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RÉSUMÉ

La diffusivité thermique d'un matériau solide peut être déterminée à l'aide de paires d'éprouvettes identiques préparées pour l'essai de détermination de la conductivité thermique dans l'appareil à plaques chaudes gardées (ASTM C-177). Les éprouvettes sont placées l'une sur l'autre avec une sonde de température entre les deux, le tout étant ensuite placé entre les plaques de dissipation de chaleur de l'appareil à plaques chaudes. On fait varier la température de ces plaques à un taux constant.

On suppose que la conductivité thermique, la chaleur spécifique et la masse volumique du matériau sont constantes aux températures d'essai. [illegible] à une température [illegible] est 8,49 × 10 [illegible]

A Procedure for Determining the Thermal Diffusivity of Materials

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ABSTRACT

The thermal diffusivity of a solid material can be determined using pairs of identical specimens prepared for a thermal conductivity determination in a Guarded Hot Plate apparatus (ASTM C-177). Specimens are placed one on top of the other with a temperature sensor between them, and the pair is placed between the heat-sink plates of a Guarded Hot Plate apparatus. The temperature of these plates is varied at a constant rate.

The thermal conductivity, specific heat and density of the material are assumed to be constant over the range of temperature used for the test. For a pair of granite slabs, at a mean temperature of 8°C, the thermal diffusivity is 8.49×10^{-7} m²/s $\pm$ 0.02×10^{-7} m²/s.

KEY WORDS

Thermal diffusivity, heat capacity, specific heat, granite.

INTRODUCTION

THE THERMAL DIFFUSIVITY of a material is the ratio of its thermal conductivity to its volumetric specific heat. This ratio, which has the dimension m^2s^{-1}, is required for calculations of heat flow and temperature profile through materials when the conditions are not at steady state. It is also an important datum for calculating the thermal transfer function coefficients for walls and roofs. The ASTM book of Standards [1] indicates that thermal

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diffusivity can be obtained by measuring thermal conductivity, density and mass specific heat separately, but no procedure is given for determining thermal diffusivity directly. McElroy, Graves, Yarbrough and Tong [2] have presented a procedure for determining thermal diffusivity using an apparatus that was developed originally to measure thermal conductivity. Their procedure requires a rather lengthy analysis of test results, and represents the test condition by a slightly simplified model.

The procedure presented in this note also utilizes an apparatus that has been developed for measuring thermal conductivity. However, it utilizes very simple procedures for data analysis. It is presented here for consideration as a possible ASTM Standard procedure for determining the thermal diffusivity of materials. It utilizes the same specimens as are used to determine thermal conductivity with a guarded hot plate [1] and thus is complementary to that standard procedure. This provides an alternative way to obtain the specific heat of a specimen, which may be preferable to the standard method of mixtures for some types of material.

THE TEST SETUP AND PROCEDURE

Two identical specimens, of thickness L, are mounted between a pair of heat-sink plates as they would be for a thermal conductivity test in the Guarded Hot Plate (ASTM C-177) Method, except that there is no heater plate between them. A thermocouple placed between the two specimens measures the temperature, T_L, and another thermocouple mounted in the heat-sink plate measures the temperature T_0. A test begins with an initial conditioning period during which the heat-sink plates are controlled at a constant temperature. This is maintained long enough for $T_0 - T_L$ to settle to zero, i.e., for the specimens to achieve a uniform initial temperature. Then the temperature T_0 of the heat-sink plates is changed at a constant rate. The temperature T_0, and the temperature difference $T_0 - T_L$ are recorded at regular intervals. The test should continue for at least 30 minutes after $T_0 - T_L$ has reached a constant value.

DATA ANALYSIS

When

$$T_0 = a \cdot t \text{ for } t \geq 0$$

$$T_0 = 0 \text{ for } t \leq 0$$

$$T_L = a\left(t - \frac{\tau}{2} + \frac{16\tau}{\pi^3} e^{-\left(\frac{\pi}{2}\right)^2 \frac{t}{\tau}} - \frac{16\tau}{(3\pi)^3} e^{-\left(\frac{3\pi}{2}\right)^2 \frac{t}{\tau}} \cdots\right) \tag{1}$$

where

t = time since the start of the variation in T_0
$\tau = L^2/\alpha$
$\alpha = \lambda/\varrho c$
α = the thermal diffusivity of the material
λ = the thermal conductivity
ϱ = the density
c = the heat capacity

(This expression is taken from Carslaw and Jaeger [3]).

Thus

$$T_0 - T_L = a\left(\frac{\tau}{2} - \frac{16\tau}{\pi^3} e^{-\left(\frac{\pi}{2}\right)^2 \frac{t}{\tau}} + \frac{16\tau}{(3\pi)^3} e^{-\left(\frac{3\pi}{2}\right)^2 \frac{t}{\tau}} - + \ldots\right)$$

When $t \geq 2\tau$ the exponential terms become insignificant and $T_0 - T_L$ has a constant value, b.

Thus

$$\tau = \frac{2b}{a}$$

and

$$\alpha = \frac{L^2 a}{2b}$$

The data analysis only involves determining the mean values of a and b, and the probable error of these mean values.

SAMPLE RESULTS

Table 1 contains a typical set of test results for a pair of granite slabs. The mean thickness, $\bar{L}$, of the two specimens is 35.94 mm. This is the mean of 17 separate measurements on each specimen. The standard deviation, σ_L, of the thickness measurements is 0.033 mm. The probable error*, r_L in $\bar{L}$ is, therefore, ± 0.0054 mm.

The value of $\bar{a}$ is obtained from the average change in T_0 over 30-minute intervals based on the 15 independent values during the period from t =

*Probable error of the mean value of n independent measurements is $0.67\ \sigma/\sqrt{n}$ [4].

1.2 ks to $t = 4.68$ ks. The value of $\bar{a}$ is $-3.68_3 \times 10^{-3}$ K/s, and the probable error, $r_{\bar{a}}$, in this value is $\pm 0.0065 \times 10^{-3}$ K/s.

The exponential terms become negligible by 2τ after the start of the ramp, and $\tau \approx 1.5$ ks, thus $\bar{b}$ is the average of the values of $T_0 - T_L$, starting at $t = 3.0$ ks, minus the value when $t \leq 0$

$$\bar{b} = -2.825 - (-0.026) = -2.80 \text{ K}$$

with a probable error, $r_{\bar{b}}$ of ± 0.005 K.

Table 1. Test results for granite, slabs.

Time ks	$T_{0,t}$ °C	$T_{0,t} - T_{0,t-1.8}$ K	$T_{0,t} - T_{L,t}$ K	NOTES
−0.60	21.28		−0.02	
−0.48	21.28		−0.02	
−0.36	21.11		−0.02	Mean −0.026
−0.24	21.20		−0.05	
−0.12	21.22		−0.02	
0.00	20.98		−0.27	Start of Ramp
0.12	20.52		−0.64	
0.24	20.15		−0.94	
0.36	19.68		−1.29	
0.48	19.28		−1.52	
0.60	18.75		−1.72	
0.72	18.42		−1.89	
0.84	17.94		−2.12	
0.96	17.48		−2.19	
1.08	17.00		−2.29	
1.20	16.62		−2.35	
1.32	16.14		−2.47	
1.44	15.72		−2.50	
1.56	15.24		−2.55	
1.68	14.84		−2.60	
1.80	14.38		−2.61	
1.92	14.00		−2.66	
2.04	13.47		−2.69	
2.16	13.12		−2.71	
2.28	12.68		−2.74	
2.40	12.24		−2.74	
2.52	11.69		−2.77	
2.64	11.24		−2.82	
2.76	10.85		−2.83	
2.88	10.42		−2.78	

(continued)

Table 1. Test results for granite, slabs (continued).

Time ks	$T_{0,t}$ °C	$T_{0,t} - T_{0,t-1.8}$ K	$T_{0,t} - T_{L,t}$ K	NOTES
3.00	9.90	−6.72	−2.83	Start to average
3.12	9.46	−6.68	−2.83	
3.24	9.12	−6.60	−2.78	
3.36	8.66	−6.58	−2.84	
3.48	8.14	−6.70	−2.84	
3.60	7.72	−6.66	−2.84	
3.72	7.32	−6.68	−2.85	for a and b
3.84	6.92	−6.55	−2.82	
3.96	6.42	−6.70	−2.85	
4.08	5.98	−6.70	−2.83	
4.20	5.60	−6.64	−2.78	
4.32	5.17	−6.52	−2.83	
4.44	4.72	−6.52	−2.81	
4.56	4.30	−6.55	−2.81	
4.68	3.81	−6.61	−2.84	
Mean Value		−6.63	−2.825	
Standard Deviation		0.068	0.021	

Thus the value of the thermal diffusivity is

$$\alpha = \frac{(\bar{L})^2 \bar{a}}{2\bar{b}} = 8.49 \times 10^{-7}\ \mathrm{m^2/s}$$

The probable error in this value is

$$r_\alpha = \pm\alpha\left[\left(\frac{2r_L}{\bar{L}}\right)^2 + \left(\frac{r_{\bar{a}}}{\bar{a}}\right)^2 + \left(\frac{r_{\bar{b}}}{\bar{b}}\right)^2\right]^{1/2}$$

$$= \pm 0.02 \times 10^{-7}\ \mathrm{m^2/s}$$

SENSITIVITY TO ERRORS IN THE DATA

The value of α obtained by this procedure is not affected by any constant bias in the values of T_0 or T_L, because it is always a difference between two values that is used. It is also insensitive to any error in the calibration of the thermocouples, provided they all have the same e.m.f. per degree over the temperature range used for the test.

The time difference between the pairs of readings that are used to calculate $\bar{a}$ is 1.8 ks. The probable error in this value is in the order of milliseconds. But even if the timing system were not so precise, variations in the time between readings would be accounted for by the variance of the differences between the pairs of temperatures. Thus the interval between the pairs of readings can be taken as exactly 1.8 ks.

HEAT CAPACITY

The heat capacity of the specimens can be calculated from the measured values of λ, α, and ϱ:

$$c = \frac{\lambda}{\varrho \cdot \alpha} \tag{4}$$

For the granite samples $\lambda = 1.74_4$ W/m·K and $\varrho = 2640$ kg/m³.
Thus

$$c = 778 \text{ J/kg} \cdot \text{K}$$

This has a probable error of about ± 3 J/kg · K.

Reference [5] gives the heat capacity of granite as 775 J/kg · K, which tends to confirm the accuracy of the value obtained with this procedure.

CONCLUSION

The proposed procedure for determining thermal diffusivity of solids is simple to perform and the data analysis involves only simple calculation procedures. The resulting value of the thermal diffusivity has a probable error of the order of 0.2 percent. The apparatus is basically the same as is used for measuring thermal conductivity; and the same test specimens can be used for thermal conductivity, thermal diffusivity and density determinations. Values for these three properties can be used to calculate the specific heat of the samples. This method of obtaining specific heat leads to a value with a probable error of about 0.4 percent.

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BIOGRAPHY

Dr. Stephenson received a BASc in Engineering Physics from the University of Toronto in 1949, and a Ph.D. in Mechanical Engineering from the University of London in 1954. He joined the staff of the Division of Building Research of the National Research Council of Canada in 1954 and has remained with that organization ever since. He was head of the Building Services Section from 1969 to 1978 and then coordinated the research on energy conservation until that program was eliminated in 1984. He is currently a Principal Research Officer in the Building Services Section and is developing testing procedures for determining the dynamic thermal characteristics of walls.

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