ISSN 0701-5232

BUILDING RESEARCH NOTE

DETERMINING THE OPTIMUM THERMAL RESISTANCE
FOR WALLS AND ROOFS

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Ottawa, January 1976

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With energy costs rising rapidly, and a growing awareness by the general public of the need to conserve energy there have been many suggestions that houses should have more insulation. There have been fewer suggestions, however, on how to decide just how much insulation should be used. This note deals with that question.

The approach suggested is to seek the situation that will be most economical for the building owner, and assume that societal concerns about pollution and depletion of nonrenewable resources will enter into consideration via the price of fuel. The main difference between this study and previous ones is that it takes account of inflation and the expected increase in the cost of fuel during the lifetime of a building. Allowing for these effects leads to the conclusion that the optimum amounts of insulation are much greater than are called for in the 1975 edition of the Residential Standards.[1]

The economic analysis used for this study is straight forward: the annual heat loss through unit area of a wall or roof is expressed as a function of the thermal resistance, and the cost of providing this amount of heat for the lifetime of a building is taken as the discounted present worth of the series of annual payments that would be required to purchase this heat. The cost of constructing walls and roofs with different resistance values is estimated and the optimum resistance is taken as the one that has a minimum total cost, i.e., the lowest sum of first cost plus the present worth of the heat that would pass through a wall or roof.

No allowance has been made for the fact that increased amounts of insulation may mean that a smaller and less expensive heating system could be used. Heating equipment only comes in certain sizes so there would be a saving only if the maximum heat

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[1] Residential Standards, Canada, 1975. National Research Council of Canada, Assoc. Com. Nat. Bldg. Code, Ottawa. 190 p., NRCC 13991.

load dropped by enough to permit the use of a smaller unit. In any case the saving connected with using a smaller furnace is quite small in comparison with the value of the fuel that is saved, so neglecting this saving has very little effect on the optimum thermal resistance for walls and roofs.

ESTIMATING ANNUAL HEAT REQUIREMENTS

Heat is lost from a building by conduction through the walls, windows, roof and floor or foundation and by the exfiltration of air. The rate of heat loss is

$$Q = G \cdot (T_i - T_o)$$

where T_i = temperature inside the building

T_o = outside temperature

G = the total heat loss coefficient for the building.

The value of G depends on the area of the different parts of the enclosure divided by its thermal resistance, and on the rate at which the air in the building is replaced by fresh air from outside. Increasing the amount of insulation in walls or ceilings decreases the value of the total heat loss coefficient and thereby reduces the amount of heat lost from the building during the heating season.

Part of the heat loss is made up by the heat that is generated by electric lights and household appliances, particularly the stove, clothes dryer and dishwasher. Some heat is generated by the occupants and a substantial amount enters the house through the windows in the form of sunshine. The difference between the rate of loss and the rate of supply from the various sources mentioned above is supplied by the heating system. Thus,

$$Q = Q^* + Q_f$$

where Q^* = the total rate of heat gain from all sources other than the heating system

Q_f = the heat delivered by the heating system.

Obviously there is no need for any heat from the heating system until Q exceeds Q*, so the heating system is needed only when

$$T_o < T_i - \frac{Q^*}{G}$$

The outside temperature below which the heating system comes into action can be designated as T*, and

$$Q_f = G\ (T^* - T_o).$$

The total heat output required from the heating system during a heating season is

$$\int_{\text{Season}} Q_f dt = G \int_{\text{Season}} (T^* - T_o) dt.$$

The degree-day method of estimating heating requirements involves approximating

$$\int Q_f dt \quad \text{by} \quad 24 \quad GD,$$

where D is the number of degree-days for a complete heating season.

For each day when the mean of the daily maximum and minimum of outside air temperature is below 65°F, the difference between this mean and 65°F is the number of degree-days for that day. The sum of all the daily values during a heating season is the annual total that is used to estimate annual heat requirement. The degree-day method implicitly assumes that T* is always 65°F and that the daily mean value of T_o is the same as the mean of the maximum and minimum temperature during the day. These assumptions may appear to be arbitrary, but it has been found that fuel consumption can be correlated reasonably well with degree-day data that are obtained in this way. Experience has shown, however, that the annual heat requirement for a well insulated house is about 25 per cent less than 24 GD, i.e., that the constant should be about 18 instead of 24.

THE COST OF HEAT

At present most buildings in Canada are heated by burning either oil or natural gas. The price of both these fuels has been rising rapidly and it seems likely that this trend will continue. This development has to be taken into account when assessing the benefit that will result from using insulation.

Forecasting the price of a commodity such as oil is extremely difficult, but an estimate must be made before the optimum amount of insulation can be determined. Figure 1 shows how No. 2 furnace oil and electricity prices may change during the next two decades. It is assumed that the price of electricity will rise at a rate of 12 per cent per year mainly as a consequence of inflation. The other basic assumption is that the price of oil will continue to increase until it catches up to the price of electricity, and that thereafter it will stay at parity with electricity or, if it rises above the price of electricity, that there would be a widespread changeover to electricity for space heating. It is possible, of course, that fuel prices may be controlled by the government rather than by free market forces, but there is no way of predicting what the prices would be nor how long the policy would last. Thus, this study is based on the basic assumption that the price of heating oil will be limited by the competition between it and electricity produced by CANDU-type nuclear power plants.

Figure 1 shows two paths for the price of oil: the higher one is for a rate of increase of 20 per cent per year for the next three years, by which time it will have reached parity with electricity, and that after 1978 it will increase at only 12 per cent per year; the lower one assumes a rate of increase of only 15 per cent per year, which means it will not reach parity with electricity until 1982. It is difficult to say which is the better estimate, but as is shown in the following sections, there is only a very small reduction in the optimum amount of insulation if one uses the lower rather than the higher curve for the price of oil. Thus, it is not very important which of the curves turns out to be the better forecast. The slope of the curve for electricity, which is taken to be the ceiling for the cost of heat, is a much more important parameter: 12 per cent per year is really just a guess but it seems to be a reasonable value for this factor.

Natural gas is currently less expensive than fuel oil, especially in Western Canada. The price of gas will probably keep on rising until it also reaches parity with an alternative fuel or at least until the price is high enough to make it economically feasible to bring gas from the Arctic to southern Canada. For the purposes of this study it has been assumed that the real value of gas as a fuel is the same as oil and thus no distinction is made so far as the optimum amounts of insulation is concerned.

PRESENT WORTH FACTOR

A problem that arises very frequently in investment calculations is how to compare an initial cash investment with the saving that it will produce during the lifetime of a piece of equipment. It cannot be claimed for example, that a \$1000 investment is paid of in 10 years if it reduces operating costs by \$100 per year. At an interest rate of 10 per cent the \$100 saving would just pay the interest, and the investment would never be paid off. How much can one justify investing in order to get a benefit of \$100 per year for a period of 10 years? If it is assumed that the appropriate interest rate is 10 per cent the answer is \$614.45. This is the present value of a series of ten annual payments of \$100 each when money is worth 10 per cent per annum. This can also be looked at in terms of a mortgage: if one borrows \$614.45 at 10 per cent interest it takes ten annual payments of \$100 to pay the interest and repay the principal. The present worth factor, P, is the ratio of the present worth of a series of annual payments to the amount of each annual payment. Thus in the example cited above the present worth factor is \$614.45 ÷ \$100 or 6.1445. The present worth factor has no units as it is the ratio of two quantities that are both expressed in dollars.

The value of P is related to the interest rate, i, and the length of the term, N, by the expression

$$P = \frac{1 - (1 + i)^{-N}}{i}$$

In the case of a fuel saving due to the use of extra insulation the annual payment is the value of the fuel that is saved. The amount of this saving increases as the price of fuel increases. If the price of fuel increases by x per cent per year as a result of inflation and the rise in the real cost of fuel, this can be taken into account by using an effective interest rate, y, in place of the nominal rate i. The effective rate is related to i and x by

$$y = \frac{i - x}{1 + x} .$$

If the nominal interest rate is 10 per cent and the cost of electricity increases by 12 per cent per year, $y = -0.01786$. This gives a present worth factor of $P = 24.30$ when $N = 20$ years. This means that the present worth of any conservation measure that will save \$1.00 worth of electricity in the first year, \$1.12 worth in the second year and so on for 20 years is \$24.30.

The present worth factors given in Table I show the sensitivity of P to the term N and to the effective interest rate y.

Table I Present Worth Factors

y, \ N, years	10	20	30	40
0.02	8.98	16.35	22.40	27.36
0.00	10.00	20.00	30.00	40.00
-0.02	11.19	24.89	41.66	62.18

It is not possible to say exactly what value of x should be assumed, but 12 per cent seems a conservative (i.e., low) guess for electricity. Using that and i = 10 per cent and a term of 30 years gives a value of P for electricity of 40.1. The values for oil have been obtained on this same basis but using the price forecasts given in Figure 1. For the higher rate of price rise the value of P is 48.7; with the lower rate of price increase P is 48.4.

It is perhaps surprising that the P values for oil are larger than for electricity. This is due to the assumption that the price of oil will rise at a higher percentage rate than electricity. It does not follow, however, that one can justify investing a larger amount of capital in order to save a Btu derived from oil than if the heat was derived from electricity. The present worth of an annual saving of one million Btu's of heat is the present worth factor multiplied by the current cost of this unit of energy. Figure 1 shows the 1975 price of one million Btu from electricity and oil as \$3.80 and \$3.10 respectively. Thus the present worth of a measure that would save one million Btu/yr of electricity would be \$3.80 x 40.1 = \$152, and if the heat were obtained by burning oil the present worth of the saving would be \$3.10 x 48.7 = \$151 for the higher oil price curve and about \$1 less for the lower one. Thus, it does not make much difference which oil price pattern is assumed and, in fact, there is not a significant difference between the values for oil and electricity.

OPTIMUM THERMAL RESISTANCE

The optimum amount of insulation for a wall or roof is that at which the saving resulting from adding any more insulation just equals the cost of adding the extra insulation. The cost per square foot of constructing a wall can be expressed as

$$A + B \cdot R$$

where A = a constant that is independent of the amount of insulation in the wall

B = the incremental cost of insulation expressed as ¢/ft^2 per unit of resistance

R = the thermal resistance value for the wall, ft^2 hr °F/Btu.

The heat loss through one square foot of wall during a heating season can be approximated, using the simple degree-day method discussed previously, by 18 D/R Btu/ft^2yr. The cost of providing this amount of heat is 18 CD/ER ¢/ft^2yr, where C is the cost of heat in ¢/Btu, D is the number of degree-days per year and E is the seasonal efficiency of the furnace.

The total payments for fuel during the lifetime of a building can be expressed as an initial lump-sum amount by multiplying the annual cost by a present worth factor P. Thus an "optimum" value of the thermal resistance is the one that makes the sum A + BR + 18 CDP/ER a minimum. This "optimum" resistance is

$$R_{OPT} = \sqrt{\frac{18 \cdot C \cdot D \cdot P}{B \cdot E}}$$

Some typical values for the factors are:

B = 1.3¢/ft^2 per unit of resistance (using glass fibre type insulation)

C = 2.4×10^{-4} ¢/Btu (i.e., 40¢/gal for No. 2 fuel oil)

D = 8000 (Montreal area)

E = 0.75 (For an oil-burning furnace)

P = 40 (to be conservative)

On the basis of these values the optimum resistance is 37.6 units.

The expression for R_{OPT} is based on the assumption that A and B are independent of the value of R. This is generally true for ceilings, where there is plenty of space in the attic to accommodate extra insulation. Thus, the simple formula is appropriate for ceilings under an attic. But for walls it is not valid, as the values indicated by the formula are far in excess of what can be obtained by adding insulation in the stud space of a conventional wood frame wall. So for walls it is necessary to proceed in a somewhat different way to find the optimum value of R.

Figure 2 shows the present worth of the heat that would be lost during the lifetime of a building through a square foot of wall with various values of R for any value of the parameter 18 CDP/E. The slopes of these lines are equal to the reciprocal of the R value, so that the higher the R value the lower the value of the heat that would pass through the wall. To get a true indication of the relative merit of one R value vs. another it is necessary to add to the costs in Fig. 2 the incremental cost of achieving the higher values of R.

Figure 3 shows the comparison of R = 12.5, R = 15 and R = 20 with the assumptions that an R = 15 wall would cost 5¢/ft^2 more to build than an R = 12.5 wall, and that an R = 20 wall would cost 20¢/ft^2 more than an R = 15 wall. These figures have been chosen simply to illustrate the approach, but are approximately correct. On the basis of these assumptions, Fig. 3 indicates that the total life-cycle cost with an R = 15 wall is less than for an R = 12.5 wall wherever the value of 18 CDP/E exceeds 370, and that an R = 20 wall would give a lower life-cycle cost than an R = 15 wall if the climate-cost parameter was greater than 1190. Lines for any other value of R could be added to Fig. 3 to see how it would conpare with these cases.

THE CLIMATE-COST PARAMETER

In order to interpret the results shown in Fig. 3 it is necessary to know the value of the parameter 18 CDP/E for different parts of Canada. For instance, in Montreal the value of D is 8000 and if the heat is obtained by burning No. 2 fuel oil costing 40¢/gal, the cost is 0.24×10^{-3} ¢/Btu. If the furnace is well maintained, the seasonal efficiency might be as much as 0.75. The present worth factor for oil heating would be about 50 if the price of oil follows the path shown on Fig. 1, but P = 40 will be used to keep the results on the conservative side. These data lead to a value of 1840 for the climate-cost parameter for Montreal. Vancouver has only 5515 degree days so if the other factors were the same the climate-cost parameter there would be 1275. Thus Fig. 3 shows that in Vancouver an R = 20 wall would have a slightly lower life-cycle cost than an R = 15 wall, and that in the colder climate of Montreal there would be a definite advantage in using the R = 20 wall. In fact, it might be advantageous in Montreal to use a wall with an R of more than 20. This would depend on the extra cost involved in constructing a wall with more resistance.

CONCLUSION

There are two key pieces of data that have to be established before the optimum heat resistance of a wall or roof can be determined:

1) The incremental cost of constructing walls or roofs with various values of R in excess of 12.5;

2) The value of the present worth factor that should be used for converting future fuel savings into a current lump-sum value.

On the basis of the data assumed in this study it appears that an R = 20 wall would be appropriate for most parts of Canada.

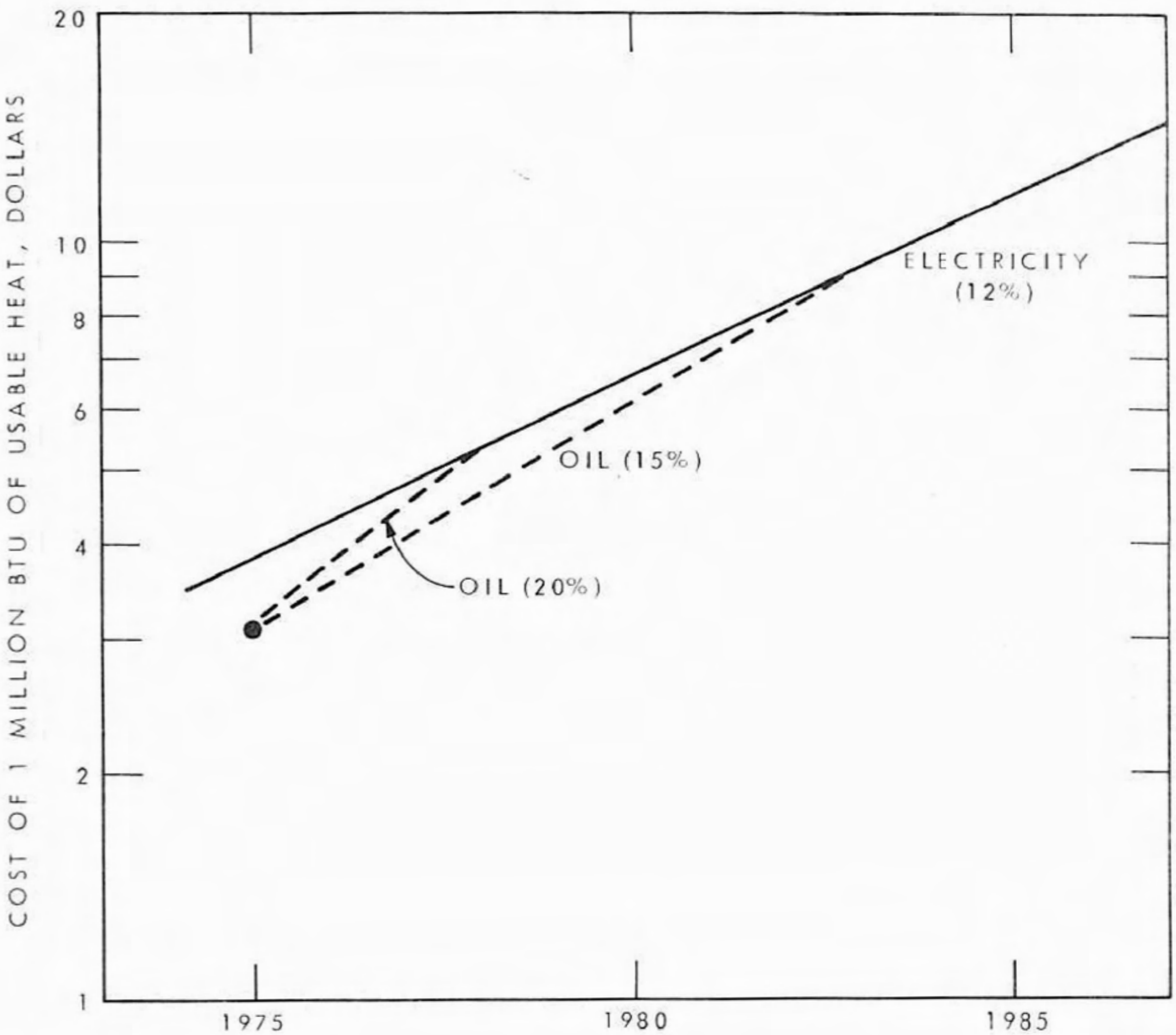

FIGURE 1 PROJECTED COST OF HEAT FROM OIL AND ELECTRICITY

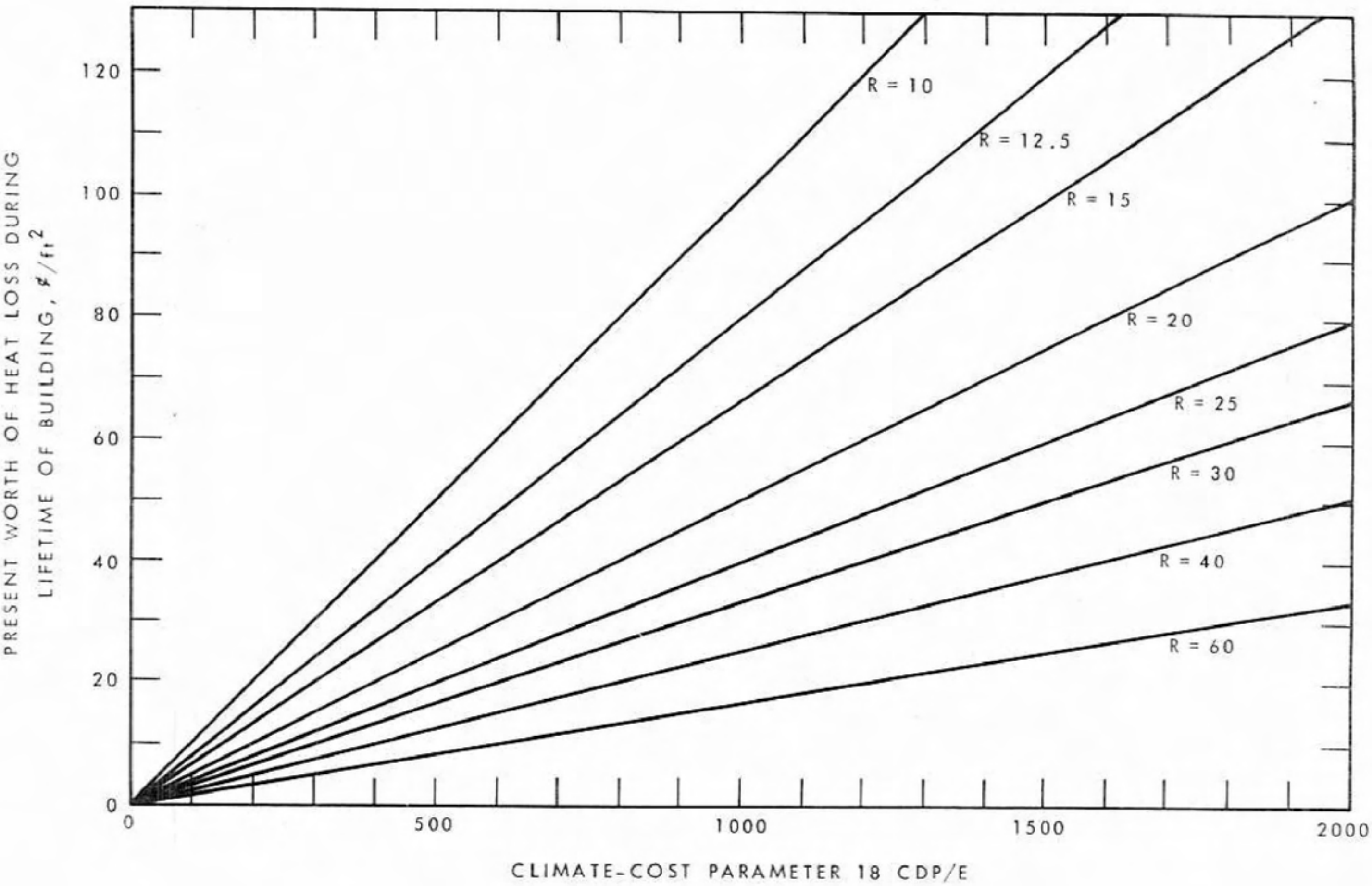

FIGURE 2 PRESENT WORTH OF HEAT LOST DURING LIFETIME OF BUILDING THROUGH 1 SQ FT OF WALL

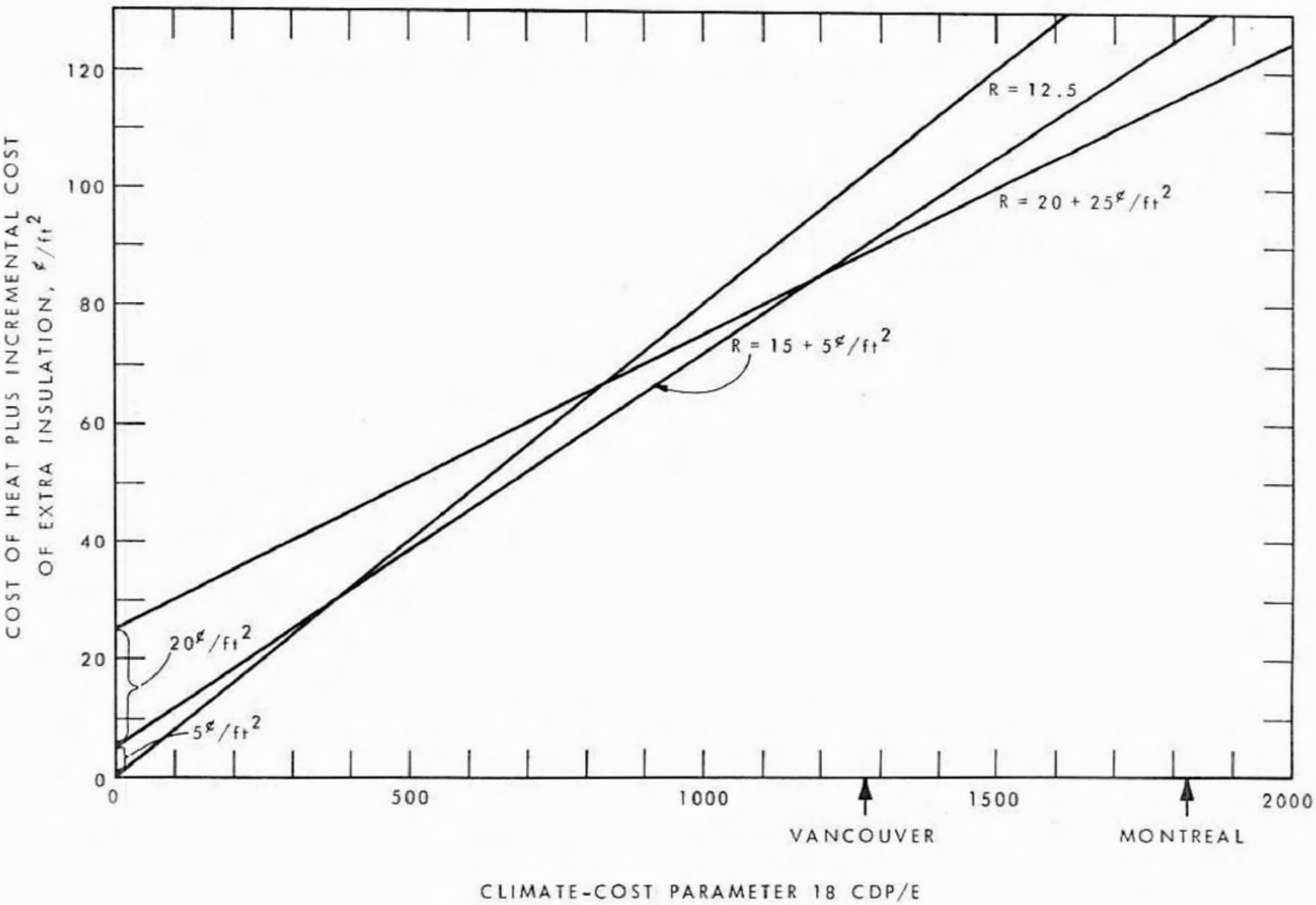

FIGURE 3 LIFE-CYCLE COST COMPARISON OF R = 12.5, R = 15, AND R = 20 WALLS