



CANADA

# Building Research Note

ANALYZED

C.S.T.B. SOLAR DIAGRAMS

by

D.G. Stephenson



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Knowledge of the apparent position of the sun at different times and dates is essential for the design of shading systems and the determination of heat gain through windows. Tables of solar altitude and azimuth angles are available (1,2), but they are less convenient for many purposes than a graphical presentation of the same data.

Nearly all the solar charts or graphs that have been published are in polar form, i.e. a vector gives solar azimuth by its direction and altitude by its length. Some charts make the length directly proportional to the altitude angle and others make the length proportional to the cotangent of the altitude angle. The Diagrammes Solaires prepared by the Centre Scientifique et Technique du Bâtiment in France are of this latter type. Each chart presents the solar position data for all times and dates for one specific latitude.

The C.S.T.B. charts can be thought of as the dial for a horizontal sun clock that has a gnomon of length  $d$  standing vertically at the principal point O. Figure 1 is a half-scale reproduction of the chart for a latitude of 48 degrees. As a sun-clock dial the chart has to be oriented so that the XY line is exactly north and south, with the X arrow pointing toward the equator. The shadow of the point of the gnomon moves along a path like those labelled Courbe A to G. These are the shadow paths for different dates; at north latitudes A is for 21 June, B for 21 May and 21 July and so on to G, which is for 21 December. These curves are referred to as the date lines. The straight lines that intersect the date lines go through points on them where the shadow of the tip of the gnomon would be at a particular time of day. For example, the intersection of the line marked 10 with the curve A indicates the position of the shadow tip at 10 a.m. on 21 June. The slightly darker lines are for the hours indicated by the numbers adjacent to them and the lighter lines are for every 10 minutes between the hours.

Whenever time is mentioned in this note, local apparent time (L.A.T.) is meant. This is the time indicated by the shadow on a sun dial. It can differ from standard time by almost an hour for places near the boundaries between time zones. The correction of standard time to L.A.T. is described in BRN 47.

The C.S.T.B. has published, in English as well as in French, a concise note on the main uses of their charts, but there are two applications not mentioned. Both derive from the cotangent type of plot used in the charts and represent part of the advantage of this type of chart over the direct altitude-angle type. This note is, therefore, intended to supplement the C.S.T.B.

brochure by giving additional uses.

### Positioning a Model

The shadows cast by shading devices or by one part of a building onto another can be determined quite easily by using a model. It is only necessary to position it in relation to a beam of light in the same way as the real building will sit with respect to the sun. One method of positioning a model is by using a heliodon (BRN 47), but the C.S.T.B. charts can also be used for this purpose.

The model is attached to a drawing board and the C.S.T.B. chart for the appropriate latitude placed beside it on the board. The chart and model are so arranged that the building has the correct orientation relative to the North-South axis of the chart. A small cone of height equal to the distance  $d$  shown on the chart is located with its apex directly above the principal point  $O$  of the chart. If the model and the chart are set in direct sunshine (or illuminated by any other parallel beam of light), the point of the shadow cast by the cone indicates the time and date when that particular shadow pattern will occur. Alternatively, if the shadows are to be found for any specified time and date, it is only necessary to move the drawing board so as to make the point of the shadow coincide with the intersection of the particular time and date lines. The shadows on and around the model are then as they would be for the real building at that particular time and date.

### Constructing Horizontal and Vertical Shadow Angles

It is often necessary to determine the position of shadows when there is no model available. This can be done quite readily using plan and elevation drawings if the horizontal and vertical shadow angles are available. The H.S.A. is the angle on a plan drawing between a line perpendicular to the wall and the projection of the sun's rays on the horizontal plane. Similarly, the V.S.A. is the angle, on a vertical section drawing of the wall, between a line perpendicular to the wall and the projection of the sun's rays on the plane of the drawing.

The H.S.A. and V.S.A. can be obtained by a simple construction on a sheet of tracing paper laid over the C.S.T.B. chart for the particular latitude. Figure 2 shows an example. The first step is to draw, through point  $O$  a straight line with the same azimuth as the wall in question. This is the "wall line". For this application the arrow,  $X$ , points to the true North and the wall line will cut the angle scale around the perimeter of the chart at the wall azimuth angle. Then the point  $P$  is located at the intersection of the time and date lines on the underlying chart corresponding to the time and date for which the shadow angles are required. A line is drawn from  $P$  perpendicular to the wall line, cutting it at  $Q$ . A line is also drawn from  $P$  through  $O$  and the angle  $OPQ$  is the horizontal shadow angle.  $R$  is located on the wall line so that the distance from  $Q$  to  $R$  is equal to the length of the line labelled "Distance  $d$ " on the upper part of the chart. The angle  $QPR$  is the vertical shadow angle.

### Finding the Position of Shadows on Vertical Walls

The boundaries of the shadows on a wall, whether from a shading device or an adjacent object, can be found by projecting the outline of the shading object on the wall, the lines of projection being parallel to the sun's rays. The shadow angle diagram obtained by the method described above can be used to find the solar projection of a point on the wall. The procedure is shown in Figure 3. Z is the shortest distance from the point A to the wall, i.e., AB is perpendicular to the plane of the wall. The solar projection of point A is at D, which is a distance X measured horizontally and Y measured vertically from B. The distances X and Y are related to the distance Z and the shadow angles by the expressions

$$X = Z \times \text{tangent H.S.A.}$$

$$Y = Z \times \text{tangent V.S.A.}$$

The angle diagram in the lower part of Figure 3 shows how X and Y can be determined directly. The distance Z is laid off from P along PQ and X and Y measured as shown.

### Insolation on a Vertical Surface

The insolation (i.e. incident solar energy), both direct and diffuse, on any wall can be found by using one of the overlay charts in conjunction with the appropriate solar diagram. The values indicated on the overlay sheets are in Watts/sq metre; these can be converted to Btu/ft<sup>2</sup>hr by multiplying them by 0.317.

The overlay sheets No. 4, 5 and 6 are based on the French standard for insolation, which is slightly less for any given condition than the standard used in the United States. The insolation on walls facing North, East, South and West has been measured at Ottawa for about three years, and these records show that the insolation frequently exceeds the values indicated by both the American and French standards. On a very clear day it can exceed the values in the charts by about 25 per cent when there is no snow on the ground. (In winter, the radiation reflected from a snow surface can increase the radiation falling on a wall by a further 25 per cent.) When more data are available, it will be possible to prepare alternative overlay charts appropriate for Canada. In the meantime, the French Diagrammes de Puissance can be used to estimate the insolation that can be expected on a typical cloudless day.

### Angle of Incidence

The angle of incidence of the solar beam on any flat surface is defined as the angle between the solar beam and a line perpendicular to the surface. The angle of incidence at a vertical surface facing in any direction can be found by using overlay No. 3 in conjunction with the solar diagram for the appropriate latitude. (It cannot be used with the charts for latitudes greater than 52 degrees because the distance d is different for the high latitude charts.)

Overlay No. 3 is laid on the solar diagram with its principal point directly over the principal point of the chart, and with the base line acting as the wall line. The incident angle, for any time and date represented by a point P on the solar diagram can be obtained by estimating the co-ordinate of P relative to the curved co-ordinate lines on the overlay. The angles marked on the chart are the complements of the incident angles; for example, if P is under the curved line marked 70 deg, the angle of incidence is  $90-70$ , or 20 degrees.

References:

1. Brown, G., and T. Tuominen. Solar Position at Various Hours, Dates and Latitudes - Tables. Rapport 75 Bygghforskningen, Stockholm, 1962.
2. U.S. Navy Hydrographic Office. Tables of Computed Altitude and Azimuth. Publication 214.

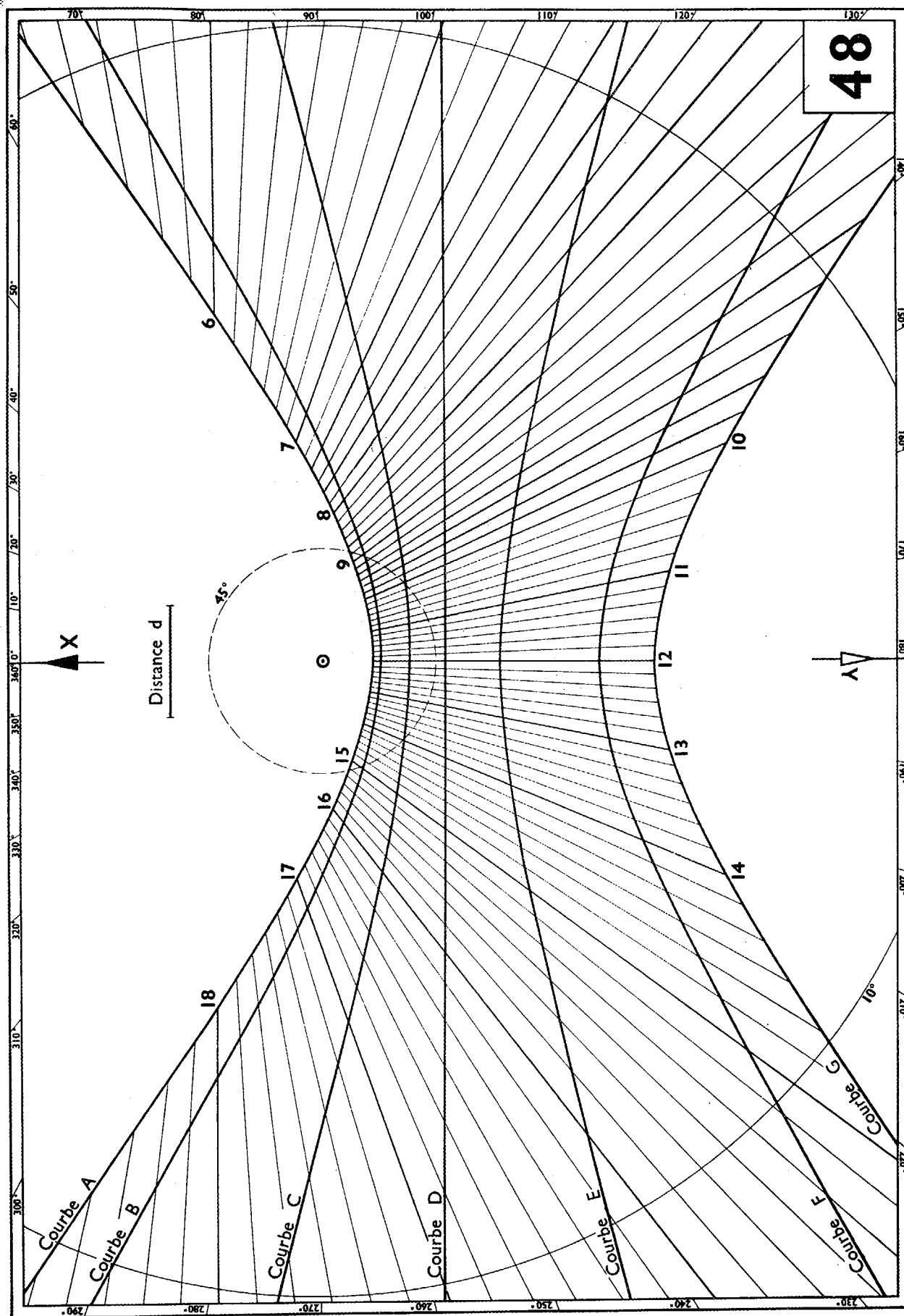


FIGURE 1 C.S.T.B. SOLAR DIAGRAM FOR LATITUDE OF 48°

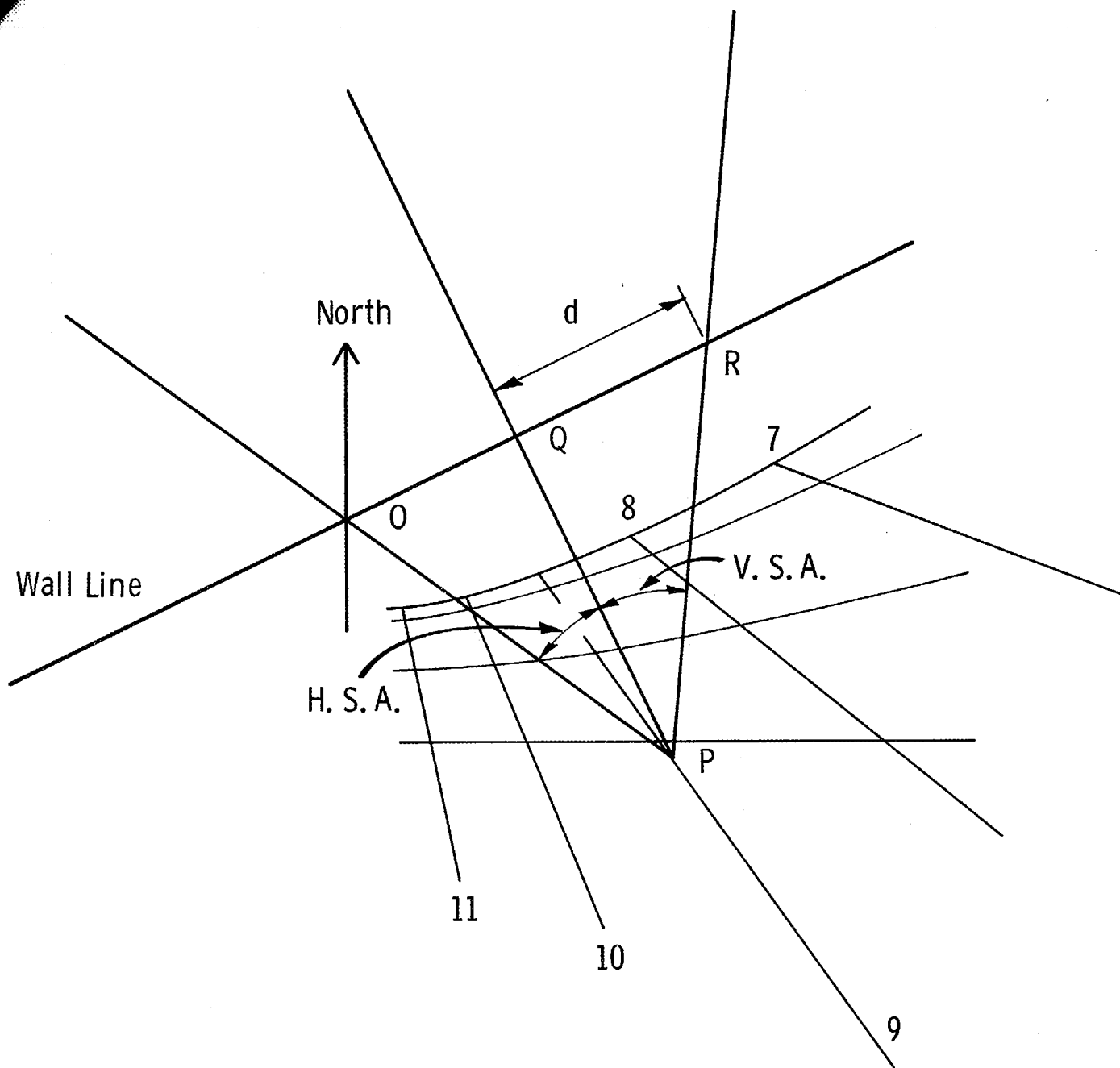


FIGURE 2  
SHADOW ANGLE DIAGRAM FROM C. S. T. B. CHART

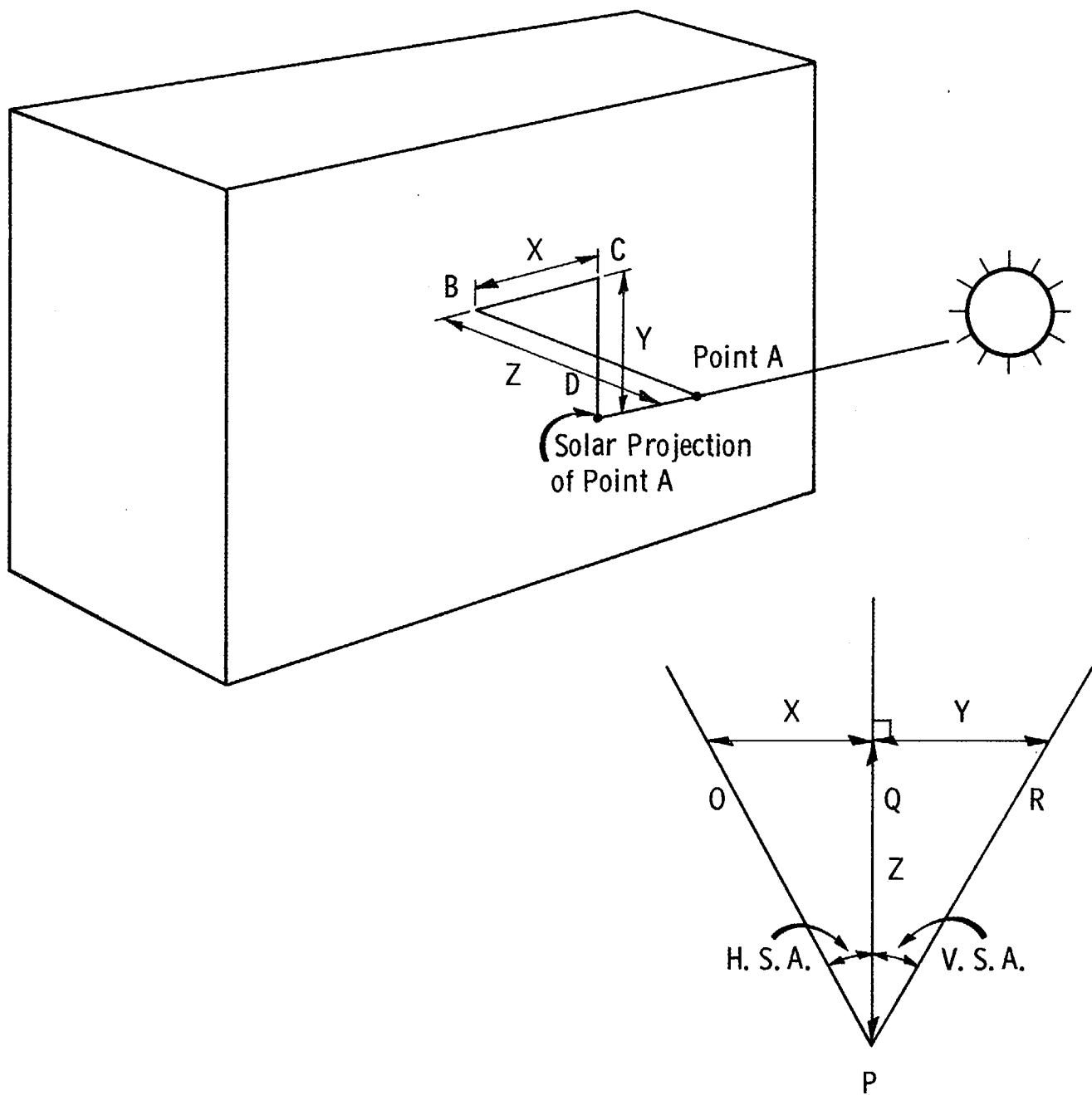


FIGURE 3

USE OF SHADOW ANGLE DIAGRAM TO MAKE SOLAR PROJECTION