The Derivation Four Poles

Solving for A_1 and B_1 gives

$$A_1 = \frac{1}{2Sj\omega \sinh \gamma L}\left(Q_o e^{\gamma L} - Q_L\right) \quad (37)$$

$$B_1 = \frac{1}{2Sj\omega \sinh \gamma L}\left(Q_L - Q_o e^{\gamma L}\right) \quad (38)$$

This gives $$P(x) = \frac{\rho_o c_o^2 \gamma}{2Sj\omega Sinh\gamma L}\left(Q_o \cosh \gamma(L-x) - Q_L \cosh \gamma x\right) \quad (39)$$

$$Q(x) = \frac{1}{\sinh \gamma L}\left(Q_o \sinh \gamma(L-x) + Q_L \sinh \gamma x\right) \quad (40)$$

The Derivation Four Poles

Evaluating $P(x)$ at $x=0$ and $x=L$ gives us the boundary values

$$P(0) = P_o = \frac{\rho_o c_o^2 \gamma}{Sj\omega \sinh \gamma L}\left(Q_o \cosh \gamma L - Q_L\right) \quad (41)$$

$$P(L) = P_L = \frac{\rho_o c_o^2 \gamma}{Sj\omega \sinh \gamma L}\left(Q_o - Q_L \cosh \gamma L\right) \quad (42)$$

Rearranging the above equations

$$Q_o = Q_L \cosh \gamma L + P_L \frac{j\omega S}{\rho_o c_o^2 \gamma} \sinh \gamma L \quad (43)$$

$$P_o = Q_L \frac{\rho_o c_o^2 \gamma}{j\omega S} \sinh \gamma L + P_L \cosh \gamma L \quad (44)$$

Eq.43 and 44 relate the conditions at one end of the tube to conditions at the other end, and in matrix for can be written as:

The Derivation Four Poles

$$\begin{Bmatrix} Q_o \\ P_o \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} Q_L \\ P_L \end{Bmatrix} \tag{45}$$

where

$$A = \cosh \gamma L = D$$

$$B = \frac{j\omega S}{\rho_0 c_0^2} \sinh \gamma L$$

$$C = \frac{\rho_0 c_0^2 \gamma}{j\omega S} \sinh \gamma L$$

Eq. 45 is called the four pole description of a uniform tube of length *L*

The Global Four Poles From Local Four Pole Element-Tubes In Series

Tubes In Series:

Four pole of the first tube is

$$\begin{Bmatrix} Q_{01} \\ P_{01} \end{Bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{Bmatrix} Q_{L1} \\ P_{L1} \end{Bmatrix}$$

Four pole of the second tube is

$$\begin{Bmatrix} Q_{02} \\ P_{02} \end{Bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{Bmatrix} Q_{L2} \\ P_{L2} \end{Bmatrix}$$

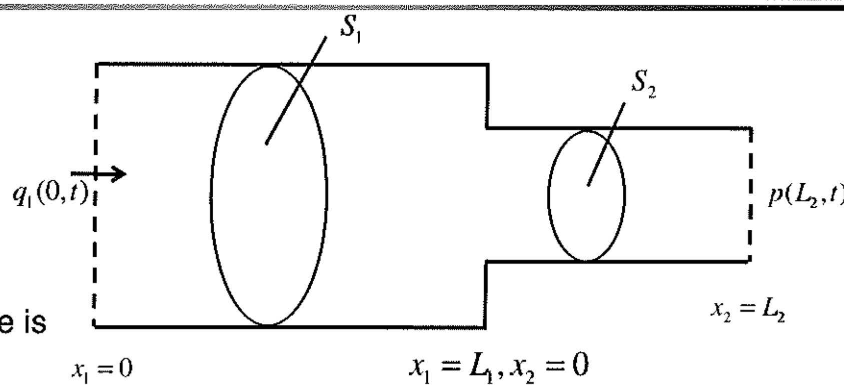

Figure 3: Combination of two tubes in series

At the boundary of the two tubes

$$\begin{Bmatrix} Q_{L1} \\ P_{L2} \end{Bmatrix} = \begin{Bmatrix} Q_{02} \\ P_{02} \end{Bmatrix}$$

The Global Four Poles From Local Four Pole Element-Tubes In Series

Thus, we get

$$\begin{Bmatrix} Q_{01} \\ P_{01} \end{Bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{Bmatrix} Q_{L2} \\ P_{L2} \end{Bmatrix} \qquad (46)$$

The same principle of connecting tubes in series can be extended to *n* tubes

$$\begin{Bmatrix} Q_{01} \\ P_{01} \end{Bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdots \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \begin{Bmatrix} Q_{Ln} \\ P_{Ln} \end{Bmatrix} \qquad (47)$$

The Global Four Poles From Local Four Pole Element-Tubes In Parallel

Branches Tubes:

The four poles for each tube are:

$$\begin{Bmatrix} Q_{01} \\ P_{01} \end{Bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{Bmatrix} Q_{L1} \\ P_{L1} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{02} \\ P_{02} \end{Bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{Bmatrix} Q_{L2} \\ P_{L2} \end{Bmatrix}$$

$$\begin{Bmatrix} Q_{03} \\ P_{03} \end{Bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{Bmatrix} Q_{L3} \\ P_{L3} \end{Bmatrix}$$

At branch junction, we have

$$Q_{L1} = Q_{02} + Q_{03} \qquad (48)$$

$$P_{L1} = P_{02} = P_{03} \qquad (49)$$

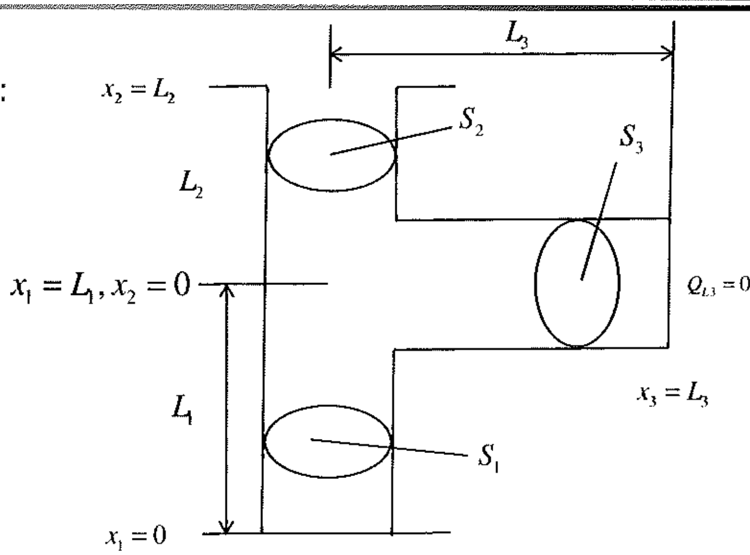

Figure 4: Branched tubes

The Global Four Poles From Local Four Pole Element-Tubes In Parallel

The impedance at point x_3 is defined as:

$$Z_{300} = \frac{P_{03}}{Q_{03}} \quad (49)$$

From branch junction condition we have

$$Q_{L1} = Q_{02} + \frac{P_{02}}{Z_{300}} \quad \text{and,} \quad P_{L1} = P_{02} \quad (50\ a\ \&\ b)$$

In matrix form we have,

$$\begin{Bmatrix} Q_{L1} \\ P_{L1} \end{Bmatrix} = \begin{bmatrix} A_1 & \frac{1}{Z_{300}} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} Q_{02} \\ P_{02} \end{Bmatrix} \quad (51)$$

$$\begin{Bmatrix} Q_{01} \\ P_{01} \end{Bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_1 & \frac{1}{Z_{300}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{Bmatrix} Q_{L2} \\ P_{L2} \end{Bmatrix} \quad (52)$$

where $P_{L2} = 0$ and Q_{01} is the input

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The Global Four Poles From Local Four Pole Element-Tubes In Parallel

The two unknown are P_{01} and Q_{L2}.

We get the impedance Z_{300} from

$$Z_{300} = \frac{P_{03}}{Q_{03}} = \frac{C_3 Q_{L3} + D_3 P_{L3}}{A_3 Q_{L3} + B_3 P_{L3}} \quad (53)$$

In this example $Q_{L3}=0$

Therefore,

$$Z_{300} = \frac{D_3}{B_3} \quad (54)$$

Example of the General Formulation of Four Pole Parameters

The derivation of the general expression is shown in the Appendix -I

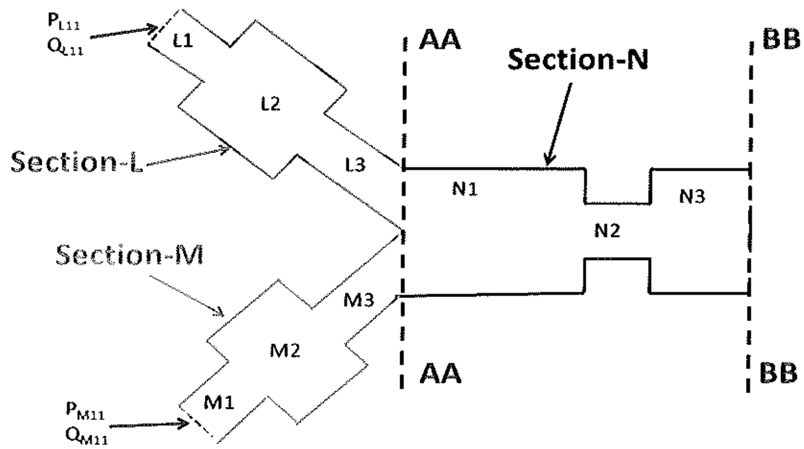


Figure 5: Schematic of a multi-cylinder compressor with each section consisting of several tubes connected in series.

The final expressions for the pressures are shown below:

$$P_{L11} = -\frac{C_L}{A_M} Q_{M11} + \left(C_L + D_L Z_{N11} + \frac{C_L}{A_M} B_M Z_{N11} \right) \left(\frac{A_M Q_{L11} + A_L Q_{M11}}{A_M A_L + A_M B_L Z_{N11} + A_L B_M Z_{N11}} \right) \qquad (55)$$

$$P_{M11} = -\frac{C_M}{A_L} Q_{L11} + \left(C_M + D_M Z_{N11} + \frac{C_M}{A_L} B_L Z_{N11} \right) \left(\frac{A_L Q_{M11} + A_M Q_{L11}}{A_M A_L + A_L B_M Z_{N11} + A_M B_L Z_{N11}} \right) \qquad (56)$$

Q_{L11}=mass flow rate at location L_{11} (given)
Q_{M11}=mass flow rate at location M_{11} (given)

$$Z_{N11} = \frac{P_{N11}}{Q_{N11}} = \frac{C_N}{A_N}$$

Implementation

Case (a): A case of simple circular tube

Note:

- Four pole parameters is a frequency domain technique and so,
- time domain data has to be converted to the frequency domain
- after calculating pressure pulsation in frequency domain, it is converted back to time domain for any further manipulations.

Simulation data: Refrigerant R410a

Length of tube=L=10 inches

Diameter of tube=d =.75 inches

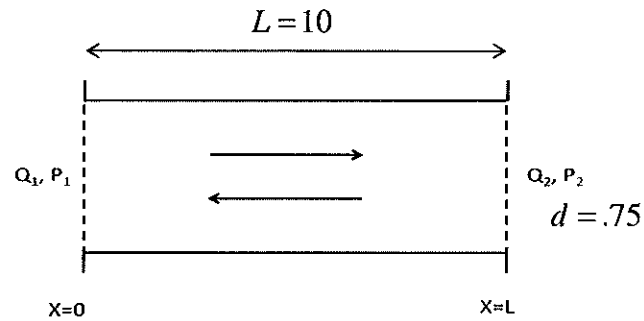
Compressor speed =N=3543.3 rpm

Fluid density =rho =2.41 lbm/ft^3

Speed of sound =c=575 ft/sec

Mean suction pressure =Ps=157 lbf/in^2

Kinematic viscosity =nu =5.12e-4 in^2/sec



Simulation Code: Simple Tube -I

```
% A program to calculate the pressure responses in a circular tube with the
% given mass flow rate vector using four pole parameters
% Author: Dr. Nasir Bilal, Purdue University; bilal@ecn.purdue.edu;nasirbilal1@yahoo.com
% Last Update: 27Jun2012
clear all;  clc; close all
aa=load('TData06Oct09.txt'); % file which contains the data provided
% details of aa matrix:
%1 col=time (sec),
%2nd col=mass flow rate (lbm/sec)
%3rd col=mean suction pressure (psi);
%4th col=nothing;
% 5th col=pressure pulsations calculated from simulation (psi)
rho=2.41; % density (lbm/ft^3): data  change to inches
rho=rho/(12^3)/386.4; % change to inches
c=575; % speed of sound (ft/sec):
c=c*12; % change speed to in/sec.
Ps=156.387; %suction pressure lbf/in^2
N=length(aa)-1; % total no. of data points
dt=aa(6000,1)/N;  % time increment
xi=((0:N-1)*dt)'; % interpolation vector
Qi=interp1(aa(1:N,1),aa(1:N,2),xi); % interpolated mass flow rate
```

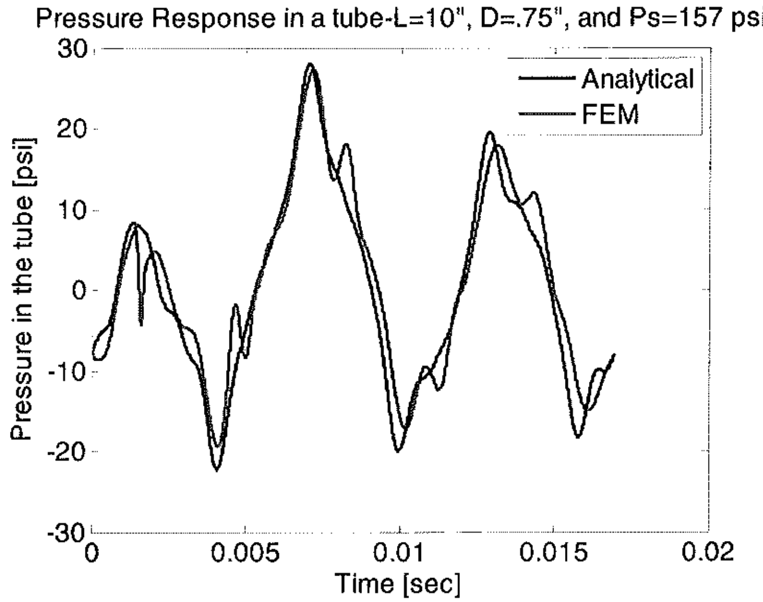
Simulation Code: Simple Tube -II

```
Q1=Qi; % suction mass flow rate
cc=(fft(Q1/386.4)./N/rho); % volume flow rate in the frequency domain
rpm=3543.3; %compressor speed
Period=60/rpm; %Time Period
T=Period; % time period
omega=2*pi/T; % rad/sec
t=((0:N-1)*dt)';
nu=5.12e-4; %kinematic viscosity in^2/s
NN=20; % no. of harmonics to be used for calculations.
 % defining the pressure vector in the in the frequency of interest
Q1w = zeros(1,fix(N/2+1));
Q1w(1:NN+1) = cc(1:NN+1);
P2=0; %absolute pressure: constant pressure boundary condition at the inlet
 % define the dimension of the pipes
L1=10;  % inches
d1=.75; % inches
S1=pi/4*(d1)^2; % area of pipe 1
n=2;
P1w = zeros(1,fix(N/2+1));
```


Simulation Code: Simple Tube -III

```
for om=(1:NN)*omega;
  gamma=0.00257+j*om/c; % gamma value in 4-pole parameters, damping values by trial
  % 4-pole parameters for 1st pipe
  A1(n)=cosh(gamma*L1);
  B1(n)=j*om*S1/rho/c^2/gamma*sinh(gamma*L1);
  C1(n)=rho*c^2*gamma/j/om/S1*sinh(gamma*L1);
  D1(n)=A1(n);
  Mat=[A1(n) B1(n);C1(n) D1(n)];
  %writing in terms of the pressure Po from 4-pole method.
  P1w(n)=(D1(n)-C1(n)./A1(n).*B1(n)).*P2 +C1(n)./A1(n).*Q1w(n);
  n=n+1;
 end
Pt = ifft([P1w conj(P1w(end-1:-1:2))])*N;
figure(1) % plot the pressure response;
plot(t,-real(Pt),'k'); hold on; % plot the pressure response.
plot(aa(1:length(Pt)),aa(1:length(Pt),5)-Ps,'r'); %plot the FEM pressure response
h1=xlabel('Time [sec]');set(h1,'FontSize',14)
h2=ylabel('Pressure in the tube [psi]');set(h2,'FontSize',14)
set(gca,'FontSize',14);
h=get(gca,'Children'); set(h,'LineWidth',[2.0]);
h3=title('Pressure Response in a tube-L=10", D=.75", and Ps=157 psi'); set(h3,'FontSize',14)
h4=legend('Analytical','FEM');set(h4,'FontSize',14)
```

Results-Simple Tube (L=10";D=.75")



Tubes In Series

Simulation data: Refrigerant R410a

Length of tube section $L_1 = L_1 = 5$ inches

Length of tube section $L_2 = L_2 = 5$ inches

Diameter of tube=d =.75 inches

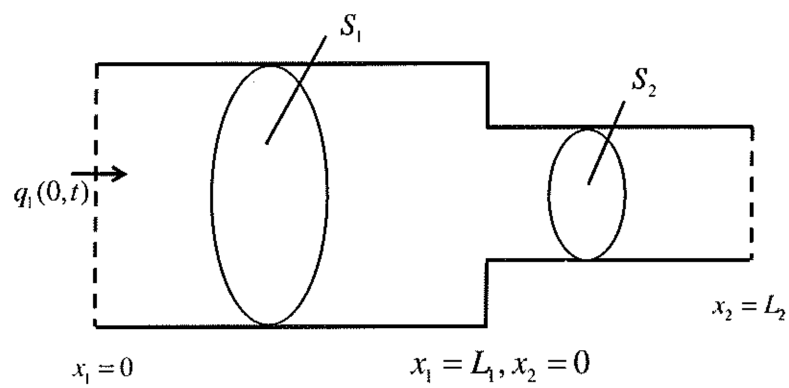
Compressor speed =N=3543.3 rpm

Fluid density =rho =2.41 lbm/ft^3

Speed of sound =c=575 ft/sec

Mean suction pressure =Ps=157 lbf/in^2

Kinematic viscosity =nu =5.12e-4 in^2/sec



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Simulation Code: Tubes Connected in Series -I

```
% A program to calculate the pressure respones in a circular tubes connected
% in  with the given mass flow rate vector and using four pole parameters
% Author: Dr. Nasir Bilal
% Last Updated: 27Jun2012
clear all;  clc; close all
aa=load('TData06Oct09.txt'); % file which contains the data provided
% details of aa matrix:
%1 col=time (sec).
%2nd col=mass flow rate (lbm/sec)
%3rd col=mean suction pressure (psi);
%4th col=nothing;
% 5th col=pressure pulsations calculated from simulation (psi)
rho=2.41; % density (lbm/ft^3): data  change to inches
rho=rho/(12^3)/386.4; % change to inches
c=575; % speed of sound (ft/sec);
c=c*12; % change speed to in/sec.
Ps=156.387; %suction pressure lbf/in^2
N=length(aa)-1;
dt=aa(6000,1)/N;  % time increment
xi=((0:N-1)*dt)'; % defining the interpolation vector
Qi=interp1(aa(1:N,1),aa(1:N,2),xi); % interpolated mass flow rate vector
```


Simulation Code: Tubes Connected in Series -II

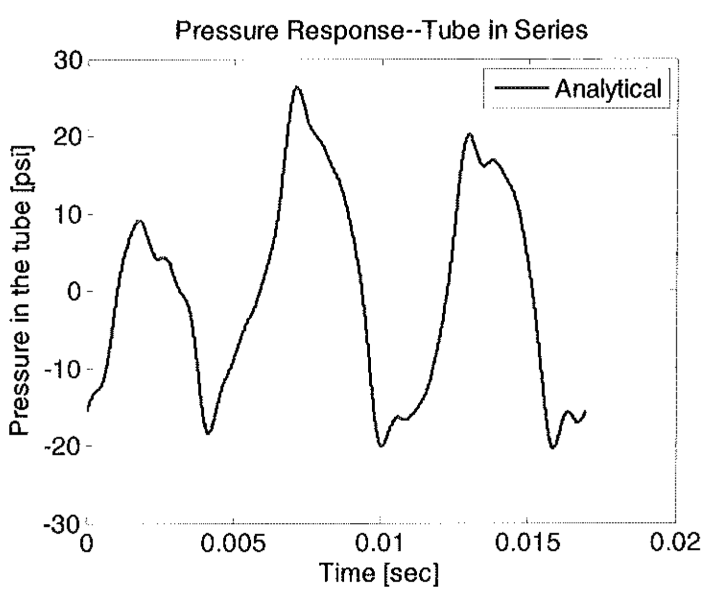
```
Q1=Qi;
cc=(fft(Q1/386.4)./N/rho); % volume flow rate in the frequency domain
rpm=3543.3;  %compressor speed
Period=60/rpm; %Time Period
T=Period; % time period
omega=2*pi/T; % rad/sec  ω
t=((0:N-1)*dt)'; %defining vector
nu=5.12e-4;  %kinematic viscosity provided by Trane. in^2/s
NN=20; % no. of harmonics to be used for calculations.
Q1w = zeros(1,fix(N/2+1));
Q1w(1:NN+1) = cc(1:NN+1);
P2=0;
% Define the dimension of the pipes
% ++++++++++++++Add the number of tube lengths (in LLL) and dia (in dl) in the lines below++++++++++++++++
% LLL=[L1 L2]; % d1=[d1 d2]; Format for lenghts and diameters of the tubes vectors.
LLL=[5 5]; %lengths of tubes in section--LL
dl=[.75 .75] ; %dia of the tube
SLL=pi/4*[dl ].^2; % area of tubes
n=2; %simulation variable
P1w = zeros(1,fix(N/2+1));        pressure vectors
```

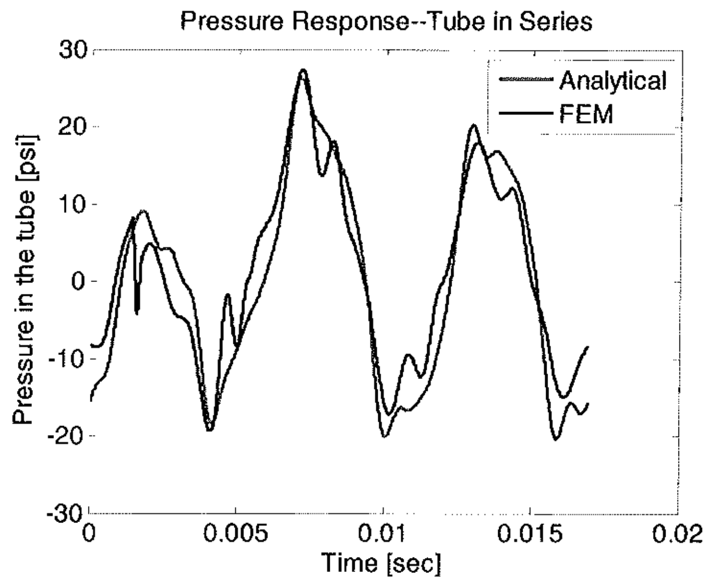
Simulation Code: Tubes Connected in Series -III

```
for om=(1:NN)*omega;
r1=2*rho/.75*sqrt(2*nu*n*omega);
   gamma1=r1/rho;
   a=gamma1/2/c; % real part or the damping term in the gamma eqn. see book Prof. Soedel pp 230.
   gamma=a+j*om/c; % gamma value in 4-pole parameters.For initial estimate ignore the real part/the damping.
   Mat=eye(2,2);
   for nn=1:length(LLL)
      Mat=Mat*[cosh(gamma*LLL(nn)) j*om*SLL(nn)/rho/c^2/gamma*sinh(gamma*LLL(nn));
rho*c^2*gamma/j/om/SLL(nn)*sinh(gamma*LLL(nn)) cosh(gamma*LLL(nn))];
   end
  P1w(n)=(Mat(2,2)-Mat(2,1)./Mat(1,1).*Mat(1,2)).*P2 +Mat(2,1)./Mat(1,1).*Q1w(n);%writing in terms of the pressure Po
from 4-pole method.
   n=n+1;
end
Pt = ifft([P1w conj(P1w(end-1:-1:2))])*N; %setup the symmetric vector for ifft
figure,  % plot the pressure response;
plot(t,-real(Pt),'r'); hold on; %plot the pressure response
plot(aa(1:length(Pt)),aa(1:length(Pt),5)-Ps,'k'); %plot the FEM pressure response with mean pressure subtracted.
h1=xlabel('Time [sec]');set(h1,'FontSize',14)
h2=ylabel('Pressure in the tube [psi]');set(h2,'FontSize',14)
set(gca,'FontSize',14);
h=get(gca,'Children'); set(h,'LineWidth',[2.0]);
h3=title('Pressure Response--Tube in Series'); set(h3,'FontSize',14)
h4=legend('Analytical','FEM');set(h4,'FontSize',14)
```

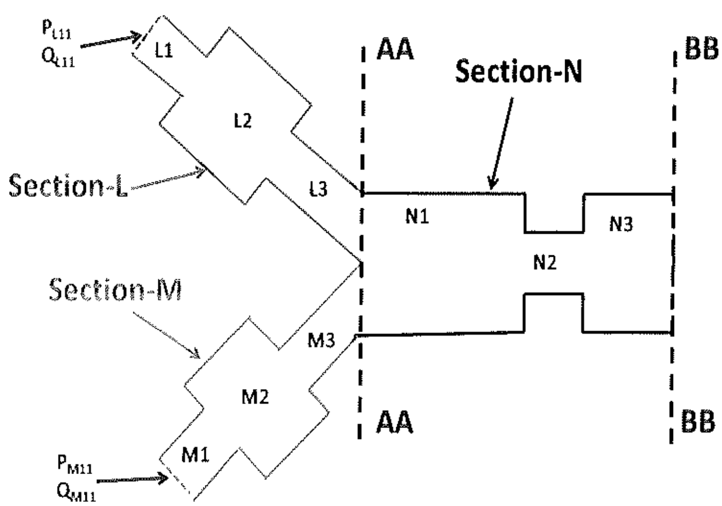
Pressure response

inverse fourier transform

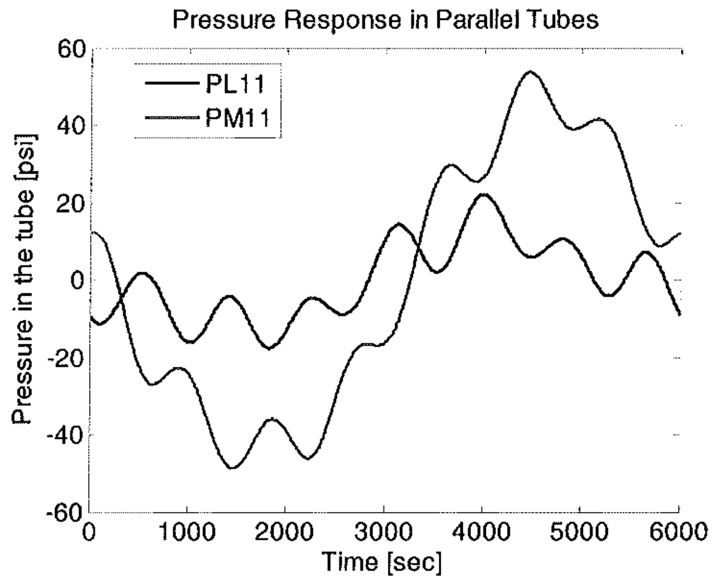




Simulation Code: Tubes Connected in Series and Parallel



Simulation Code: Tubes Connected in Series and Parallel -I



Simulation Code: Tubes Connected in Series and Parallel -I

```
% A program to calculate the pressure respones in a circular tube with  two
% tube in parallel having different multiple input of mass flow rate.
% there are multiple pipes in each leg of the pipes and there derivation
% is explained in the separate work.
% Here the Boundary condition at the far end is assumed to by anechoic
% termination.
% For configure/drawing of the tubes see the attached pdf file
% Author:Nasir Bilal %27Jun2012
clear all;  clc; close all
aa=load('TData06Oct09.txt'); % file which contains the data provided
% details of aa matrix:
%1 col=time (sec).
%2nd col=mass flow rate (lbm/sec)
%3rd col=mean suction pressure (psi);
%4th col=nothing;
% 5th col=pressure pulsations calculated from simulation (psi)
rho=2.41; % density (lbm/ft^3): data  change to inches
rho=rho/(12^3)/386.4; % change to inches
c=575; % speed of sound (ft/sec):
c=c*12; % change speed to in/sec.
nu=5.12e-4;  %kinematic viscosity provided by Trane. in^2/s
Ps=156.387; % mean suction pressure lbf/in^2
```

```
P2=0; % boundary condition at the free end
rpm=3543.3; %compressor speed
Period=60/rpm; %Time Period
T=Period; % time period
omega=2*pi/T; % rad/sec
N=length(aa)-1;
NN=10; % no. of harmonics to be used for calculations.
dt=aa(6000,1)/N; % time increment
t=((0:N-1)*dt)'; %defining vector
xi=((0:N-1)*dt)'; % defining the interpolation vector
Qi=interp1(aa(1:N,1),aa(1:N,2),xi); % interpolated mass flow rate vector
Q1=Qi;
cc=(fft(Q1/386.4)./N/rho); % volume flow rate in the frequency domain
delay=(0:N/2)'; %defining a vector for time delay
cc1=cc(1:N/2+1,1).*exp(-j*delay*omega*T/36*0); %define a time delay at inc of 10 deg. 1st one has no delay;
cc2=cc(1:N/2+1,1).*exp(-j*delay*omega*T/36*3); %define a time delay at inc of 10 deg. It has a delay of 30 deg.
QL11w = zeros(1,fix(N/2+1));
QM11w = zeros(1,fix(N/2+1));
QL11w(1:NN+1) = cc1(1:NN+1);
QM11w(1:NN+1) = cc2(1:NN+1);
Qw=[QL11w;QM11w];
```

$$\mathbb{F}[f(t-t_0)](\omega) = e^{-j\omega t_0} f(\omega)$$

Ref: O' Neil, 2003

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Simulation Code: Tubes Connected in Series and Parallel -I

```
% ++++++++++++++++Add the number of tube lengths (in LLL) and dia (in dl) in the lines below+++++++++++++++++
LLL=[10 10]; %lengths of tubes in section--LL
dl=[.75 .5] ; %dia of the tube
SLL=pi/4*[dl ].^2; % area of tubes
% +++++++++++++++Add the number of tube lengths (in MMM) and dia (in dm) inthe lines below+++++++++++++++++
MMM=[10 10]; %lengths of tubes in section--MM
dm=[.75 .5]; % dia of the tube
SMM=pi/4*[dm ].^2; % area of tubes
% +++++++++++++++Add the number of tube lengths (in NNN) and dia (in dn) inthe lines below++++++++++++++++
NNN=[20]; %lengths of tubes in section--NN
dn=[1.5]; % dia of tube
SNN=pi/4*[1.5].^2; %area of tubes
PL11w = zeros(1,fix(N/2+1));
PM11w = zeros(1,fix(N/2+1));
n=2; %simulation variable
for om=(1:NN)*omega;
    %r1=2*rho/SLL(.75)*sqrt(2*nu*n*omega);
    r1=2*rho/.75*sqrt(2*nu*n*omega);
    gamma1=r1/rho;
    a=gamma1/2/c; % real part or the damping term in the gamma eqn. see book soedel pp 230.
    gamma=a+j*om/c; % gamma value in 4-pole parameters.For initial estimate ignore the real part/the damping.
```

```
ML=eye(2,2);    MM=eye(2,2);    MN=eye(2,2);
for nn=1:length(LLL)
    ML=ML*[cosh(gamma*LLL(nn)) j*om*SLL(nn)/rho/c^2/gamma*sinh(gamma*LLL(nn));
rho*c^2*gamma/j/om/SLL(nn)*sinh(gamma*LLL(nn)) cosh(gamma*LLL(nn))];
end
for nn=1:length(MMM)
    MM=MM*[cosh(gamma*MMM(nn)) j*om*SMM(nn)/rho/c^2/gamma*sinh(gamma*MMM(nn));
rho*c^2*gamma/j/om/SMM(nn)*sinh(gamma*MMM(nn)) cosh(gamma*MMM(nn))];
end
for nn=1:length(NNN)
    MN=MN*[cosh(gamma*NNN(nn)) j*om*SNN(nn)/rho/c^2/gamma*sinh(gamma*NNN(nn));
rho*c^2*gamma/j/om/SNN(nn)*sinh(gamma*NNN(nn)) cosh(gamma*NNN(nn))];
end
  ZN33=rho*c/SNN(length(SNN)); % defining for anechoic termination
  ZN11(n)=(MN(2,1)+MN(2,2)*ZN33)/(MN(1,1)+MN(1,2)*ZN33); % the ZN equation has to be rewritten. See derivation
by hand for explanation.
   %ZN11(n)=MN(2,1)/MN(1,1)
   % calculating the pressure at the valve in section-LL
  PL11w(n)=-ML(2,1)/MM(1,1)*Qw(2,n)+(ML(2,1)+ML(2,2)*ZN11(n)+ML(2,1)/MM(1,1)*MM(1,2)*ZN11(n))*...
     ((MM(1,1)*Qw(1,n)+ML(1,1)*Qw(2,n))/((MM(1,1)*ML(1,1)+MM(1,1)*ML(1,2)*ZN11(n)+ML(1,1)*MM(1,2)*ZN11(n))));
   % calculating the pressure at the valve in section MM
  PM11w(n)=-MM(2,1)/ML(1,1)*Qw(1,n)+(MM(2,1)+MM(2,2)*ZN11(n)+MM(2,1)/ML(1,1)*ML(1,2)*ZN11(n))*...
     ((ML(1,1)*Qw(2,n)+MM(1,1)*Qw(1,n))/(MM(1,1)*ML(1,1)+ML(1,1)*MM(1,2)*ZN11(n)+MM(1,1)*ML(1,2)*ZN11(n)));
  n=n+1;
end
```


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Simulation Code: Tubes Connected in Series and Parallel -I

```
PtL11 = ifft([PL11w conj(PL11w(end-1:-1:2))])*N; %setup the symmetric vector for ifft
PtM11 = ifft([PM11w conj(PM11w(end-1:-1:2))])*N; %setup the symmetric vector for ifft
figure, % plot the pressure response;
plot(-real(PtL11),'k'); hold on; %plot the pressure response
plot(-real(PtM11),'b'); hold on; %plot the pressure response
 %plot(aa(1:length(Pt)),aa(1:length(Pt),5)-Ps,'r'); %plot the FEM pressure response with mean pressure subtracted.
h1=xlabel('Time [sec]');set(h1,'FontSize',14)
h2=ylabel('Pressure in the tube [psi]');set(h2,'FontSize',14)
set(gca,'FontSize',14);
h=get(gca,'Children'); set(h,'LineWidth',{2.0});
h3=title('Pressure Response in Parallel Tubes'); set(h3,'FontSize',14)
h4=legend('PL11','PM11');set(h4,'FontSize',14)
```

Expression for two pressure locations P_{L11} and P_{M11} are calculated below.

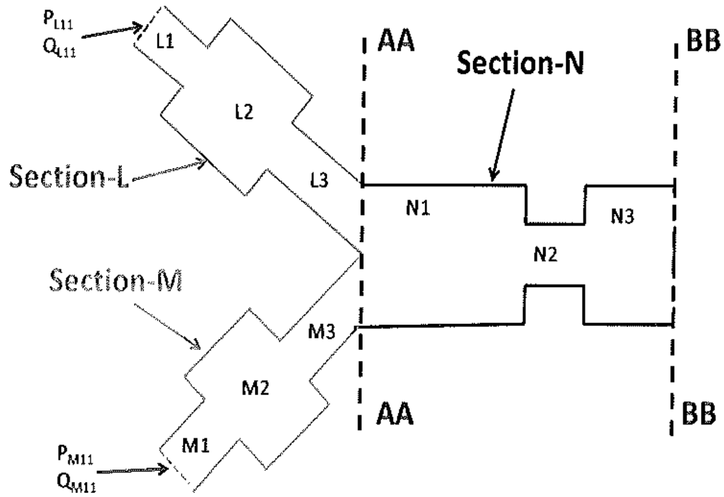

Figure 5.7: Schematic of a multi-cylinder compressor with each section consisting of several tubes connected in series.

A schematic of section L of the tube is used to identify all the variable at various locations along the tube.

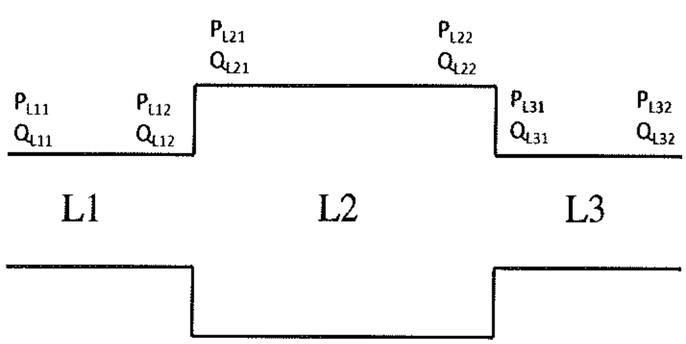

Figure A.2: Section L of tube with pressure and flow variables location

The equation for each section of the tube in terms of four poles is given by

$$\begin{Bmatrix} Q_{L11} \\ P_{L11} \end{Bmatrix} = \begin{bmatrix} A_{L1} & B_{L1} \\ C_{L1} & D_{L1} \end{bmatrix} \begin{Bmatrix} Q_{L12} \\ P_{L12} \end{Bmatrix} \tag{A.1}$$

$$\begin{Bmatrix} Q_{L21} \\ P_{L21} \end{Bmatrix} = \begin{bmatrix} A_{L2} & B_{L2} \\ C_{L2} & D_{L2} \end{bmatrix} \begin{Bmatrix} Q_{L22} \\ P_{L22} \end{Bmatrix} \tag{A.2}$$

$$\begin{Bmatrix} Q_{L31} \\ P_{L31} \end{Bmatrix} = \begin{bmatrix} A_{L3} & B_{L3} \\ C_{L3} & D_{L3} \end{bmatrix} \begin{Bmatrix} Q_{L32} \\ P_{L32} \end{Bmatrix} \tag{A.3}$$

At junction (1), the boundary conditions are:

$$\begin{Bmatrix} Q_{L12} \\ P_{L12} \end{Bmatrix} = \begin{Bmatrix} Q_{L21} \\ P_{L21} \end{Bmatrix} \tag{A.4}$$

At junction (2), the boundary conditions are:

$$\begin{Bmatrix} Q_{L12} \\ P_{L12} \end{Bmatrix} = \begin{Bmatrix} Q_{L21} \\ P_{L21} \end{Bmatrix} \tag{A.5}$$

Therefore,

$$\begin{Bmatrix} Q_{L11} \\ P_{L11} \end{Bmatrix} = \begin{bmatrix} A_{L1} & B_{L1} \\ C_{L1} & D_{L1} \end{bmatrix} \begin{bmatrix} A_{L2} & B_{L2} \\ C_{L2} & D_{L2} \end{bmatrix} \begin{bmatrix} A_{L3} & B_{L3} \\ C_{L3} & D_{L3} \end{bmatrix} \begin{Bmatrix} Q_{L22} \\ P_{L22} \end{Bmatrix} \tag{A.6}$$

$$\begin{Bmatrix} Q_{L11} \\ P_{L11} \end{Bmatrix} = \begin{bmatrix} A_{L} & B_{L} \\ C_{L} & D_{L} \end{bmatrix} \begin{Bmatrix} Q_{L32} \\ P_{L32} \end{Bmatrix} \tag{A.7}$$

where, $$\begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} = \begin{bmatrix} A_{L1} & B_{L1} \\ C_{L1} & D_{L1} \end{bmatrix} \begin{bmatrix} A_{L2} & B_{L2} \\ C_{L2} & D_{L2} \end{bmatrix} \begin{bmatrix} A_{L3} & B_{L3} \\ C_{L3} & D_{L3} \end{bmatrix} \quad \text{(A.8)}$$

Similarly, for section M and N

$$\begin{Bmatrix} Q_{M11} \\ P_{M11} \end{Bmatrix} = \begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix} \begin{Bmatrix} Q_{M32} \\ P_{M32} \end{Bmatrix} \quad \text{(A.9)}$$

$$\begin{Bmatrix} Q_{N11} \\ P_{N11} \end{Bmatrix} = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \begin{Bmatrix} Q_{N32} \\ P_{N32} \end{Bmatrix} \quad \text{(A.10)}$$

The resultant equivalent system for the two tubes after applying four poles is

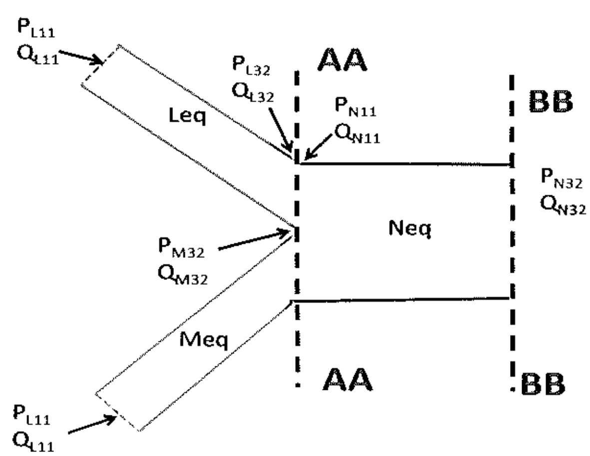

Figure A.3: Equivalent simplified model

The boundary conditions at section AA are

$$P_{N11} = P_{L32} = P_{M32} \qquad \text{(A.11)}$$

$$Q_{N11} = Q_{M32} + Q_{L32} \qquad \text{(A.12)}$$

Two boundary conditions at the far end, i.e. section-BB will be considered

Zero pressure at section-BB, i.e. $P_{N32}=0$

Anechoic termination, i.e. $\frac{P_{N32}}{Q_{N32}} = \frac{\rho c}{s}$

where ρ, c and S are fluid density, speed of sound and cross-sectional area

Case (a): Zero Pressure Boundary Condition

Let $P_{N32}=0$, then Equation (A.10) becomes

$$\begin{Bmatrix} Q_{N11} \\ P_{N11} \end{Bmatrix} = \begin{bmatrix} A_N \\ C_N \end{bmatrix} \{Q_{N32}\} \qquad \text{(A.13)}$$

$$\frac{P_{N11}}{Q_{N11}} = \frac{C_N}{A_N} = Z_{N11} \qquad \text{(A.14)}$$

Sub Eq. (A.11) and (A.12) into (A.7),

$$\begin{Bmatrix} Q_{L11} \\ PL \end{Bmatrix} = \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} \left(\begin{Bmatrix} Q_{N11} \\ P_{N11} \end{Bmatrix} - \begin{Bmatrix} Q_{M32} \\ 0 \end{Bmatrix} \right) \qquad \text{(A.15)}$$

$$\begin{Bmatrix} Q_{L11} \\ P_{L11} \end{Bmatrix} = \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} \begin{bmatrix} 1 \\ Z_{N11} \end{bmatrix} (Q_{N11}) - \begin{bmatrix} A_L \\ C_L \end{bmatrix} (Q_{M32}) \qquad \text{(A.16)}$$

From Eq. A.9 and A.11

$$\begin{Bmatrix} Q_{M11} \\ P_{M11} \end{Bmatrix} = \begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix} \begin{Bmatrix} Q_{M32} \\ Z_{N11} Q_{N11} \end{Bmatrix} \qquad \text{(A.17)}$$

$$Q_{M11} = A_M Q_{M32} + B_M Z_{N11} Q_{N11} \quad \text{(A.18)}$$

$$Q_{M32} = \left(\frac{Q_{M11} - B_M Z_{N11} Q_{N11}}{A_M} \right) \quad \text{(A.19)}$$

sub Q_{M32} in equation (A.16),

$$\begin{Bmatrix} Q_{L11} \\ P_{L11} \end{Bmatrix} = \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} \begin{Bmatrix} 1 \\ Z_{N11} \end{Bmatrix} (Q_{N11}) - \begin{bmatrix} A_L \\ C_L \end{bmatrix} \left(\frac{Q_{M11} - B_M Z_{N11} Q_{N11}}{A_M} \right) \quad \text{(A.20)}$$

Form (A.20a),

$$Q_{L11} = (A_L + B_L Z_{N11}) Q_{N11} - \frac{A_L}{A_M} (Q_{M11} - B_M Z_{N11} Q_{N11}) \quad \text{(A.21)}$$

$$Q_{N11} = \left(\frac{A_M Q_{L11} + A_L Q_{M11}}{A_M A_L + A_M B_L Z_{N11} + A_L B_M Z_{N11}} \right) \quad \text{(A.22)}$$

Sub in equation (A.20b), we get

$$P_{L11} = -\frac{C_L}{A_M} Q_{M11} + \left(C_L + D_L Z_{N11} + \frac{C_L}{A_M} B_M Z_{N11} \right) \left(\frac{A_M Q_{L11} + A_L Q_{M11}}{A_M A_L + A_M B_L Z_{N11} + A_L B_M Z_{N11}} \right) \quad \text{(A.23)}$$

A similar expression for pressure in tube M can be derived and is given by

$$P_{M11} = -\frac{C_M}{A_L} Q_{L11} + \left(C_M + D_M Z_{N11} + \frac{C_M}{A_L} B_L Z_{N11} \right) \left(\frac{A_L Q_{M11} + A_M Q_{L11}}{A_M A_L + A_L B_M Z_{N11} + A_M B_L Z_{N11}} \right) \quad \text{(A.24)}$$

Case (b): Anechoic Termination

For an anechoic termination, the impedance at the far end, i.e. at section BB Fig. (A.3) is given by

$$Z_{N32} = \frac{P_{N32}}{Q_{N32}} = \frac{\rho c}{S_3} \tag{A.25}$$

where ρ, c, S_3 are the density, the speed of sound, and cross-sectional area of tube N_3

$$P_{N32} = Z_{N32} Q_{N32} \qquad \text{sub in Eq. (A.10)}$$

$$\begin{Bmatrix} Q_{N11} \\ P_{N11} \end{Bmatrix} = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \begin{Bmatrix} 1 \\ Z_{N32} \end{Bmatrix} \{Q_{N11}\} \tag{A.26}$$

$$Z_{N11} = \frac{P_{N11}}{Q_{N11}} = \frac{C_N + D_N Z_{N32}}{A_N + B_N Z_{N32}} \tag{A.27}$$

$$P_{N11} = Z_{N11} Q_{N11} \tag{A.28}$$

Using the value of from (A.27) in equation (A.23) and (A.24) will give the pressure at the suction valve with the anechoic termination