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Heat Transport Through Thermal Insulation: An Application of the Principles of Thermodynamics of Irreversible Processes

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RÉSUMÉ

Les auteurs utilisent les principes de la thermodynamique des processus irréversibles pour analyser la transmission de chaleur à travers les isolants thermiques secs. Ils considèrent le flux de chaleur comme la combinaison des modes de transfert de chaleur par conduction et par rayonnement, et ils en tirent des équations phénoménologiques. Les mesures réalisées en laboratoire leur permettent de déterminer le flux de chaleur traversant un spécimen d'isolant en fibre de verre pour vérifier la validité de cette méthode. Ils formulent une expression précisant la dépendance de la conductibilité thermique apparente de l'isolant sec à l'égard de la température. Les données expérimentales concernant un panneau de céramique sont bien représentées par l'expression d'une gamme de température de 800 K. Les auteurs proposent une extension de la méthode théorique pour décrire le transport simultané de chaleur et d'humidité à travers l'isolant thermique.

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ABSTRACT

The principles of irreversible thermodynamics are used to interpret heat transfer through dry thermal insulation. The heat flux is treated as coupled conductive and radiative modes of heat transfer and phenomenological equations are accordingly derived. Laboratory measurements of heat flux through a specimen of glass fibre insulation are used to check the validity of the approach. An expression is derived for the temperature dependence of the apparent thermal conductivity of dry thermal insulation. The experimental data for a ceramic board are well represented by the expression for a temperature range of 800 K. An extension of the theoretical method to describe simultaneous heat and moisture transport through thermal insulation is suggested.

KEY WORDS

Heat transfer, thermal insulation, thermodynamics of irreversible processes, rate of entropy production, phenomenological equations, moisture.

INTRODUCTION

Heat is transferred through dry thermal insulation by a combination of conduction and radiation. Models described in the literature interpret the heat transfer as the sum of solid conduction, gas conduction and radiation. In engineering applications, the interaction between conduction and radiation is usually neglected and the apparent thermal conductivity ($\lambda_{apparent}$) of a thermal insulation is written as

$$\lambda_{apparent} = \lambda_s + \lambda_g + \lambda_r \qquad (1)$$

where (λ_s) is the contribution from solid conduction, (λ_g) from gas conduction and (λ_r) from radiation. These models use Fourier relation and Rosseland approximation to write expressions for the heat flux (q) as

$$q = -\lambda'\frac{dt}{dx} - \lambda''T^3\frac{dt}{dx} \qquad (2)$$

where ($\frac{dt}{dx}$) is the temperature gradient in the direction of the heat transfer and (λ') and (λ'') are proportionality constants that describe conductive (solid + gas) and radiative modes of heat transfer, respectively. The use of Fourier relation and Rosseland's approximation, together with the theory of irreversible processes (T.I.P.) (1-6) can provide a method to look at the interaction between conductive and radiative modes of heat transfer in thermal insulation. This paper describes such an application of T.I.P. to heat transfer through thermal insulation. A three-term expression is derived for the heat flux through the insulation. One term represents the conductive part, the second, the radiative part and the third, the interaction between the two parts.

The basic postulates of T.I.P. are:

1) The rate (σ) of entropy production per unit volume is given by the sum of the products of fluxes (J_i) and associated forces (ϕ_i) i.e.

$$\sigma = \sum_i J_i\phi_i \qquad (3)$$

2) The components of the flux are linear functions of the components of forces and are given by phenomenological equations as

$$J_i = \sum_{k=1}^{i} L_{ik}\phi_k \qquad (4)$$

where the coefficients L_{ik} are called phenomenological coefficients.

3) The cross phenomenological coefficients, subject to certain mathematical restrictions, obey the Onsager reciprocal relations (7,8) i.e.

$$L_{ij} = L_{ji} \qquad (5)$$

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Thus for an irreversible process, associated with two simultaneous fluxes, J_1 and J_2

$$\sigma = J_1\phi_1 + J_2\phi_2 \tag{6}$$

where ϕ_1 and ϕ_2 are the forces corresponding to J_1 and J_2, respectively and

$$J_1 = L_{11}\phi_1 + L_{12}\phi_2$$

$$J_2 = L_{12}\phi_1 + L_{22}\phi_2$$

or

$$\begin{vmatrix} J_1 \\ J_2 \end{vmatrix} = \begin{vmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{vmatrix} \cdot \begin{vmatrix} \phi_1 \\ \phi_2 \end{vmatrix} \tag{7}$$

__Heat Transport and Entropy Production__

The entropy flux through any material is:

$$S = \frac{q}{T} . \tag{8}$$

The direction of the S vector is the same as the direction of the heat flux (q), since the temperature (T) is a scalar. If we make the x axis parallel to the entropy and heat flux vectors, the derivative $\frac{dS}{dx}$ is

$$\frac{dS}{dx} = \frac{1}{T} \cdot \frac{dq}{dx} - \frac{q}{T^2} \cdot \frac{dT}{dx} . \tag{9}$$

The second term of Eq. (9) is the rate at which entropy is being produced in unit volume of the material; i.e.,

$$\sigma = - \frac{q}{T^2} \cdot \frac{dT}{dx} . \tag{10}$$

When heat is transferred by conduction,

$$q_c \propto - \frac{dT}{dx} \tag{11}$$

Thus for heat conduction, from Eqs. (3), (4), (10) and (11)

$$J_c = \frac{q_c}{T} \tag{12}$$

$$\phi_c = - \frac{1}{T}\frac{dT}{dx} . \tag{13}$$

If the heat is transferred by radiation

$$q_r \propto - T^3 \frac{dT}{dx} \tag{14}$$

(this is the Rosseland approximation for thermal radiation). In this case

$$J_r = \frac{q_r}{T^{2.5}} \tag{15}$$

$$\phi_r = - T^{0.5} \cdot \frac{dT}{dx} . \tag{16}$$

When heat conduction and heat transfer by radiation occur together, the flux and force vectors have two components, and the phenomenological equation that interrelates these components becomes:

$$\begin{vmatrix} \frac{q_c}{T} \\ \frac{q_r}{T^{2.5}} \end{vmatrix} = - \begin{vmatrix} a\ , & b \\ b\ , & c \end{vmatrix} \cdot \begin{vmatrix} \frac{1}{T}\frac{dT}{dx} \\ T^{0.5}\frac{dT}{dx} \end{vmatrix} . \tag{17}$$

where a, b and c are the phenomenological coefficients.

Since q_c and q_r are in the same direction, they may be added algebraically to obtain the total heat flux

$$q = q_c + q_r = - (a + 2\ bT^{1.5} + cT^3)\frac{dT}{dx} . \tag{18}$$

The first term in the bracket gives the conductive part of the heat transfer, the third term, the radiative part and the second term, the interactive part. This indicates that the apparent thermal conductivity should be a quadratic function of $T^{1.5}$.

The objective of this paper is to check this hypothesis.

A TEST OF THE HYPOTHESIS

The form of the temperature dependence of the apparent thermal conductivity that is indicated by Eq. (18) can be checked by measuring the total rate of heat flow through semi-transparent materials. It is convenient to define a quantity Θ as

$$\Theta = a{\cdot}T + \frac{2b{\cdot}T^{2.5}}{2.5} + \frac{c{\cdot}T^4}{4} . \tag{19}$$

Then $$\frac{d\Theta}{dx} = (a + 2b{\cdot}T^{1.5} + c{\cdot}T^3)\frac{dT}{dx} . \tag{20}$$

Hence from (18) and (20),

then $$q = - \frac{d\Theta}{dx} . \tag{21}$$

If one measures the total heat flux (q_{1-2}) through a sample of thickness t, when the surfaces are kept at T_1 and T_2,

$$q_{1-2} = \frac{\Theta_1 - \Theta_2}{t} \tag{22}$$

$$= \frac{a}{t}(T_1 - T_2) + \frac{2b}{2.5t}(T_1^{2.5} - T_2^{2.5}) + \frac{c}{4t}(T_1^4 - T_2^4) . \tag{23}$$

Similar equations can be written when the surfaces are maintained at T_3 and T_4, T_5 and T_6, etc. These equations can be combined into a matrix form

$$\begin{vmatrix} (T_1-T_2), & (T_1^{2.5}-T_2^{2.5}), & (T_1^4-T_2^4) \\ (T_3-T_4), & (T_3^{2.5}-T_4^{2.5}), & (T_3^4-T_4^4) \\ \vdots & \vdots & \vdots \end{vmatrix} \cdot \begin{vmatrix} \frac{a}{t} \\ 0.8\ \frac{b}{t} \\ 0.25\ \frac{c}{t} \end{vmatrix} = \begin{vmatrix} q_{1-2} \\ q_{3-4} \\ \vdots \end{vmatrix} \tag{24}$$

and solved for $\frac{a}{t}$, $\frac{b}{t}$ and $\frac{c}{t}$.

Test Specimens

Two test specimens were prepared from a sheet of medium density glass fiber board produced by Johns Manville Co. for the National Bureau of Standards, Washington, D.C. The average density of the sample was 51 $kg \cdot m^{-3}$. Specimens were cut and placed in wooden frames. The dimensions of the specimens were 58.42 × 58.57 × 2.64 cm. The two specimens weighed 461.2 g and 461.3 g, and were identical in all other respects. These specimens were dried in an oven at 323 K for one week and then they were mounted on a 60 cm guarded hot plate apparatus built according to ASTM specifications (9). Steady state measurements were made at ten different sets of boundary temperatures.

EXPERIMENTAL RESULTS

The test results and the derived values of a, b and c are given in Table 1, along with values of heat flux calculated using these values of a, b and c and the boundary temperatures for the various tests. The standard deviation between the measured and calculated values of heat flux is 0.04 W/m^2 when the flux ranged from 23.4 to 52.6 W/m^2.

Table 1

Heat flow through a dry sample of medium density glass fiber insulation (51 $kg \cdot m^{-3}$); T_1 and T_2 are surface tempertures and q_{1-2} is the heat flux

T_1 (K)	T_2 (K)	q_{1-2} Measured W/m^2	q_{1-2} Calculated* W/m^2
293.9_2	273.9_7	23.47	23.43
303.1_1	283.1_6	24.24	24.29
312.6_3	293.4_8	24.21	24.24
322.0_8	303.2_5	24.75	24.78
331.6_1	313.6_2	24.72	24.66
343.1_3	323.3_0	28.47	28.43
354.0_2	333.8_8	30.21	30.24
363.4_6	343.2_1	31.65	31.66
314.2_2	286.3_3	34.90	34.93
333.6_1	293.8_6	52.61	52.60

$a = 1.896 \times 10^{-2}$ W/m•K
$b = 1.764 \times 10^{-7}$ $W/m \cdot K^{2.5}$
$c = 4.520 \times 10^{-10}$ $W/m \cdot K^4$
t = 26.4 mm

$$*q_{1-2} = \frac{a}{t}(T_1 - T_2) + \frac{2b}{2.5t}(T_1^{2.5} - T_2^{2.5}) + \frac{c}{4t}(T_1^4 - T_2^4)$$

As the NRC guarded hot plate apparatus is limited to temperatures between 273 K and 365 K, the validity of Eq. (16) could only be established over this limited range of temperature. However, some high temperature data were available from a set of round-robin measurements made on a ceramic material by seven laboratories in Canada and the United States (10). These data are in the form of the apparent thermal conductivity at four mean temperatures ranging from 533 K to 1363 K. These data were fitted by a quadratic in $T^{1.5}$. Table 2 gives these data and the values of a, b and c that were derived from them, and shows the calculated values of the apparent thermal conductivity. The quadratic in $T^{1.5}$ represents the data very well over this relatively wide range of temperatures. The values of the coefficients a, b, and c depend on the nature of the material, and may also depend on the emissivity of the bounding surfaces and the thickness of the sample. Measurements on similar samples of different thicknesses are needed to check on these possibilities. These results show, however, that these characteristic coefficients are not functions of temperature or temperature gradient, and that the analysis based on T.I.P. is valid for this type of insulation material. The interaction between the heat conduction and radiation is taken into account without having to establish the temperature distribution through the material.

Table 2

The apparent thermal conductivity ($\lambda_{apparent}$) of a ceramic board at different temperatures.

T (K)	$\lambda_{apparent}$ (W/m•K) Measured	$\lambda_{apparent}$ (W/m•K) Calculated*
533	0.073	0.073
813	0.114	0.113
1088	0.189	0.189
1363	0.312	0.311

$a = 5.40 \times 10^{-2}$ W/m•K
$b = 1.92 \times 10^{-7}$ $W/m \cdot K^{2.5}$
$c = 9.40 \times 10^{-11}$ $W/m \cdot K^4$

$$*\lambda_{apparent} = a + 2bT^{1.5} + c\,T^3$$

An Extension to Heat Transfer Through Moist Materials

The principal advantage of the TIP approach is that it can be extended to include additional components in the flux vector. An enclosed porous system that has moist material adjacent to a warm boundary will have vapour flowing from the warm region to the cooler region. The heat flux (q_v) associated with this vapour flow can be represented as,

$$q_v = -k\left(h \cdot \frac{dP_v}{dT}\right)\frac{dT}{dx}$$

where k = vapour permeability of the material,
h = enthalpy of saturated vapour at T,
P_v = saturation vapour pressure of water at T.

Around 300 K the product $h \cdot \frac{dP_v}{dT}$ can be approximated by $A \cdot T^{15.4}$.[1]

[1]This approximation has been derived from tabulated values of enthalpy and vapour pressure of water at saturation vs temperature. These data are published in ASHRAE Handbook of Fundamentals, 1985, Ch. 6, Table 2: p. 6.6.

This means that the flux (J_v) should be

$$J_v = \frac{q_v}{T^{8.7}} \qquad (25)$$

$$\phi_v = - T^{6.7} \frac{dT}{dx} . \qquad (26)$$

The phenomenological equation for heat transport by conduction, radiation and moisture migration becomes

$$\begin{vmatrix} \frac{q_c}{T} \\ \frac{q_r}{T^{2.5}} \\ \frac{q_v}{T^{8.7}} \end{vmatrix} = - \begin{vmatrix} a & b & d \\ b & c & e \\ d & e & f \end{vmatrix} \cdot \begin{vmatrix} T^{-1} \frac{dT}{dx} \\ T^{0.5} \frac{dT}{dx} \\ T^{6.7} \frac{dT}{dx} \end{vmatrix} . \qquad (27)$$

The coefficients a, b and c can be determined from tests on a dry sample and the coefficients d, e and f from additional tests on a moist sample of the same material. Of course, the component q_v of heat flux will only occur when there is moisture in the warm parts of the material. As soon as all the moisture has migrated to the coldest region, the moisture flux and associated heat flux will stop; the conduction and radiation will continue as long as a temperature gradient exists.

CONCLUSION

The postulates of the Thermodynamics of Irreversible Processes simplify the analysis of heat transfer in situations where more than one mode of heat transfer occurs. This approach takes account of energy shifting from one mode of transport to another within the material without having to determine the temperature distribution through the material. The approach, which has been shown to be applicable to heat transfer through thermal insulation, may also be applicable to heat transport through semi-transparent gases in the combustion chambers of furnaces and internal combustion engines.

It appears that the interaction between conductive and radiative modes of heat transfer amounts to approximately 6% of the heat flux through the two specimens, for the range of temperature reported in this paper. Further testing is needed to check the validity of Eq. (27) when moisture migration contributes to the heat transport through porous materials. The current study has demonstrated that heat transport through a semi-transparent medium like glass fiber insulation can be calculated by using three coefficients that characterize the material. The values of these characteristic coefficients can be determined from measurements of the total heat flow through a sample of the material with various combinations of the boundary temperatures. These characteristic coefficients appear to be independent of temperature over a wide range. In this respect, they are preferable to the value of the apparent thermal conductivity, which is a function of temperature. The apparent thermal conductivity can be calculated for any value of mean temperature when the values of the characteristic coefficients are known.

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