adaptive RK4/RK5
- Kreyszig, E., 2006, *Advanced Engineering Mathematics – 9th Edition*, John Wiley & Sons
  - » Also provides solid background in numerical methods

Overall Energy Balance

Overall Energy Balance

- Basic Idea:
  - » Apply a steady-state energy balance to certain components, or grouped components in a lumped model, to be used in further heat transfer analysis

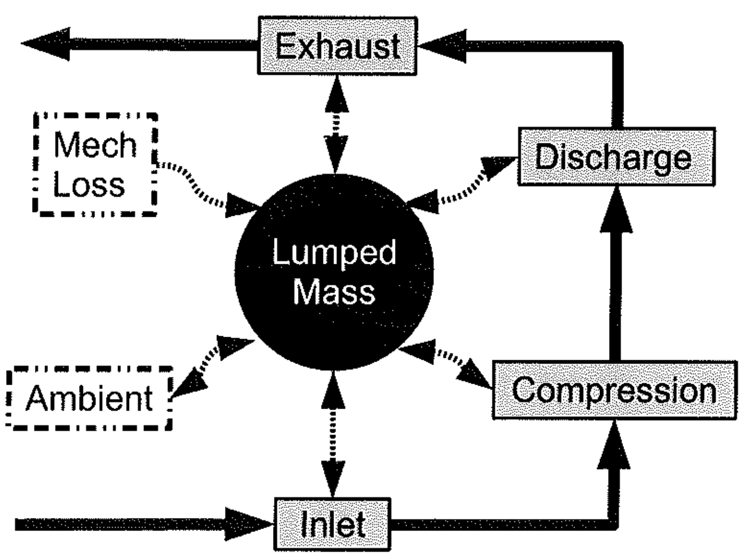
Overall Energy Balance



use newton-Raphson for multiple lumps

Energy Balance

- Single Lump



Energy Balance

- Energy balance for lump

$$r_{EB} = Q_{inlet}(T_{lump}) + Q_{outlet}(T_{lump}) + Q_{comp}(T_{lump}) + Q_{amb}(T_{lump}) + W_{loss}$$

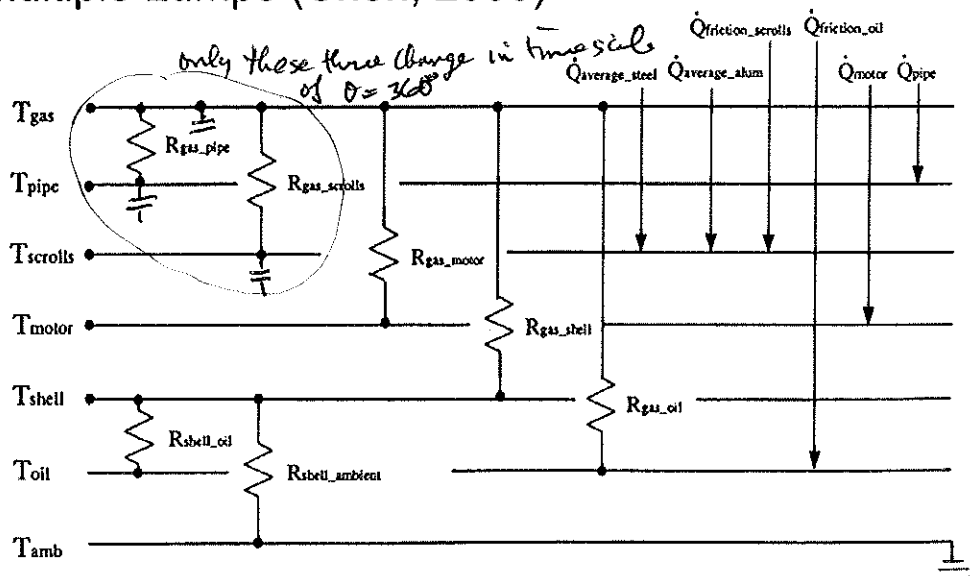
Error in energy balance

- Use secant method to find T_{lump} *by driving* r_{EB} *to zero*

Energy Balance

Q
A1: Wall penetration depth is probably negligible
A2: changing ... over 10:1 has little effect on η_s

- Multiple Lumps (Chen, 2000)



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Conclusion

- Presentation covered the fundamentals of compressor modeling:
  - Discussed the mass and energy balance equations that govern the state of the refrigerant during the compression process
  - Introduced models for leakage (including flow through valves), heat transfer, and mechanical losses that improve the accuracy of the temperature and pressure predictions
  - Explained numerical solution methods for calculating the temperature and pressure variations over a crankshaft rotation
  - Discussed an overall energy balance on the compressor to predict the temperatures of different components that will influence the refrigerant temperature
- Compressor modeling concepts were applied to a simplified reciprocating compressor to demonstrate the modeling process

Conclusion

- For a real compressor design, experimental testing would be required to validate the reciprocating compressor model accuracy
- However, the reciprocating compressor model illustrates that modeling is a relatively fast and cost effective tool for predicting compressor performance
  - Model can be used to study the effect of design modifications and operating conditions on compressor performance
  - Compressor model can be incorporated into a larger system model
- The example model can be modified to fit any compressor design by updating the geometry equations, the leakage paths under consideration, and the mechanical loss model