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THE USE OF PIPE FLOW CHARACTERISTICS IN CHECKING FLUID FLOWMETERS

by

J. G. Racine and J. R. Sasaki

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PREFACE

The accurate calibration of fluid metering devices poses many problems, yet there are many kinds of laboratory studies which depend on the ability to measure fluid flow accurately. The Division has encountered such a requirement recently in the calibration of simple plate orifice meters for use in the measurement of window infiltration. Recognizing that the establishment of a suitable primary standard could be both difficult and time-consuming it was decided to accept as an interim laboratory standard a series of commercial flowmeters offered with special calibration. These were not found to be accurate and in the course of checking them it was possible to examine the flow characteristics of steel pipes with a view to using them as a calibration device. The results obtained are now reported.

The first author, Mr. Racine, carried out much of the measurement work as a summer student with the Division. Mr. Sasaki, a research officer with the Building Services Section, is now engaged in the study of window infiltration.

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Five commercially available calibrated variable-area flowmeters were purchased by the Division of Building Research to serve as a laboratory standard for air flow measurement. The rotameters were intended for use in the calibration of sharp-edged orifice meters. These flowmeters were selected to cover a range of flows and there was some overlap in range between each pair taken in order of ranges. It was thus possible to compare the reading given by one meter at the top of its range with that given by the next larger meter operating near the bottom of its range. Discrepancies of up to 5 per cent were found, but it could not be determined how much discrepancy could be attributed to each meter of the pair.

The poor correspondence of the readings from two successive flowmeters was attributed to two possible sources:

a) that the accuracy of the purchased rotameters was not ± 1 per cent, as guaranteed by the manufacturer; or

b) the density corrections applied to the metered flow were incorrect.

Subsequent tests showed the density corrections to be valid and the conclusion was, therefore, that the rotameters did not achieve an accuracy of ± 1 per cent.

The objective in determining the friction factor Reynolds' number characteristic of the three steel pipes using the rotameters was threefold. The first was to check the discrepancies between the individual rotameters by obtaining the characteristic for a particular pipe over a range of Reynolds' number common to two meters with an overlapping flow range, and comparing the friction factor values obtained with the different meters. The second was to compare the pipe characteristics obtained with the rotameters with published pipe characteristics such as those presented by Moody(2). This would give some idea of the relative accuracy of the whole set of rotameters in addition to the regions of overlapping flow. The third purpose was to determine whether the flow rates obtained by applying the generally-accepted friction factor characteristic for smooth pipe to the galvanized iron conduits available were sufficiently accurate for calibration purposes.

Description of the Apparatus

The apparatus for the tests was arranged as shown in Fig. 1. Air flow was obtained from the laboratory-compressed air supply main and was reduced from 100 psi in the main to between 30 and 80 psi at the inlet side of the throttling valves. Immediately ahead of the pressure-reducing valve the air passed through a Logan aridifier which eliminated any droplets of water or oil, as well as particles of pipe scale or rust entrained in the compressed air. The moisture content of the air which passed through the test conduit was measured using a Burton dew point apparatus. The quantity of air flowing through the system was controlled by the throttling valves on the low-pressure side of the pressure-reducing valve.

From the flow control panel the air passed through a 20-ft length of $1\frac{1}{4}$-in. galvanized iron pipe into a $1\frac{1}{4}$-in. header pipe and thence through any one of the three 40-ft runs of test conduit of galvanized iron. The test conduits were those used in a previous study, described in DBR Internal Report No. 99 (3). From the conduit the air passed through a rotameter to atmosphere.

Six static pressure holes 1/16 in. in diameter had been drilled at 60° intervals around the conduit approximately 4 in. from one end of each 10-ft section. These holes were covered by brass piezometer rings, soldered to the outside of the conduit to make an airtight seal.

The inside of the conduit had been reamed for approximately 6 in. at the end where the piezometer ring was fitted and 2 in. at the other end. This reaming ensured that the internal area of the flow passage was the same at each pressure tapping and that the inside diameter of the conduits were the same at the joints. The differential pressure was measured with a 250 mm Betz water manometer and when necessary with a 100-in. vertical Meriam manometer using coloured water. Gauge pressures through the system were measured using another 100-in. vertical Meriam manometer and when necessary with a Bourdon gauge. A pressure-switching panel was used to connect the manometers to any of the twelve piezometer rings. With this arrangement, the Betz or Meriam manometers could readily be made to read the differential pressure between any two piezometer rings.

Before assembly of the piping, the internal cross-sectional area of each length of conduit had been determined. The pipe diameters are given in Table 1.

TABLE 1

Test Conduit Sizes

Conduit Number		Diameter($\bar{d}$)(ft)
3/4 in.	I	6.908×10^{-2}
	II	6.869×10^{-2}
	III	6.828×10^{-2}
	IV	6.919×10^{-2}
1 in.	I	8.744×10^{-2}
	II	8.671×10^{-2}
	III	8.712×10^{-2}
	IV	8.728×10^{-2}
1 1/4 in.	I	11.575×10^{-2}
	II	11.413×10^{-2}
	III	11.578×10^{-2}
	IV	11.586×10^{-2}

The temperatures inside the pipe were measured with thermocouples read on an electronic temperature indicator. Thermocouples were placed in the header pipe, at the ends of each pipe, and at the outlet of each rotameter.

The air flows through the pipe were measured using four rotameters. The flow ranges are shown in Table 2.

TABLE 2

Rotameter Flow Ranges

Meter	Flow Range(cfm)
1	0.29 - 1.60
2	1.15 - 6.3
3	4.0 - 22.5
4	18.0 - 98.0

Theory of Flow through a Pipe with Friction

Air flowing through a pipe, in which friction occurs, can be dealt with as either a compressible or incompressible fluid depending on the magnitude of flow, the length of the pipe, and the pipe diameter. When the flow is small, the pipe length short, and the pipe diameter relatively large, the pressure drop and consequently the density change is very small and incompressible flow theory can be applied. If, however, the flow is large, the pipe length long and the diameter of the pipe relatively small, the density change can no longer be ignored, and the flow must be considered compressible.

Reference No. 1 states that investigators have found the friction factor, f, to be a physical characteristic of a pipe, independent of the Mach number and dependent only on the Reynolds' number and the pipe roughness, no matter what type of flow exists in the pipe. The problem is to obtain an expression for flow that represents the flow conditions actually existing in the pipe. A choice of three expressions presented themselves; the incompressible equation and the isothermal and adiabatic equations for compressible flow. Their developments are shown in Appendix A. Considering the maximum flow and the minimum pipe diameter to be encountered in the study it was decided that the pressure ratio, p_2/p_1, would be sufficiently low to necessitate the use of the compressible flow equation.

Reference No. 1 states further, that only in very long pipes with a large temperature difference between the inside and outside of the pipe will isothermal flow conditions be approached. In short pipes where the gas temperature is near the room temperature, adiabatic conditions are more nearly approached.

The conditions during the tests on the pipe were such that the actual test conditions probably approached adiabatic more closely than isothermal. A sample calculation of the friction factor is shown in Appendix A using all three expressions. It was found that the results obtained with the isothermal expression varied very little from that obtained with the adiabatic expression. Therefore, although realizing that the test conditions more closely approached adiabatic, the isothermal compressible flow expression was used to determine the friction factor from the test results for its simplicity. The isothermal compressible flow expression for friction factor is

$$f = \frac{d_o}{L}\left[\frac{g\ p_1 v_1}{V_1^{\,2}}\left\{1 - \left(\frac{p_2}{p_1}\right)^2\right\} - \ln\left(\frac{p_1}{p_2}\right)^2\right]$$

The corresponding pipe Reynolds' number is

$$R_e = \frac{d_o \; V}{g \; \nu_m \mu}$$

The above symbols are defined in Appendix A.

Test Procedure

The rotameter was connected to the particular pipe to be tested. The air from the laboratory supply was turned on and governed by the control valve until the appropriate flow was indicated on the rotameter. For the given flow, the pressure drop across one, two or three lengths of the conduit being tested was measured using the Betz manometer for the small values, and the 100-in. Meriam manometer for the larger values. When the pressure drop was small, it was measured over the longest available length, i.e. three 10-ft sections together. The gauge pressure at the pipe inlet was measured with the 100-in. manometer and with the Bourdon gauge at the larger pressures, and the pipe inlet and outlet and rotameter outlet temperatures were measured with thermocouples. The above readings were made at each flow reading.

The barometric pressure, room temperature and the line air dew-point temperature were measured at the start and finish of each test. Four tests were made using meter 1 and three tests each were made with meters 2, 3 and 4 on the 3/4 in. pipe. Three tests each were made with meters 2, 3 and 4 on the 1 in. pipe, and three tests each were made with meters 2, 3 and 4 on the $1\frac{1}{4}$ in. pipe.

Data Processing

The data processing was performed by the G-15 Bendix computer for which a program was written. The program computed values of friction factor and the corresponding Reynolds' numbers. A plotter subroutine was written into the program to provide two forms of output: type-out in fixed point notation and punched paper tape. The punched paper tape result was inputted to a Mosely plotter which yielded a plot of log (f) versus log (R_e). The processed results of the tests appear in Figs. 2 to 8, inclusive. In addition, several test points were re-calculated with the value of Q_m modified by $\pm$ 1 per cent to $\pm$ 5 per cent.

The inaccuracy introduced into the final results due to instrument errors is outlined in Appendix B.

Discussion of Results

In the subsequent discussion, all discrepancies in f are expressed as percentage differences in flow.

I. Pipe Friction Factor Characteristic

(a) 3/4 in. Ø Conduit (Figs. 2 and 3)

The figures show very poor correlation between the characteristics as determined by different meters. The poor correlation between the characteristics was attributed to the calibration of the meters. At $R_e = 2240$, the difference is equivalent to a difference of $4\frac{1}{2}$ per cent in the values of flow as measured by meters 1 and 2. At $R_e = 12{,}000$, the difference in terms of flow is $2\frac{1}{2}$ per cent. At $R_e = 42{,}600$, the difference in terms of flow is $2\frac{1}{2}$ per cent.

From Fig. 2 it can be seen that in the turbulent and transition regions a difference of 1 per cent in Q_m produces a 1 per cent increase in R_e and a 2 per cent decrease in f which can be seen quite easily. However, a change in Q_m of 1 per cent in the laminar region can scarcely be discerned as the increase in R_e and the decrease in f follows closely the slope of the laminar flow line.

In Fig. 3 published pipe friction factors are plotted for comparison: curve 2 falls from the smooth pipe characteristic by 2 to 5 per cent of flow, curve 3 by 2 to 3 per cent of flow, and curve 4 by 1 to 4 per cent of flow.

If the characteristic of a pipe with roughness 0.0004 is taken as the mean, curve 2 falls off from 0 to 4 per cent, curve 3 falls off from 1 to 0 per cent and curve 4 falls off by 4 per cent. Therefore, if the published pipe characteristics are assumed correct, and the method of calculating f and R_e are assumed correct, the results show at least 2 of the 3 meters as having errors in the calibration greater than 2 per cent.

(b) 1 in. Ø Conduit (Figs. 4 and 5)

The poor correlation between the characteristics, as determined by meters 2, 3 and 4, was again evident, and in the same manner. The difference between curves 2 and 3 increased a small amount and the difference between curves 3 and 4 decreased.

In Fig. 5 it is seen that the characteristic for a relative roughness of .0004 is a good approximation for curves 3 and 4 which are only about $+1\frac{1}{2}$ and -2 per cent off. However, curve 2 is off by as much as 2 to 4 per cent.

(c) $1\frac{1}{4}$ in. Ø Conduit (Figs. 6 and 7)

Very similar in characteristic to the 3/4 and 1 in. pipes, except that the difference between curve 2 and 3 is quite small. The presence of the lower points on curve 3 is due to an error of 0.45 per cent in the diameter that was used in the calculation. Figure 7 compares the test results with the curve for .0004 relative roughness. Curve 4 falls below by 1 per cent, curve 3 lies above by 2 per cent and curve 2 lies above by 3 to 5 per cent.

II. Figure 8 shows all the measured characteristics plotted on a single graph. Except for the transition and high turbulence regions, all the curves fall very nearly one on top of the other, an indication that all three conduits tested had similar relative roughness.

Conclusions

I. The pipe friction factor characteristics as determined with the four rotameters were at variance with one another in the overlapping ranges of flow. The calibration of meter 2 was lower than that of meter 1 by as much as $4\frac{1}{2}$ per cent of the flow value. The calibration of meter 3 was higher than that of meter 2 by $2\frac{1}{2}$ per cent of the flow, and the calibration of meter 4 was higher than that of meter 3 by $2\frac{1}{2}$ per cent of the flow.

II. The pipe friction factor characteristics presented by Moody were taken as a reference. When the experimental characteristics were superimposed, they did not lie on any one line of a given roughness but crossed several. The results obtained for the 3/4 in. Ø pipe can be taken as an example. Curve 3 lay along the general characteristic for a pipe with a relative roughness of 0.0004. Curve 2 lay above this line, indicating that the whole calibration on meter 2 may be low by up to 4 per cent. Curve 4 lay below the line, an indication that the whole calibration of meter 4 may be high by 4 per cent.

III. The study shows that there is a distinct possibility that for $R_e > 6000$ the published pipe friction factor characteristics can be used as a calibrating means, provided the general curves are accurate and the pipe roughness can be closely estimated. The friction factor is an inverse function of the square of the flow, and an error of 2 per cent in determining the friction factor will result in only 1 per cent error in flow. Therefore, if one is able to choose a published pipe friction factor characteristic to within 2 per cent of the true characteristic, by assuming no experimental error, flow values accurate to 1 per cent can be obtained.

In the laminar region, there is only one characteristic and this is independent of roughness. However, this characteristic must be very accurate since a small error in f introduces a very large error in the determined flow.

Acknowledgements

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References

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3. Stephenson, D. G. Pressure loss associated with air flow in electrical conduits. National Research Council, Division of Building Research, Internal Report No. 99, August 1956, 41 p., 15 Figs.

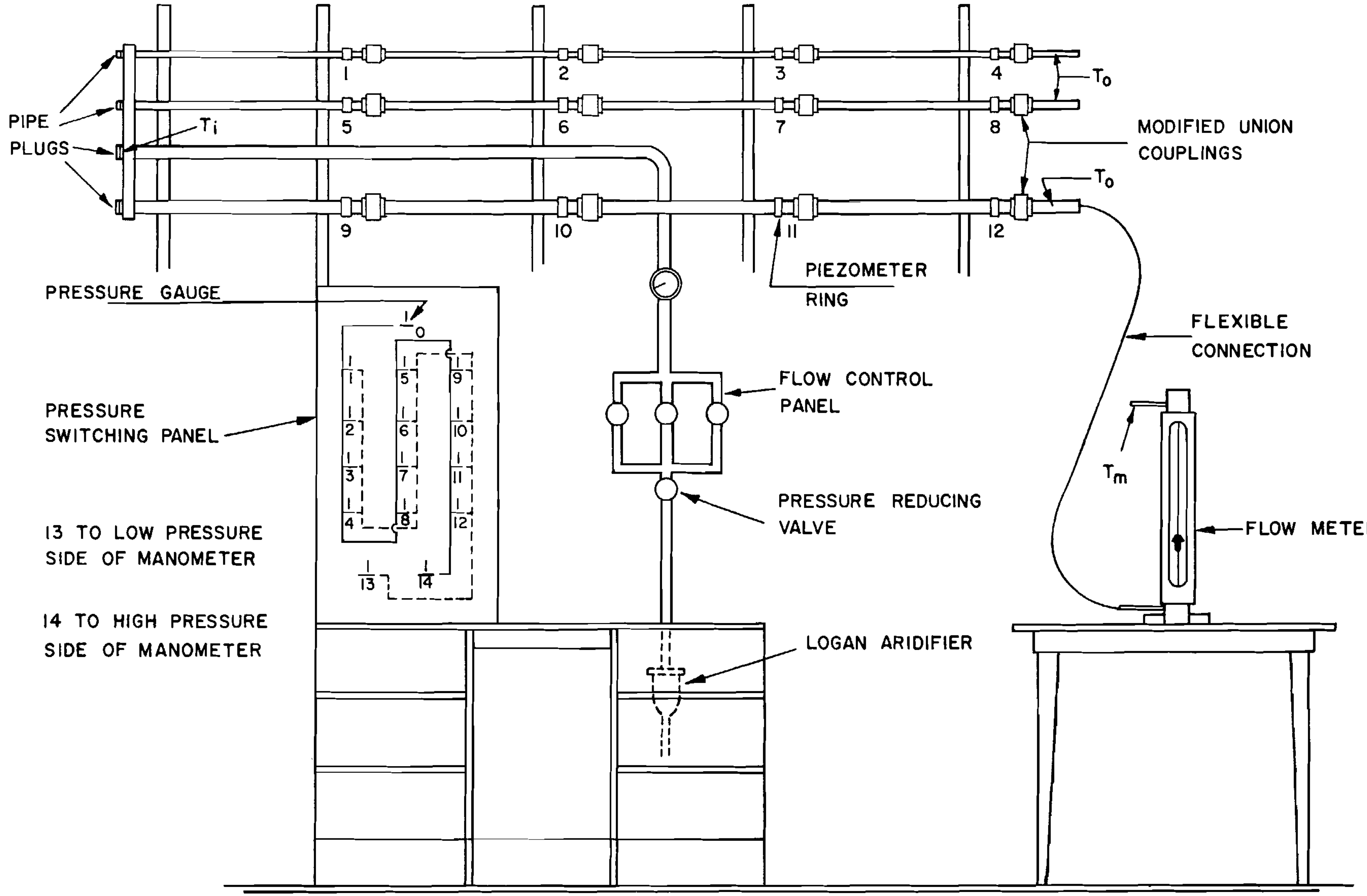

FIGURE I LAYOUT OF APPARATUS

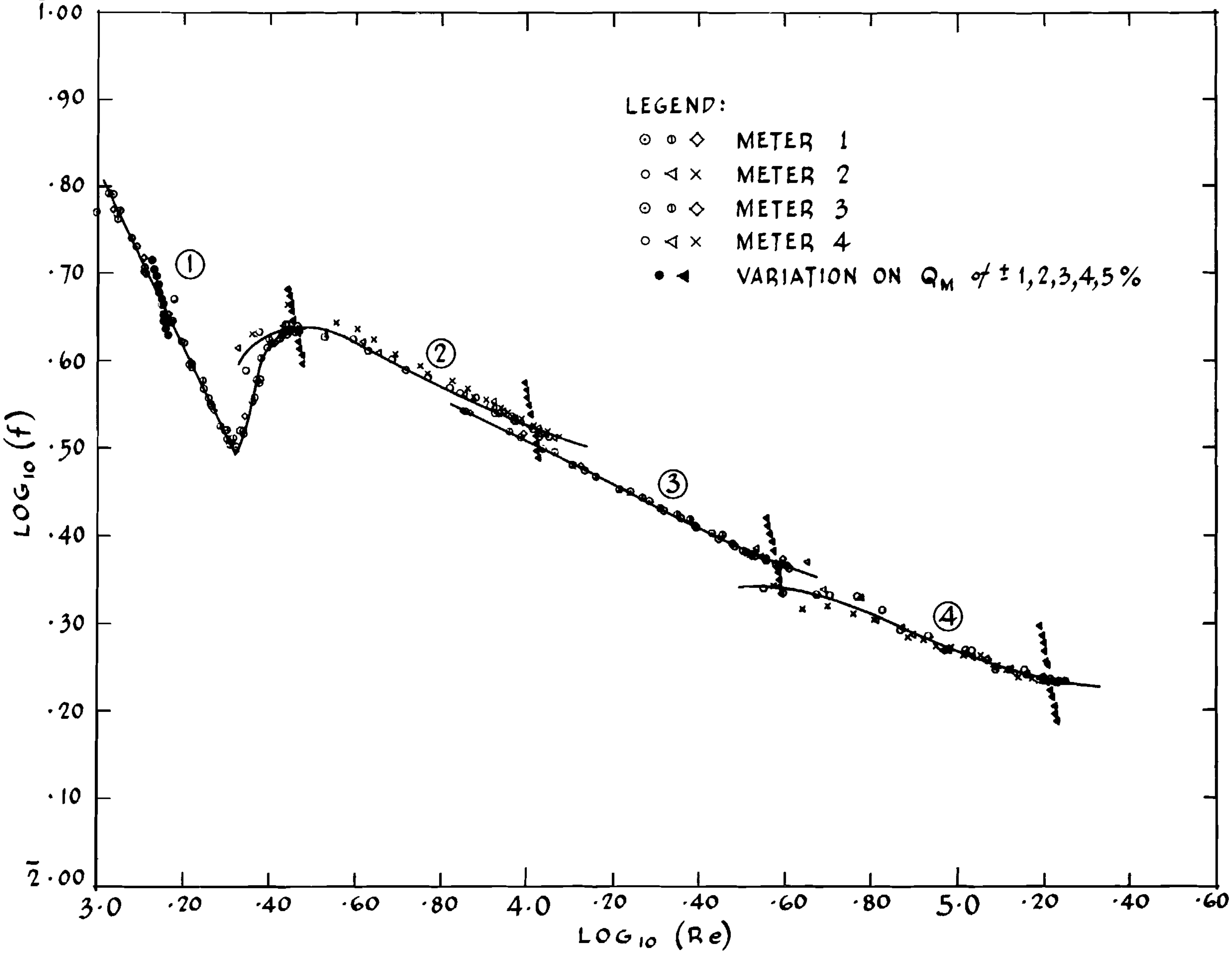

FIGURE 2

FRICTION FACTOR vs. REYNOLDS NUMBER   3/4" CONDUIT SECTIONS

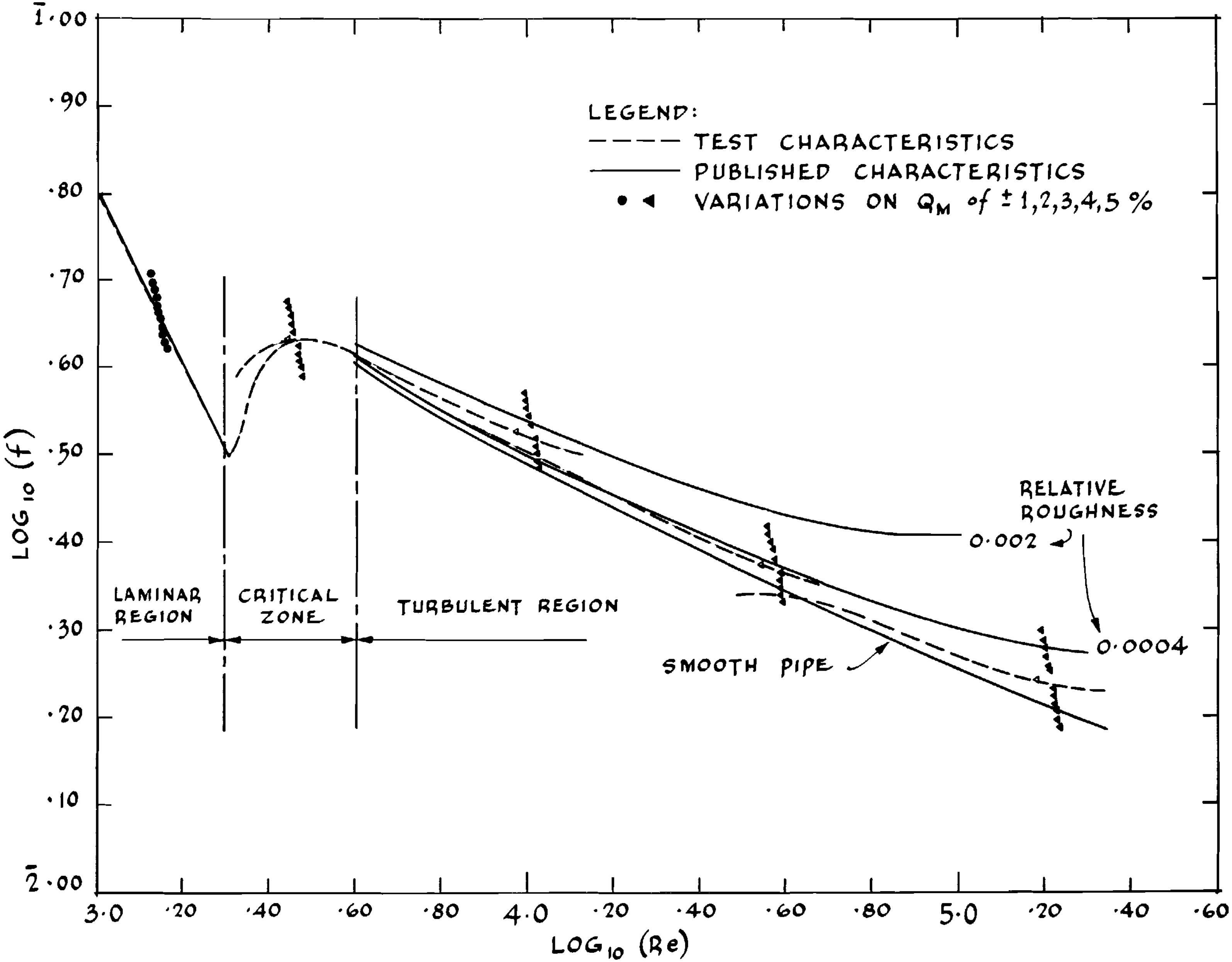

FIGURE 3

FRICTION FACTOR vs. REYNOLDS NUMBER 3/4" CONDUIT SECTIONS
COMPARISON of TEST AND PUBLISHED CHARACTERISTICS

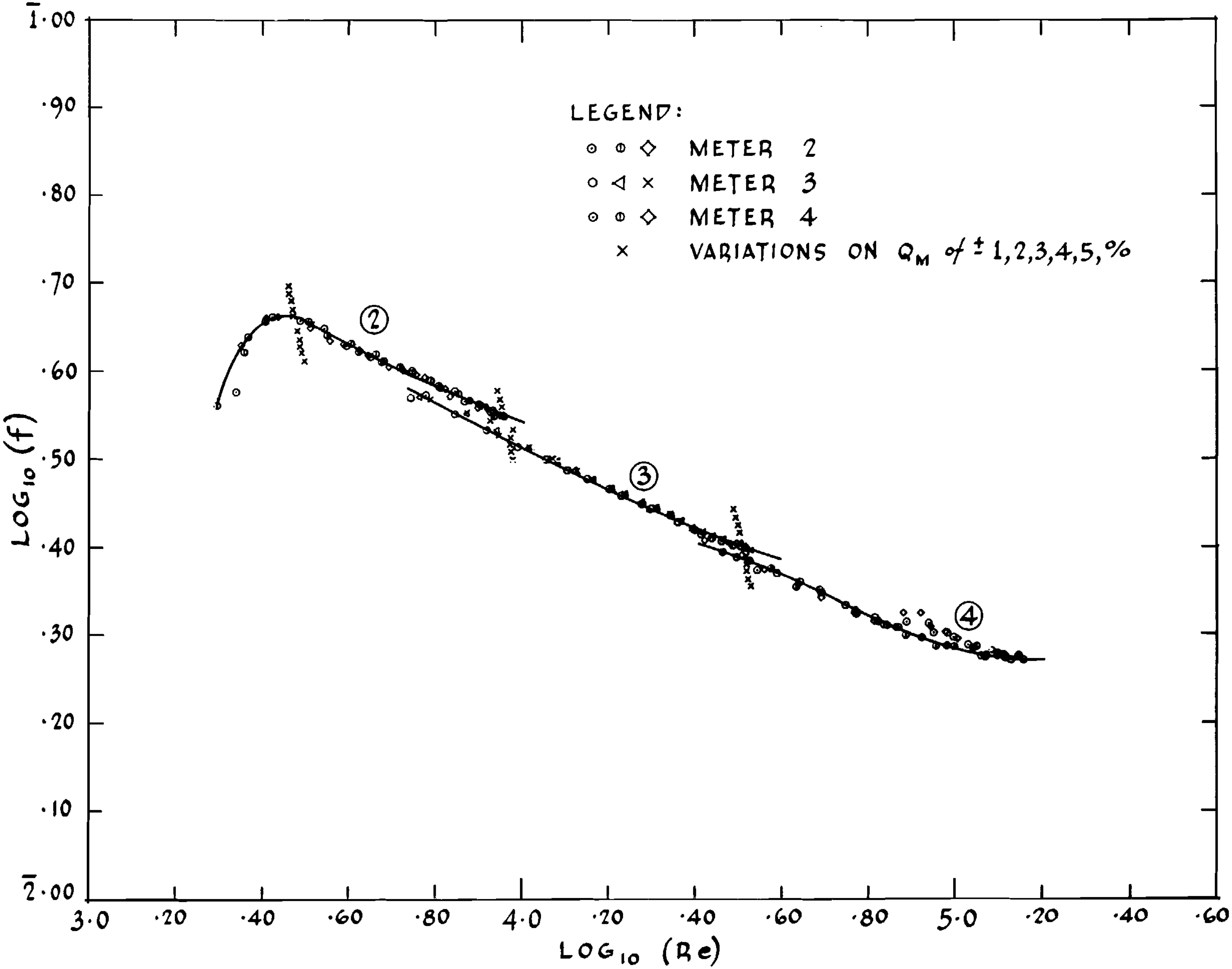

FIGURE 4

FRICTION FACTOR vs. REYNOLDS NUMBER 1" CONDUIT SECTIONS

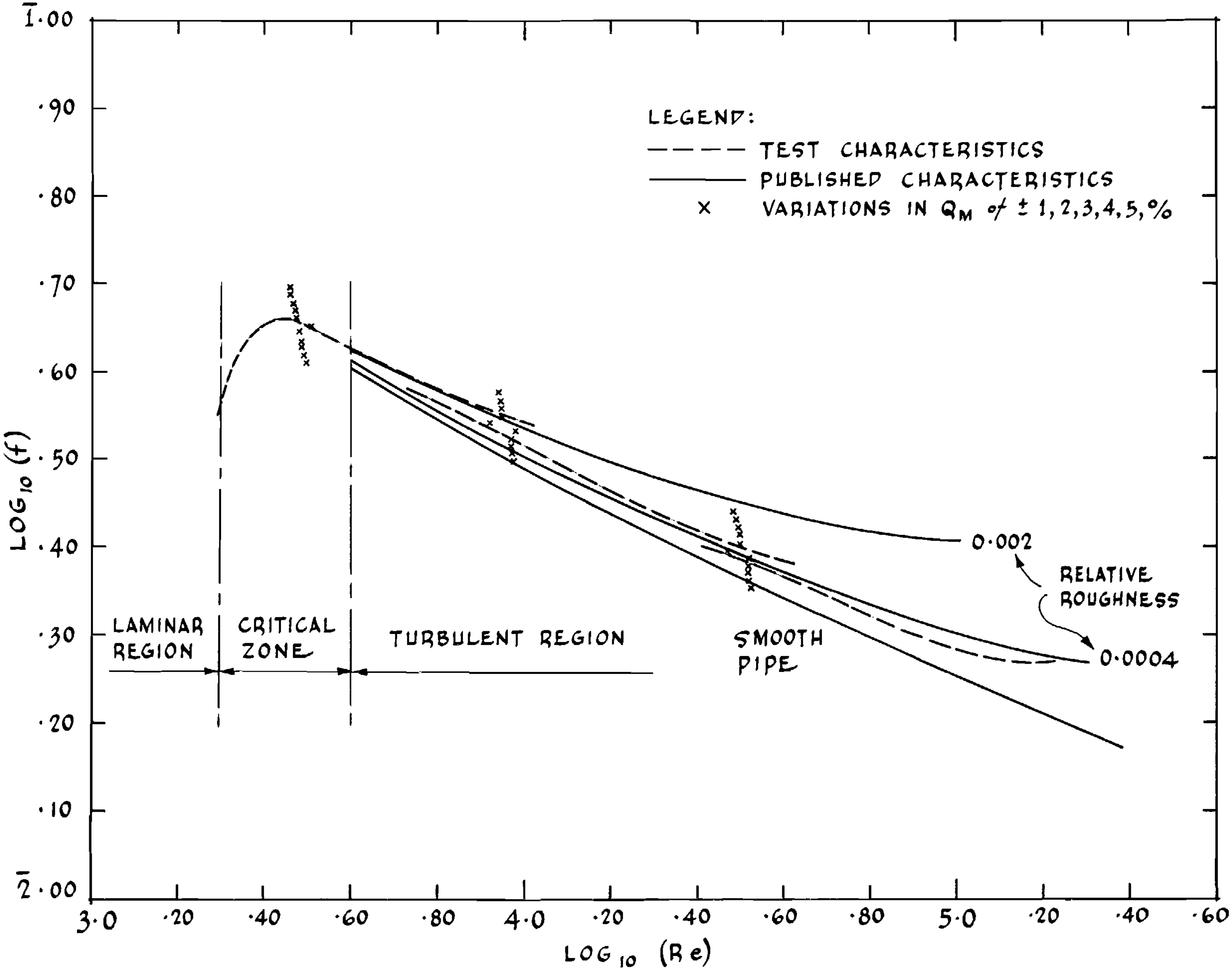

FIGURE 5

FRICTION FACTOR vs. REYNOLDS NUMBER 1" CONDUIT SECTIONS
COMPARISON of TEST AND PUBLISHED CHARACTERISTICS

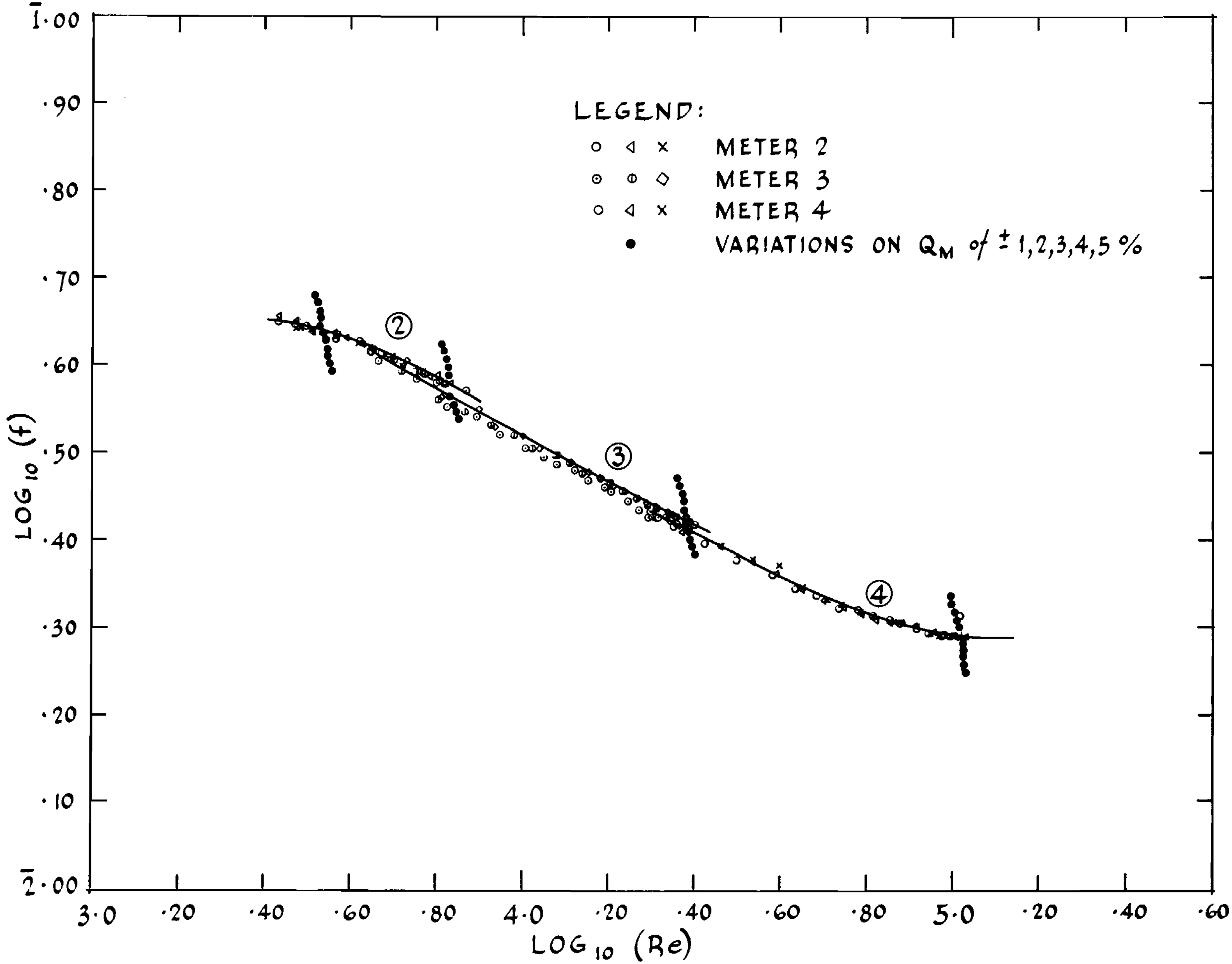

FIGURE 6

FRICTION FACTOR vs. REYNOLDS NUMBER 1¼" CONDUIT SECTIONS

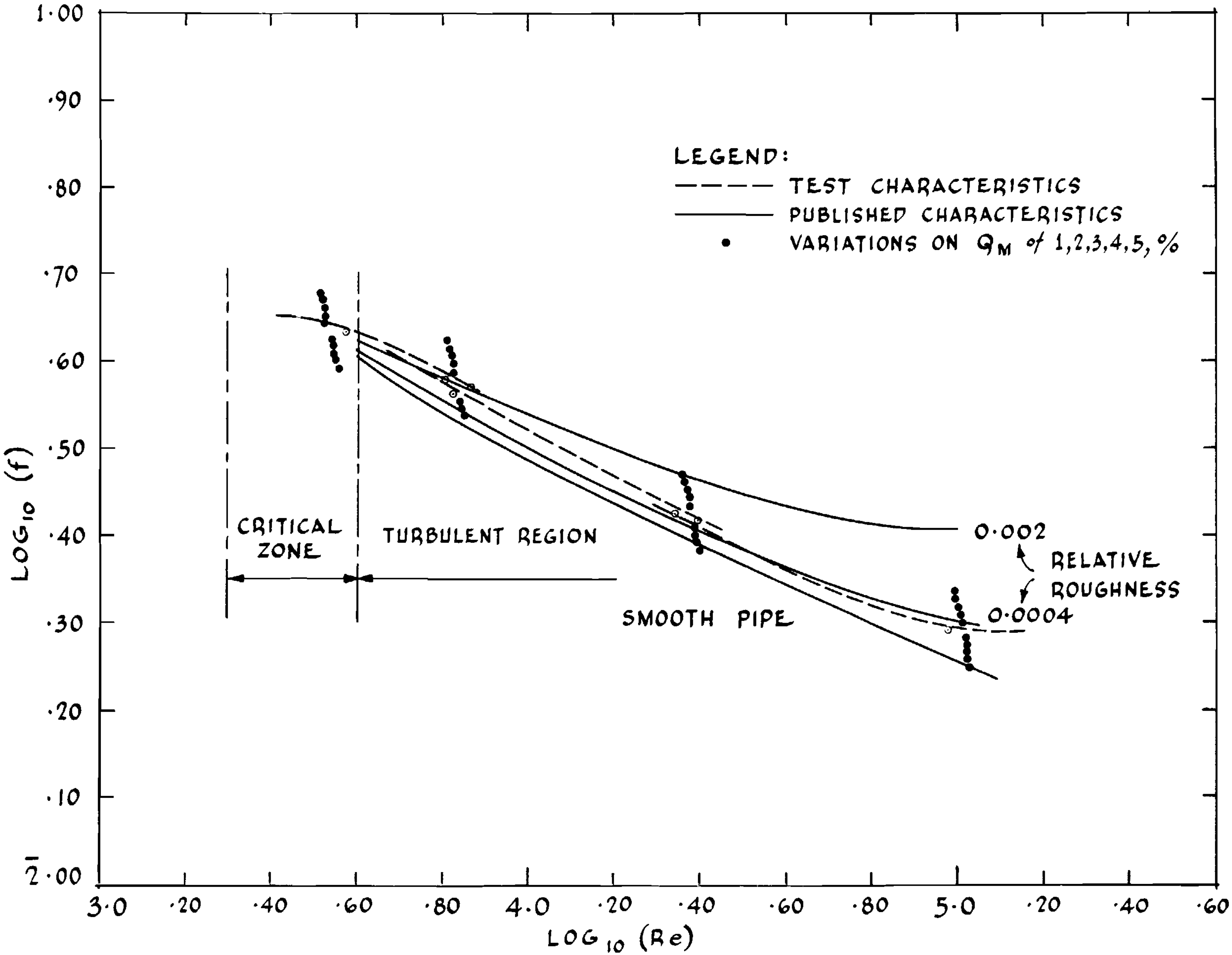

FIGURE 7

FRICTION FACTOR vs REYNOLDS NUMBER 1¼" CONDUIT SECTIONS
COMPARISON of TEST AND PUBLISHED CHARACTERISTICS

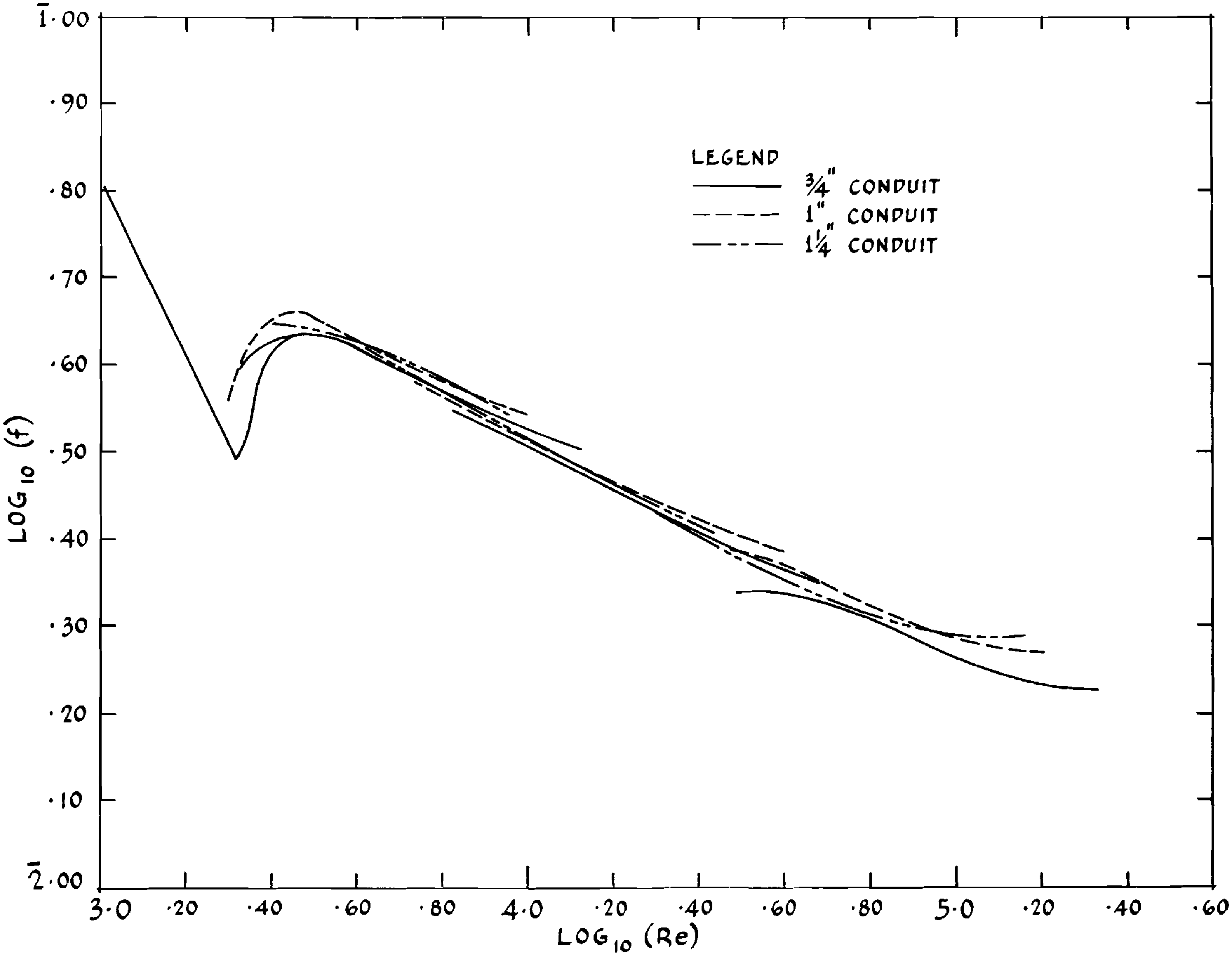

FIGURE 8 FRICTION FACTOR vs. REYNOLDS NUMBER
COMPARISON of THE THREE PIPE CHARACTERISTICS

APPENDIX A

STEADY FLOW OF AIR THROUGH A PIPE WITH FRICTION

An explanation of the symbols used in the equations to follow will be found at the end of Appendix A.

For the steady flow of air through a pipe, the following relations will be assumed to hold:

(i) Continuity equation

$$W = \rho AV = \frac{AV}{v} = \text{constant}$$

and $\frac{V_1}{v_1} = \frac{V}{v}$ = constant for a constant area pipe;

(ii) Perfect gas law

$$pv = RT$$

The steady flow of a fluid through a pipe can be completely described by the following equations:

(a) the equation for the condition of state;

(b) the Energy Equation of flow.

The General Energy Equation is the algebraic expression of the first law of thermodynamics which states that in a conversion of thermal energy to mechanical energy, the amount of mechanical energy developed is equal to the amount of thermal energy which disappears and vice versa. The equation can be expressed as follows, for horizontal flow, in which no work is done:

$$J\,dq - J\,du = d\left(\frac{V^2}{2g}\right) + d\,(pv) \qquad (1)$$

When friction is present in the pipe, part of the mechanical energy developed is reduced by the friction to heat.

An energy equation involving the friction can be developed from the dynamic equation. Consider an element of fluid, dm, moving from point 1 to point 2 along the pipe. Acting on the upstream face of the element is a pressure (p) and on the downstream face, (p+dp). The velocity at the upstream face is V and at the downstream face V + dV.

Resisting the movement of the element is the friction shear stress, τ_o, where $\tau_o = \frac{\rho f\ V^2}{8g}$.

If the pipe has a diameter, d_o, and the element has length, dL,

$$dm = \frac{\pi}{4}\ d_o^{\ 2} \ x \ dL \ x \ \frac{\rho}{g}$$

The dynamic equation states that the sum of all the forces acting on the element causes the element to accelerate.

Acceleration is $a = \frac{dV}{dt}$.

Since the flow is steady, the acceleration is a function of length only.

$$\frac{dV}{dt} = \frac{dV}{dL} \cdot \frac{dL}{dt} = \frac{VdV}{dL}$$

$$\left[p - (p+dp)\right] \frac{\pi}{4}\ d_o^{\ 2} - \tau_o \cdot \pi . d_o . dL = dm. \ \frac{VdV}{dL}$$

which is $- \frac{dp}{\rho} - \frac{f V^2}{2gd_o} .dL = \frac{VdV}{g}$

or $$v\,dp + \frac{VdV}{g} + \frac{f V^2}{d_o 2g} .dL = 0 \qquad (2)$$

This is the energy equation, including friction.

The equations for incompressible flow and isothermal and adiabatic compressible flows can be developed from the energy equation (2) by incorporating the particular conditions of state.

(a) <u>Incompressible Flow</u>

The condition of state for incompressible flow is

v = constant

and since $\frac{V}{v}$ = constant, V = constant

Equation (2) reduces to

$$v\,dp = - \frac{f\ V^2}{d_o\ 2g} \ . \ dL$$

Upon integration, the expression for friction factor is given as

$$f = \frac{d_o}{L} \cdot 2g \frac{v_m}{V^2} \quad (p_1 - p_2) \qquad (3)$$

where

v_m for true incompressible flow is equal to v_1 and v_2, but for air flow is equal to the mean.

The corresponding pipe Reynolds' number is

$$R_e = \frac{d_o \quad V}{g v_m \mu}$$

Since v_m is constant, and further assuming that the temperature from point 1 to point 2 is nearly constant, μ is constant and therefore R_e is constant along the pipe length.

(b) Isothermal Compressible Flow

The condition of state for isothermal flow is

$$dT = 0$$

The Perfect Gas Relation

$$d \quad (pv) = RdT = 0$$

From the continuity equation, $\frac{V}{v}$ = constant

Equation (2), re-arranged, gives

$$2 \frac{dV}{V} + 2g \frac{v dp}{V^2} + \frac{f}{d_o} \cdot dL = 0$$

Reduces to $\frac{2dV}{V} + 2g \frac{v_1}{V_1^{\,2}} \, pdp + \frac{f}{d_o} \cdot dL = 0$

Integration gives,

$$f = \frac{d_o}{L} \left[\frac{g \; p_1 v_1}{V_1^{\,2}} \left\{ 1 - \left(\frac{p_2}{p_1} \right)^2 \right\} - 2 \ln \left(\frac{p_1}{p_2} \right) \right] \qquad (4)$$

(c) Adiabatic Compressible Flow

The condition of state is

$dq = 0$

and Equation (1) becomes

$$d(p\nu) + \frac{d}{2g}(V^2) + J\,d\,u = 0$$

Since $Jdu = J\,c_V\,dT = \frac{1}{(k-1)} \,.d\,(p\nu)$

The equation of the condition of state becomes:

$$d(p\nu) + \frac{(k-1)}{k} \cdot \frac{d\,V^2}{2g} = 0 \qquad (5)$$

The energy equation is Equation (2)

$$\frac{VdV}{g} + \nu dp + dF = 0 \qquad (6)$$

Solving Equations (5) and (6) simultaneously and obtaining an expression for F in terms of p_1 and p_2 would give proper expression for f. However, the resulting equation is difficult to solve and normally the Equations (5) and (6) are expressed separately.

Equation (5) integrates to

$$p\nu + \frac{(k-1)}{k} \cdot \frac{V^2}{2g} = \text{constant}$$

Since $M = \frac{V}{\sqrt{kg\,p\nu}}$

$$\left(\frac{p_2}{p_1}\right) = \frac{\nu_1}{\nu_2}\left[1 + \frac{(k-1)}{2} \cdot M_1^{\,2}\left\{1 - \left(\frac{\nu_2}{\nu_1}\right)^2\right\}\right] = \text{equation}$$

of state.

Equation (6) can be re-written as

$$\frac{VdV}{g} + d\,(p\nu) - \frac{(p\nu)\,d\nu}{\nu} + \frac{f\,V^2}{d_o 2g}\,.dL = 0 \qquad (6a)$$

Substituting

$$d(p\nu) = -\frac{(k-1)}{k} \cdot \frac{d\,V^2}{2g}$$

and $(p\nu) = p_1 \nu_1 + \frac{(k-1)}{k} \cdot \frac{V_1^{\,2}}{2g} - \frac{(k-1)}{k} \cdot \frac{V^2}{2g}$

into (6a) and integrating, gives the resulting energy equation:

$$f = \frac{d_o}{L}\left[\left\{\frac{2+(k-1)\,M_1^{\,2}}{2\,k\,M_1^{\,2}}\right\}\left\{1 - \left(\frac{\nu_1}{\nu_2}\right)^2\right\} - \frac{(k+1)}{k} \cdot \ln\left(\frac{\nu_2}{\nu_1}\right)\right].$$

By solving the equations of state and energy together for a particular value of p_2/p_1, the value of friction factor is obtained.

SAMPLE CALCULATION USING THE THREE EXPRESSIONS

Test Condition -	10 ft of uninsulated iron pipe
	$d_o = 6.828 \times 10^{-2}$ ft (3/4 in. Ø)
B_a = 29.67 in.M.C.	t_{dp} = 17°F
B_c = 29.54 in.M.C.	$h_1 - h_2$ = 63.6 in. W.C. = 2.30 psi
P_1 = 13.25 psig	Q_m = 97.1 cfm
T_i = 541.2°R	T_o = 538.6°R
T_r = 538.3°R	T_a = 536.6°R

Solution

$$\rho_m = \frac{1.317}{T_m}\left[B_c - .378\,p_v\right] = \frac{1.317}{538.3} \times 29.51 = 0.07220 \text{ pcf}$$

$$W_a = Q_m\sqrt{\rho_m \cdot \rho_s} = 7.16 \text{ lb/min}$$

$$A = \frac{\pi}{4} \times d_o^2 = \frac{\pi}{4} \times (6.828)^2 \times 10^{-4} = 36.62 \times 10^{-4} \text{ ft}^2$$

p_a = 2085 psf

p_1 = 3993 psf

p_2 = 3661 psf

v_1 = 7.210 cfp

v_{12} = 7.519 cfp

(a) Incompressible Flow

$$f = \frac{d_o}{L} \cdot 2g \frac{v_{12}}{V^2} \quad (p_1 - p_2)$$

$$V_{12} = \frac{W_a \times v_{12}}{A \times 60} = 244.5 \text{ ft/sec}$$

$$f_a = \frac{6.828 \times 10^{-2} \times 64.4 \times 7.519 \times 332}{10 \times (244.5)^2} = 0.01835$$

$$R_e = \frac{d_o \cdot V_{12}}{g v_{12} \mu_{12}} = 1.80 \times 10^5$$

(b) Isothermal Compressible Flow

$$f_b = \frac{d_o}{L} \left[\frac{p_1 \, v_1}{V_1^2} \left\{ 1 - (p_2/p_1)^2 \right\} - 2 \ln\left(\frac{p_1}{p_2}\right) \right]$$

$$= 0.01835 - .001176 = 0.01715$$

(c) Adiabatic Compressible Flow

$$\frac{L\, f_c}{d_o} = \left(\frac{1}{k}\right) \left[\frac{2 + (k-1)\, M_1^2}{2\, M_1^2} \right] \left[1 - \left(\frac{v_1}{v_2}\right)^2 \right] - \frac{(k+1)}{k} \cdot \ln \frac{v_2}{v_1}$$

$$p_2/p_1 = \frac{v_1}{v_2} \left[1 + \left\{ \frac{(k-1)\, M_1^2}{2} \right\} \left\{ 1 - \left(\frac{v_2}{v_1}\right)^2 \right\} \right]$$

$$M_1^{\,2} = \frac{V_1^{\,2}}{kg\ RT_1} = 0.0422$$

$p_2/p_1 = 0.9144;$

By iteration $\frac{v_2}{v_1} = 1.0883$, and

$f_c = 0.01710$.

Checking the Isothermal Condition From Heat Transfer Consideration

The general energy equation for the isothermal condition is

$$J\left({}_1q_2\right) = \frac{\left(V_2^{\,2} - V_1^{\,2}\right)}{2g} = \frac{v_1^{\,2}}{2g}\left[\left(\frac{v_2}{v_1}\right)^2 - 1\right]$$

For isothermal,

$$\frac{v_2}{v_1} = \frac{p_1}{p_2} = 1.0907 \text{ from previous example.}$$

$${}_1q_2 = \frac{(234.5)^2}{778 \times 64.4} \times \left[(1.0907)^2 - 1\right] = 0.21 \quad \text{Btu/lb.}$$

Thus, for a flow of 7.16 lb/min, the heat flow required to maintain isothermal conditions is = 7.16 x 60 x 0.21
≅ 90 Btu/hr.

Therefore, isothermal conditions would have required a heat flow from the surroundings into the pipe. During the test, the room temperature was less than the temperature in the pipe, making it virtually impossible for heat to flow in, and with the small temperature difference existing, the pipe was effectively insulated, approaching the adiabatic condition rather than the isothermal.

Symbols Used in Appendix A

ρ =	air specific weight	lb/ft^3
v =	air specific volume	ft^3/lb
A =	area	ft^2

d_o = pipe diameter ft

V = air velocity ft/sec

W = air weight flow lb/min

T = absolute temperature °R

q = heat flow Btu/lb

u = internal energy Btu/lb

J = mechanical equivalent of heat = 778 ft lb/Btu

F = friction energy loss = $\frac{f\, V^2}{d_o\, 2g} \cdot dL$

f = pipe friction factor

μ = air dynamic viscosity lb sec/ft^2

k = ratio of specific heats = $\frac{c_p}{c_v}$ = 1.4 for air

c_v = specific heat at constant volume

c_p = specific heat at constant pressure

L = length of pipe ft

d = differential of,

R = gas contact for air = 53.3

R_e = Reynolds' number

P = gauge pressure psig

p = absolute pressure psfa

g = acceleration due to gravity = 32.2 ft/sec^2

ln = natural logarithm of,

a = acceleration ft/sec

p_v = vapour pressure in. M.C.

τ_o = friction shear stress

Q_m = volume flow cfm

t_{dp}	= dew point temperature	°F
M	= Mach number	
T_i	= absolute air temperature at pipe inlet	°R
T_r	= absolute air temperature at flowmeter	°R
T_m	= absolute mean air temperature	°R
h_1-h_2	= pressure difference between positions 1 and 2	psig
ρ_m	= mean air specific weight	pcf
ρ_s	= standard air specific weight = .075	pcf

APPENDIX B

ACCURACY OF THE APPARATUS

All the measuring apparatus showed fluctuations in readings, and certain inherent errors.

(a) Rotameters

Readings made on the rotameters were generally stable. Meter 1 was readable to ± 0.001 cfm, meter 2 to ± 0.005 cfm, meter 3 to ± 0.01 cfm, and meter 4 to ± 0.10 cfm.

The uncertainty in the readings averaged ± ½ per cent of the lowest flow measured by each meter. However, sufficient readings were taken to virtually eliminate this uncertainty.

(b) Thermocouple - Indicator Combination

The indicator scale could be read to ± 0.1°F. However, the thermocouples themselves were susceptible to error when wires of dissimilar diameters were joined together or when the thermocouple junctions were improperly installed in the pipes. The thermocouples were checked with a calibrated thermometer and found to be accurate to ± 1°F.

(c) Manometers and Pressure Gauge

(i) Betz. - At the low flows, the readings were very stable and could be read to ± 0.02 mm W.C. but under turbulent flow, the readings were unstable. Small amplitude, high-frequency fluctuations were superimposed on a large amplitude, low-frequency fluctuation, making the readings difficult to take. The low-frequency fluctuation appeared to follow the fluctuation in the rotameter reading. Therefore, the two readings were made as nearly simultaneous as possible, which left only the small amplitude high-frequency fluctuations as error in reading. This error reached a magnitude of approximately ± 0.7 per cent.

(ii) 100 in. Manometers. - These manometers were normally stable, but at the high flows, reached fluctuations of nearly ± 1-in. water column, or approached an uncertainty of ± 1 per cent in reading.

(iii) Bourdon Gauge. - The Bourdon gauge readings were generally stable and were readable to ± 1/8 psig introducing a maximum uncertainty of ± 1.2 per cent.

(d) Dew Point Apparatus

The dew point temperature was determined to within ± 3°F, but since its effect on density is very small, this error was considered negligible.

Extraneous leakages from the apparatus were reduced until they were less than approximately 1×10^{-3} cfm.

Maximum Possible Errors in the Determination of f and R_e Due to Instrumentation

The readings that were made during the test were all subject to systematic and random errors.

The measured flow was considered as having a systematic error, and the random error was assumed negligible. The pressure measurements were considered to be subject to only random errors.

The random errors could be considered self-eliminating when a sufficiently large number of points were taken. On the other hand, the systematic errors would persist no matter how many readings were taken.

The flow was considered as having a systematic error of $\pm$ 1 per cent; the temperatures, as having a systematic error of $\pm$ 0.2 per cent. Since the pressure measurement errors were considered random, even though large, their effects may be ignored, provided a sufficiently large number of readings were made.

The specific weight of the air, which is a function of temperature and pressure, will have a maximum possible systematic error of $\pm$ .2 per cent.

The weight flow of air

$$W_a = Q_m \sqrt{\rho_s \cdot \rho_m}$$

and $$\frac{\Delta W_a}{W_a} = \frac{\Delta Q_m}{Q_m} + \frac{1}{2}\frac{\Delta \rho_m}{\rho_m} = (\pm 1\%) + (\pm \tfrac{1}{2} \times 0.2) = \pm 1.1\%$$

By assuming the error in μ and $\underline{d}$ as being negligible,

$$\frac{\Delta R_e}{R_e} = \frac{\Delta W_a}{W_a} = \pm 1.1\%$$

The error in $\ln \frac{p_1}{p_2}$ is neglected, and

$$\frac{\Delta f}{f} = \frac{2\Delta W_a}{W_a} + \frac{\Delta \rho}{\rho} = (\pm 2.2) + (\pm 0.2) = \pm 2.4\%$$

Therefore, when the errors in the pressure measurement are considered as being only random, the maximum possible error in R_e is $\pm$ 1.1 per cent, and the maximum possible error in f is $\pm$ 2.4 per cent. However, if there were systematic errors in the pressure measurements as well, the maximum possible errors would be greater. It can be assumed that the probable error will be less than the maximum values determined, the determining factor being the signs of the flow and temperature errors.