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EFFICIENCY OF EXTENDED SURFACES WITH SIMULTANEOUS HEAT AND MASS TRANSFER

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RÉSUMÉ

Ce document présente un algorithme en vue de définir l'efficacité de surfaces étendues (ailettes circulaires ou longitudinales ayant une épaisseur uniforme) lorsque les transferts thermique et de masse se produisent simultanément. La répartition de la température sur la surface des ailettes est obtenue par la résolution d'une équation différentielle non linéaire de second ordre.

L'efficacité globale de la surface humide d'une ailette est déterminée en utilisant les écarts de température et les écarts hygrométriques particuliers agissant comme moteurs du transfert thermique et du transfert de masse respectivement. La notion d'efficacité d'une ailette à surface humide est semblable à la définition fondamentale d'une ailette à surface sèche.

Unique et exact, [illegible] la surface de l'ail[illegible] peut également servir [illegible] de refroidissement [illegible] fins précises.

No. 2744

Efficiency of Extended Surfaces with Simultaneous Heat and Mass Transfer

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ABSTRACT

An algorithm is presented to determine the efficiency of extended surfaces (circular or longitudinal fins with a uniform thickness) when simultaneous heat and mass transfer occur. The temperature distribution over the fin surface is calculated by solving numerically a nonlinear, second-order differential equation.

The overall fin efficiency of a wet surface is evaluated using the temperature and specific humidity differences as the driving forces for heat and mass transfer, respectively. The concept of efficiency of a wet surface fin is similar to the basic definition of a dry surface fin.

The presented algorithm is unique and accurate in determining the condition of the fin surface with regard to being dry or wet. This model could also be used to optimize the cooling and dehumidifying coil design for specific operating conditions.

INTRODUCTION

Air-to-water heat exchangers generally form the thermodynamic link between the water side (boilers and chillers) and the airside of heating, ventilating, and air-conditioning systems (HVAC) in buildings. Among the factors affecting the thermal performance of such heat exchangers are geometry and materials, air and water flow rates, and thermodynamic conditions. If latent heat transfer exists in cooling and dehumidifying coils, considerable change in the coil performance will occur as a result of the change in the extended surface efficiency.

Energy requirements for heating and cooling of buildings can be calculated by means of energy analysis computer programs, which simulate the performance of buildings and HVAC components.[1-3] An accurate analytical representation of all components of the building's energy system is necessary, however, to predict the building's energy requirements with accuracy.

The variation of extended surface efficiency must be known to model cooling-coil performance.[4,5] When phase change occurs (in dehumidification), the corresponding change in extended surface efficiency becomes an influential factor in determining coil performance under both full- and part-load conditions.[6]

This paper provides an analytical technique to be used in determining the efficiency of circular or continuous fins under conditions of simultaneous sensible and latent heat transfer. The concept of fin efficiency for dry fins is outlined and that for wet surface conditions determined using a similar approach.

DRY SURFACE FIN EFFICIENCY

In this section, fin efficiency, η, is defined and the basic approach for determining it is outlined. The coil fin will be dry when its surface temperature is higher than the dew-point temperature of the air adjacent to it. The driving force for heat transfer is then the temperature difference between the fin surface and the air. In most cases the temperature at the base of the fin, t_b, will be very close to the tube surface temperature. Owing to the thermal resistance of the fin material, there is a temperature gradient along the fin surface from base to tip. Thus, the heat-transfer potential of the fin is reduced in comparison with that of a "perfect" fin of infinite thermal conductivity.

The inescapable loss in performance of the real fin is characterized as fin efficiency, η, defined as the ratio of actual heat transfer through a fin surface to the heat transfer of an ideal fin at t_b.

Consider the constant-thickness circular fin shown in Fig. 1. A heat balance on a circumferential control volume leads to the following differential equation:

$$r^2 \frac{d^2t}{dr^2} + r \frac{dt}{dr} - r^2M_r^2 t = 0 \tag{1}$$

where

$$M = \sqrt{\frac{2f_o}{k\delta}}$$

$$t = t_r - t_a$$

Eq 1 can be solved to determine the temperature distribution over the fin surface using these boundary conditions:

$$t = t_b - t_a \qquad \text{at} \qquad r = r_o,$$

and

$$\frac{dt}{dr} = 0 \qquad \text{at} \qquad r = r_e$$

Fin efficiency can be obtained using the basic definition and temperature distribution over the fin surface resulting from Eq 1.

$$\eta_d = \frac{q}{q_{max}} = \frac{\int_A f_o \, t dA}{f_o(t_b - t_a)A} \tag{2}$$

The temperature distribution obtained by solving Eq 1 when substituted into Eq 2 gives the following expression for η_d:[7-10]

$$\eta_d = \frac{2r_o}{M\left(r_e^2 - r_o^2\right)} \left(\frac{I_1\ (Mr_e)\ K_1\ (Mr_o) - I_1\ (Mr_o)\ K_1\ (Mr_e)}{I_o\ (Mr_o)\ K_1\ (Mr_e) + I_1\ (Mr_e)\ K_o\ (Mr_o)} \right) \tag{3}$$

Eq 3 shows that fin efficiency is a function of fin geometry, the film heat-transfer coefficient, f_o, and fin material thermal conductivity, k, i.e.,

$$\eta_d = \psi\ (\phi,\ f_o) \tag{4}$$

where

ϕ is a nondimensional parameter defined as

$$\phi = (r_e - r_o) \sqrt{\frac{2\ f_o}{k\delta}} \tag{5}$$

WET SURFACE FIN EFFICIENCY

Condensation of moisture on the fin surface occurs whenever the fin temperature is below the dew-point temperature of the air adjacent to the surface. The difference between the fin surface temperature and the surrounding air temperature is the driving force for sensible heat transfer, whereas the difference between the specific humidity of saturated air at the fin surface temperature and that of the adjacent air is the potential for latent heat transfer.

Fin efficiency is considerably influenced by the presence of moisture and by the additional potential for heat transfer. Attempts have been made to determine η using a modified form of the factor M. For example, the wet surface efficiency of a straight longitudinal fin was studied by McQuiston.[11] He assumed that the difference between the enthalpy of air adjacent to the fin and that of saturated air at the fin surface temperature is the potential for combined heat and mass transfer. By approximating the saturation curve on the psychrometric chart by a straight line over a small range of temperatures, the slope of the saturation line between two temperatures, t_1 and t_2, may be expressed as

$$b_s = \frac{W_{s,2} - W_{s,1}}{t_2 - t_1} \tag{6}$$

The author presented only an expression of η for a longitudinal fin of length L, perimeter P, and area A

$$\eta = \frac{\tanh(M_1 L)}{M_1 L} \tag{7}$$

where

$$M_1 = \sqrt{\frac{f_o P}{kA}\left(1 + \frac{b_s\, i_{fg}}{c_p}\right)} \tag{8}$$

For a dry surface fin, b_s is equal to zero and Eq 8 reduces to the conventional form given in the literature.

Ware and Hacha assumed[12] that if the dry surface fin efficiency is expressed as

$$\eta_d = \psi(f_o) \tag{9}$$

then the wet surface fin efficiency can be written in terms of the slope of the saturated air temperature enthalpy curve, m, at the mean fin surface temperature, as follows:

$$\eta_w = \psi\left(f_o \frac{m}{c_p}\right) \tag{10}$$

Previous attempts to evaluate η_w failed to give an accurate expression based on the determination of the temperature distribution over the fin surface. As well, the complexity of the mathematical equations of various fin geometries has resulted in restricting the solution to longitudinal fins.

This paper presents a method of calculating the wet surface fin efficiency of a circular or a continuous fin of uniform thickness. Although the final results do not provide a closed form solution, they can easily be solved numerically with this technique.

Applying the first law of thermodynamics to the incremental area dA_f ($=2\pi r dr$) of a circular fin of uniform thickness, shown in Fig. 1,

$$dq = f_o (t_r - t_a)\, dA_f + f_m\, i_{fg} (W_{r,st} - W_a)\, dA_f \tag{11}$$

Rewriting Eq 11 in terms of radius r and integrating over the entire fin surface yields

$$q = \int_{r_o}^{r_e} 4\pi (f_o r t + f_m i_{fg} W r)\, dr \tag{12}$$

where

$$t = t_r - t_a$$

and

$$W = W_{s,r} - W_a$$

The maximum heat that would be transferred to the fin surface under the given conditions is

$$q_{max} = 2 \pi \left(f_o t_o + f_m W_o i_{fg} \right) \left(r_e^2 - r_o^2 \right) \tag{13}$$

where

$$t_o = t_b - t_a$$

and

$$W_o = W_{s,b} - W_a$$

By definition, the fin efficiency is

$$\eta_w = \frac{q}{q_{max}} \tag{14}$$

Substituting for q and q_{max} from Eqs 12 and 13 in Eq 14, then

$$\eta_w = \frac{2 \int_{r_o}^{r_e} \left(tr + \frac{i_{fg}}{Le\ c_p} Wr \right) dr}{\left(t_o + \frac{i_{fg}}{Le\ c_p} W_o\right) \left(r_e^2 - r_o^2\right)} \tag{15}$$

where

$$Le = f_o / f_m c_p \tag{16}$$

The temperature distribution over the fin surface is required to calculate η_w. The following differential equation of the temperature distribution can be derived:[13]

$$r \frac{d^2 t}{dr^2} + \frac{dt}{dr} - \frac{2f_o}{k\delta} \left[t + \frac{i_{fg}}{Le\ c_p} W \right] = 0 \tag{17}$$

The air specific humidity difference, W, is a function of the local fin surface temperature, t_r. Assuming a linear relation between W_s and t_r over a small temperature range simplifies the numerical integration of Eq 17 without sacrificing the accuracy of calculation, provided the temperature range is kept appropriately small. Therefore

$$W_{s,r} = a + b_s t_r \tag{18}$$

The constants a and b_s are to be determined from the psychrometric data for the range of water temperatures considered.

Substituting Eq 17 in Eq 18,

$$r \frac{d^2 t}{dr^2} + \frac{dt}{dr} - r \frac{2f_o}{k\delta} \left[t + \frac{i_{fg}}{Le\ c_p} \left\{ a + b_s (t + t_a) - W_a \right\} \right] = 0 \tag{19}$$

Thus, the temperature distribution over the fin surface can be determined by numerical integration of Eq 19. The resulting temperature distribution, together with Eq 15, can be used to determine the wet surface efficiency, η_w.

Eq 19 has been used to determine the temperature distribution over a fin surface for a range of conditions. The fin base temperature has been held constant at about 7°C (44.6°F), whereas the dry-bulb temperature of the air was varied between 16°C (60.8°F) and 32°C (89.6°F) and the air relative humidity varied between 0 and 100 per cent.

The dimensionless fin surface temperature, T, is plotted against the nondimensional fin radius, R, for various air psychrometric conditions and ϕ = 0.82 on Figs. 2 and 3. T and R are defined as

$$T = \frac{t_r - t_b}{t_a - t_b} \tag{20}$$

and

$$R = \frac{r - r_o}{r_e - r_o} \tag{21}$$

Fin efficiency is plotted as a function of ϕ on Figs. 4 and 5 for various specific humidities. The average film heat-transfer coefficient for wet surface fins is calculated using the method described in Refs 13 and 14.

The method described in this paper can be used to calculate the fin efficiency of a wet surface fin of a circular or continuous flat plate type. A continuous plate fin can be considered to be composed of discrete circular fins having the same surface area. This approximation does not result in a serious error on the evaluation of fin efficiency.[15,16]

Figs. 2 and 3 show that the temperature profiles of a wet surface fin lie below those of a dry surface fin. As the air relative humidity increases, the departure of the temperature profile from the dry surface curve becomes greater.

For a given set of conditions (ϕ, f_o, t_a, and t_b), the potential for heat and mass transfer increases as the air relative humidity increases. This results in an increase in the theoretical maximum heat transfer through the fin base, as expressed in Eq 13. The corresponding actual heat transfer through the fin surface, however, does not increase by the same amount, resulting in a decrease in fin efficiency. This is illustrated in Figs. 4 and 5 when plotting η vs ϕ.

The algorithm presented in this paper is a detailed and accurate model for determining the fin efficiency of a wet surface fin. It can be used to predict η_w for various fin geometries, different air psychrometric conditions, and fin base temperatures. Hence, an empirical formula of η_w could be developed as a function of all other variables using regression analysis techniques. This has been done successfully to predict heat exchanger performance.[4,5,14]

Another important result of this study is that coil designs could be optimized for specific operating conditions. The optimum fin length (radius) increases as the amount of dehumidification increases.

CONCLUSION

An algorithm to calculate the efficiency of a wet surface fin is presented for a circular or continuous flat plate fin with uniform thickness. This new approach, however, could be applied to other fin geometries.

The analytical results showed that a significant reduction in fin efficiency occurs with increasing amounts of dehumidification. The departure of fin efficiency from the dry surface condition depends on the difference between the specific humidity of the air surrounding the fin and air at fin base temperature.

The technique presented in this paper could be used to predict the performance of existing heat exchangers, and during the design stage to determine the optimum fin area for dehumidification processes.

NOMENCLATURE

Symbol	Description	Units
a	Constant in Eq 16	kg_w / kg_{air} (lb_w / lb_{air}
A	Cross-section area of a straight fin	m^2 (ft^2)
b_s	Slope of the saturation curve on the psychrometric chart	kg_w / $kg_{air} \cdot °C$ (lb_w / $lb_{air} \cdot °F$)
c_p	Moist air specific heat at constant pressure	$J/(k_g \cdot K)$ ($Btu/lb_g °F$)
f_m	Average mass transfer coefficient	k_g / $(m^2 \cdot s)$ ($lb/ft^2 \cdot sec$)
f_o	Average film heat-transfer coefficient on the air side	$W/(m^2 \cdot K)$ ($Btu/ft^2 \cdot hr \cdot °F$)
i_{fg}	Latent heat of evaporation of water	J/k_g (Btu/lb)
I_o, I_1	Modified Bessel functions of zero and first order, respectively	
k	Fin material thermal conductivity	$W/(m \cdot K)$ ($Btu/ft \cdot hr \cdot °F$)
K_o, K_1	Modified Bessel function of zero and first order, respectively	
ℓ	Straight fin length	m (ft)
Le	Lewis number	
m	Slope of $i - t$ saturation curve on the psychrometric chart	$J/(k_g \cdot °C)$ ($Btu/lb \cdot °F$)
M	Parameter defined by Eq 1	
M_1	Parameter defined by Eq 8	
p	Straight fin perimeter	m (ft)
q	Rate of heat transfer	W (Btu/hr)
R	Dimensionless fin radius, Eq 21	
r	Fin radius measured from the tube centerline	m (ft)
r_e, r_o	Outer and inner fin radius, respectively	m (ft)
T	Dimensionless temperature, Eq 20	
t_a	Air dry-bulb temperature	°C (°F)
t_b	Fin base temperature at $r = r_o$	°C (°F)
t_r	Fin surface temperature at radius r	°C (°F)
W_a	Specific humidity of air	kg_w / kg_{air} (lb_w / lb_{air})
$W_{s,r}$	Specific humidity of saturated air at temperature t_r	kg_w / kg_{air} (lb_w / lb_{air})

$W_{s,b}$ Specific humidity of saturated air at temperature t_b — kg_w / kg_{air} (lb_w / lb_{air})

ψ Function

ϕ Fin parameter defined by Eq 5

δ Fin thickness, m (ft)

η_d, η_w Dry and wet surface fin efficiency, respectively

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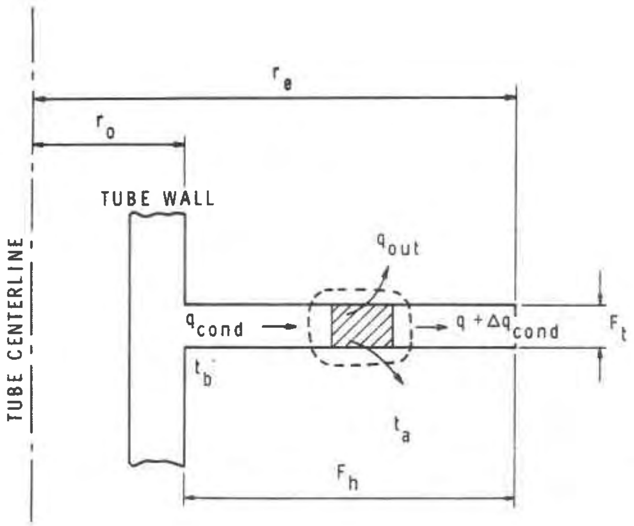

Figure 1. Schematic diagram of fin

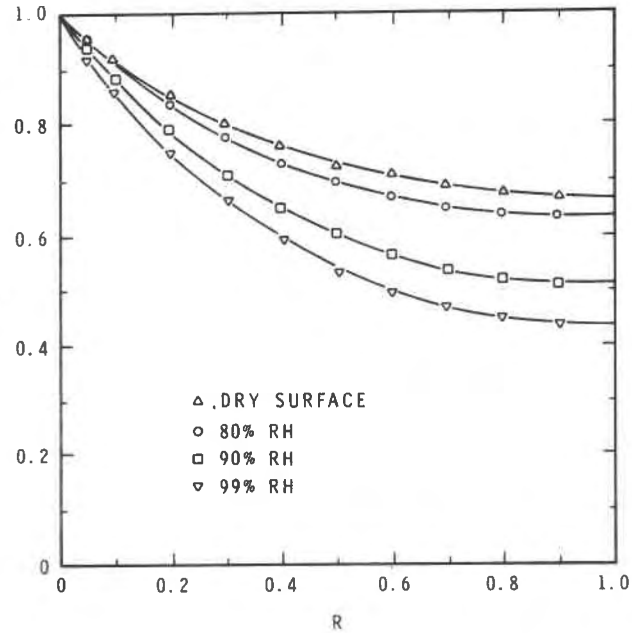

Figure 2. Fin surface temperature at $t_a = 60^\circ F$, $f_o = 10$ Btu/ft^2 - h$^\circ$F, $\phi = 0.82$, $t_b = 45^\circ F$

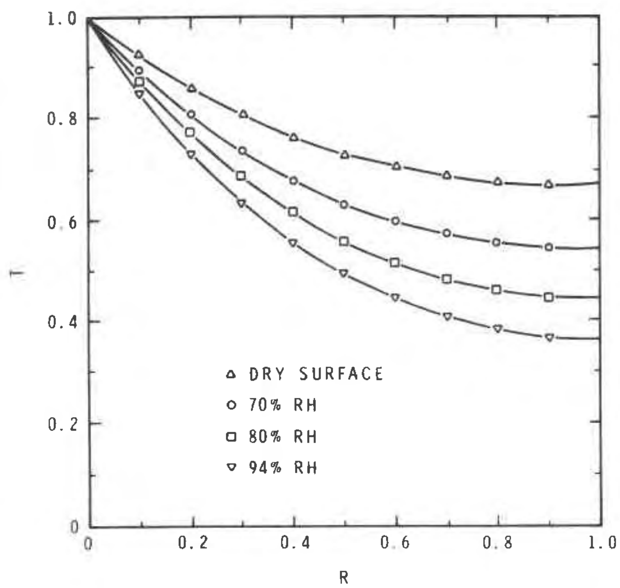

Figure 3. *Fin surface temperature at $t_a = 90^\circ F$, $f_o = 10$ Btu/ft^2 - h $^\circ$F $\phi = 0.82$, $t_b = 45^\circ F$*

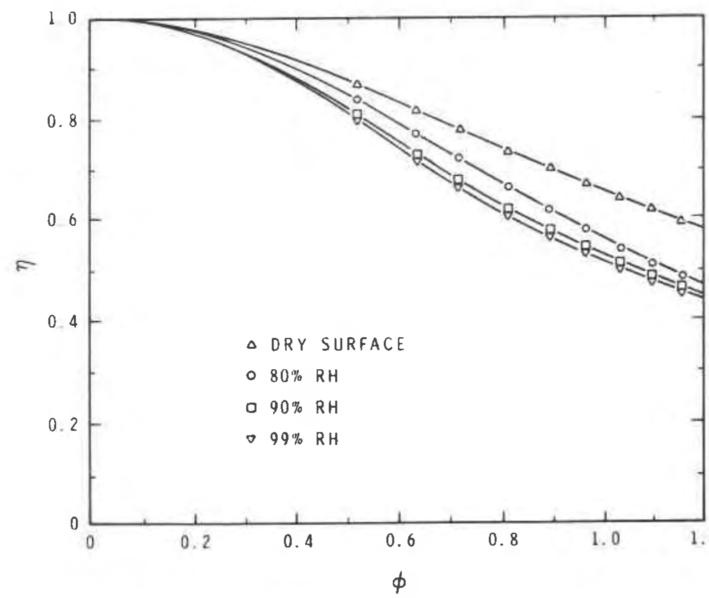

Figure 4. Fin efficiency at $t_a = 60^\circ F$, $t_b = 45^\circ F$

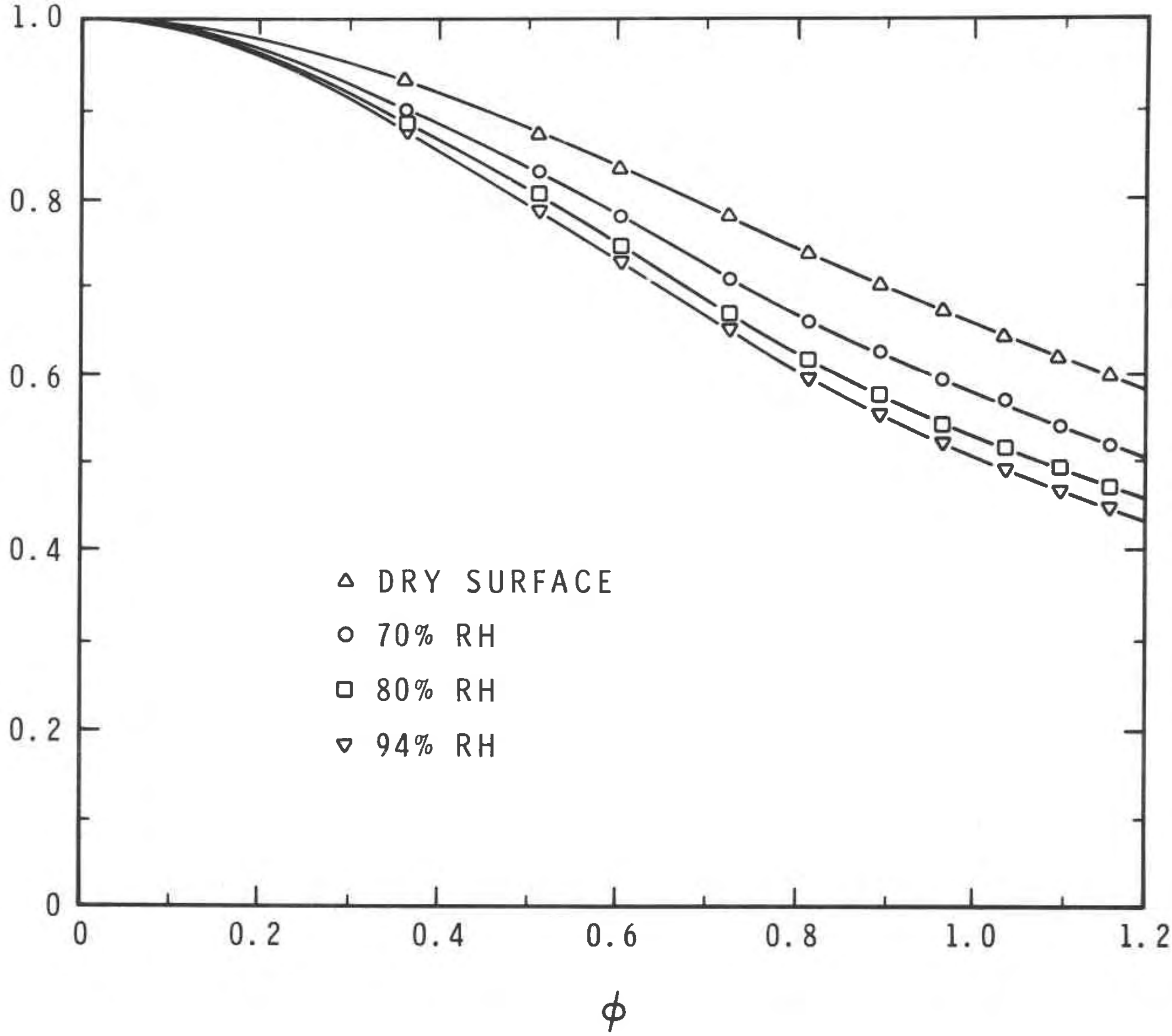

Figure 5. Fin efficiency at $t_a = 90^\circ F$, $t_b = 45^\circ F$