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RÉSUMÉ

On montre que le flux de chaleur à travers les matériaux isolants fibreux peut être représenté par un ensemble de trois constantes matérielles dans une large gamme de températures d'intérêt pratique. Ces constantes peuvent être déterminées à l'aide de mesures standard faites au moyen de l'appareil à plaque chaude gardée.

# Heat Transport through Fibrous Insulation Materials

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**ABSTRACT**

It is shown that heat flux through fibrous insulation materials can be represented by a set of three material constants over a wide range of temperature of practical interest. These constants can be determined from standard guarded-hot-plate apparatus measurements.

**KEY WORDS**

Fibrous insulation, heat flux.

## INTRODUCTION

THE STANDARD PROCEDURE for measuring the thermal conductivity of thermal insulation is based on Fourier's equation,

$$q = -\lambda(dT/dx) \tag{1}$$

but recast as

$$\lambda = (q \cdot L)/(T_H - T_C) \tag{2}$$

In the above equations, $q$ is the heat flux at a steady state through a specimen of thickness $L$, $(dT/dx)$ is the temperature gradient at a location in the direction of the heat flux, and $\lambda$ the thermal conductivity at that location. The temperature gradient is approximated in Equation (2) by $-(T_H - T_C)/L$

where $T_H$ and $T_C$ are respectively the hot and cold surface temperatures of a specimen. The thermal conductivity so calculated is usually referred to as the apparent thermal conductivity at the mean temperature $T_M$ defined as $(T_H + T_C)/2$.

Equation (2) gives the right value for $\lambda$ when it is either independent of temperature or linearly dependent on $T_M$. The latter condition is often fulfilled for small values of $(T_H - T_C)$. Unfortunately this constraint means that errors in measuring the values of $T_H$, $T_C$ and $q$ have a relatively large effect on the accuracy of $\lambda$. It would be desirable, therefore, to be able to use larger values of $(T_H - T_C)$ and consequently larger values of $q$. It would also be beneficial to be able to determine how $\lambda$ varies with temperature. This note presents a way of achieving both of these objectives using the results from standard types of apparatus for measuring $\lambda$.

For fibrous insulating materials, it has been shown recently [1] that the heat flux is related to $T_H$ and $T_C$ by

$$q = (a/L)(T_H - T_C) + (b/2.5L)(T_H^{2.5} - T_C^{2.5}) + (c/4L)(T_H^4 - T_C^4) \quad (3)$$

Equation (3) may be written in differential form as

$$q = -(a + b \cdot T^{1.5} + c \cdot T^3)(dT/dx) \quad (4)$$

Comparison of Equations (1) and (4) gives, for the above materials,

$$\lambda = a + b \cdot T^{1.5} + c \cdot T^3 \quad (5)$$

As practical problems usually involve calculating $q$, and $\lambda$ is just a datum used in the calculation, it is better to utilize the $a$, $b$ and $c$ constants directly in Equation (3). In fact, the concept of an apparent thermal conductivity that is a function of mean temperature is of limited value when dealing with materials like fibrous insulation for a wide range of temperature.

Equation (3) was derived on the basis of the theory of irreversible processes [1]. The constant $(a)$ was identified as representing the conductive part of the heat transport, $(c)$ as the radiative part and $(b)$ as an interaction between radiation and conduction. It was implied [1] that these constants have the status of a set of material properties, independent of temperature and size. The experimental results reported in this note confirm that this is a valid assumption.

## MATERIALS AND METHOD

Six slabs of medium density glass fibre insulation were used in this in-

*Table 1. Heat flux through the combinations of specimen pairs I-A/B and II-A/B at various sets of hot surface temperature $T_H$ and cold surface temperature $T_C$; q(exp) is the measured heat flux and q(cal) the heat flux calculated according to Equation (3).*

| $T_H$ (K) | $T_C$ (K) | q(exp) ($W \cdot m^{-2}$) | q(cal) ($W \cdot m^{-2}$) |
|---|---|---|---|
| *combination of I-A/B; thickness 5.28 cm* | | | |
| 308.19 | 286.33 | 13.53 | 13.52 |
| 318.40 | 286.75 | 19.97 | 20.00 |
| 326.73 | 287.08 | 25.46 | 25.51 |
| 336.98 | 287.56 | 32.31 | 32.53 |
| *combination of I-A/B and II-A/B; thickness 10.54 cm* | | | |
| 306.98 | 285.87 | 6.50 | 6.52 |
| 317.08 | 286.09 | 9.77 | 9.77 |
| 327.89 | 286.31 | 13.38 | 13.42 |
| 337.93 | 286.53 | 16.80 | 16.95 |

*Table 2. Heat flux through the specimen pair III-A/B at various sets of hot surface temperature $T_H$ and cold surface temperature $T_C$; q(exp) is the measured heat flux and q(cal) is the heat flux calculated according to Equation (3).*

| $T_H$ (K) | $T_C$ (K) | q(exp) ($W \cdot m^{-2}$) | q(cal) ($W \cdot m^{-2}$) |
|---|---|---|---|
| | $(T_H - T_C) < 25$ K* | | |
| 304.32 | 282.09 | 19.75 | 19.70 |
| 308.00 | 286.30 | 19.54 | 19.59 |
| 309.41 | 287.65 | 19.84 | 19.77 |
| 322.97 | 302.39 | 19.85 | 19.98 |
| 323.24 | 302.88 | 19.87 | 19.80 |
| 336.84 | 316.97 | 20.63 | 20.62 |
| 356.88 | 337.32 | 22.45 | 22.32 |
| 370.72 | 348.07 | 27.35 | 27.39 |
| | $(T_H - T_C) \gg 25$ K** | | |
| 415.64 | 326.04 | 113.7 | 115.0 |
| 430.76 | 326.26 | 139.4 | 139.4 |
| 445.49 | 326.24 | 163.3 | 165.0 |
| 469.95 | 327.55 | 209.3 | 210.0 |
| 494.44 | 326.80 | 258.1 | 262.6 |
| 518.88 | 327.31 | 376.5 | 383.3 |
| 566.85 | 327.55 | 445.6 | 450.6 |

* Only these measurements were used to estimate the constants a, b and c for III-A/B:

$a = 1.337 \times 10^{-2}\ W \cdot m^{-1} \cdot K^{-1}$

$b = 2.066 \times 10^{-6}\ W \cdot m^{-1} \cdot K^{-2.5}$

$c = 4.112 \times 10^{-10}\ W \cdot m^{-1} \cdot K^{-4}$

** These measurements were done at the Thermal Testing Laboratory of Fibreglas Canada Inc., Sarnia.

vestigation. These six slabs formed three pairs of test specimens as follows:

1. Specimen pair I-A/B, each 58.4 cm × 58.6 cm and thickness 2.64 cm; they weighed 461.2 g and 461.3 g respectively when dried.
2. Specimen pair II-A/B, each 58.4 cm × 58.4 cm and thickness 2.63 cm; they weighed 482.0 g and 483.0 g respectively when dried.
3. Specimen pair III-A/B, each 57.9 cm × 57.9 cm and thickness 3.85 cm; they weighed 626.8 g and 628.1 g respectively when dried.

The pairs I-A/B and II-A/B were prepared from the same batch of material, and the specimen pair III-A/B was prepared from a different batch of material.

The experimental quantity determined was always the steady state heat flux through the slabs for known values of hot and cold surface temperatures. The measurements were done either on a guarded hot plate (GHP) apparatus [2] or on a heat flow meter (HFM) apparatus [3].

As reported earlier [1], the specimen pair I-A/B was used in ten GHP measurements, in different ranges of $T_H$ and $T_C$. Analysis of those results gave,

$$a = 1.896 \times 10^{-2}\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$$
$$b = 3.528 \times 10^{-7}\ \mathrm{W \cdot m^{-1} \cdot K^{-2.5}}$$
$$c = 4.520 \times 10^{-10}\ \mathrm{W \cdot m^{-1} \cdot K^{-4}}$$

Subsequently, this pair of specimens was placed one over the other in the HFM apparatus to form a specimen of total thickness, 5.28 cm and the heat flux through the combined slab was determined at four pairs of hot and cold surface temperatures.

Yet another specimen of total thickness, 10.54 cm was obtained by placing the specimen pair II-A/B over the above combined slab and the heat flux through the combination of four slabs was determined at four pairs of hot and cold surface temperatures. The results from the above eight sets of measurements are listed in Table 1.

The specimen pair III-A/B was used in several GHP measurements in the temperature range 282 to 567°K. In an initial set of measurements, in the temperature range 282 to 371°K, the temperature difference ($T_H - T_C$) was always lower than 25°K. But in a subsequent set of measurements, the temperature difference was as high as 239.3°K. The results from these measurements are summarized in Table 2.

## DISCUSSION

The constants $a$, $b$ and $c$ evaluated from the earlier measurements [1] and quoted above for the specimen pair I-A/B were used to calculate the heat fluxes using Equation (3) through the combination of two or four slabs at the different sets of $T_H$ and $T_C$ given in Table 1. These calculated values are also listed in Table 1. It can be seen that the agreement between the measured and calculated values is excellent in all the cases. This suggests that the constants *a*, *b* and *c* may be treated as true material constants independent of the thickness of the specimen or temperature.

The results in Table 2, at small differences in $T_H$ and $T_C$ alone, were used in Equation (3) to evaluate the constants *a*, *b* and *c* for the specimen pair III-A/B. These constants were then used to calculate the heat flux in the whole experimental range of temperature. The values so calculated are also listed in Table 2. The largest deviation between the measured and calculated heat fluxes is 1.8%. A deviation of this magnitude is well within the precision of the experimental technique. This agreement once again confirms that for glass fibre insulation an equation like Equation (3) with three coefficients can represent heat flux through the insulation over a wide range of temperature of practical interest.

The material in specimens I-A/B and II-A/B differed by about 6% in density, but were made of the same type of glass fibre and binder, and have essentially the same values for the characteristic constants *a*, *b* and *c*, as illustrated in Table 1. Specimens III-A/B, on the other hand, had nearly the same density, but were made of smaller diameter fibres and possibly had a different binder as well. Its characteristic constants are different from the other specimens, particularly the *b* coefficient, which is nearly six times larger than for the other specimens. Probably, the values of these coefficients reflect the fibre size and binder characteristics.

The heat flux under steady-state conditions through a layer of material can be calculated directly using Equation (3), or it can be obtained using Equations (2) and (5). This latter procedure is valid when $T_H - T_C$ is small, but

*Table 3. Comparison of heat fluxes, q, calculated from two different equations at various hot and cold surface temperatures; the value of λ corresponds to the mean temperature, $T_M$.*

| $T_H$ (K) | $T_C$ (K) | $T_M$ (K) | $\lambda$ ($W \cdot m^{-1} \cdot K^{-1}$) | $q = (\lambda/L)(T_H - T_C)$ ($W \cdot m^{-2}$) | q from Equation (3) ($W \cdot m^{-2}$) |
|---|---|---|---|---|---|
| 300 | 250 | 275 | 0.02997 | 14.98 | 15.02 |
| 350 | 250 | 300 | 0.03300 | 33.00 | 33.34 |
| 400 | 250 | 325 | 0.03654 | 54.81 | 56.08 |
| 450 | 250 | 350 | 0.04065 | 81.30 | 84.51 |

$a = 1.896 \times 10^{-2}\ W \cdot m^{-1} \cdot K^{-1}$
$b = 3.528 \times 10^{-7}\ W \cdot m^{-1} \cdot K^{-2.5}$
$c = 4.520 \times 10^{-10}\ W \cdot m^{-1} \cdot K^{-4}$
$L = 0.100$ m
$\lambda = a + bT_M^{1.5} + cT_M^3$

is not appropriate for large values of $T_H - T_C$. The error due to using the thermal conductivity for the mean temperature is illustrated by the values in Table 3. This shows that when the temperature drop across a layer of material is large, Equation (2) underestimates $q$ even though the value of $\lambda$ that is used is correct for the mean temperature. Equation (3), on the other hand, as shown above, gives the correct value of $q$ for any values of $T_H$ and $T_C$.

The principal advantage, however, of Equation (3) is that the coefficients $a$, $b$ and $c$, are independent of the temperature. Thus they can be determined from the results of a series of tests with temperatures in a range that is convenient for testing, and then used for temperatures that are outside of that range.

Another advantage is that the values of $b$ and $c$ can also be used to calculate the value of $d\lambda/dT$ for any temperature. This is required as a datum when solving the differential equation that describes the temperature and heat flux under non-steady-state conditions.

## CONCLUSIONS

The ASTM Standard Test Methods C177-85 and C518-85 can give accurate values of the thermal conductivity at the mean of the hot and cold plate temperatures, provided the difference between these temperatures is not too large. But this conductivity should only be used when the temperature difference across a layer of material is small. These limitations can be overcome by using Equation (3) with the three coefficients that are properties of the material.

It would be desirable to revise the ASTM Standard Test Methods to give the $a$, $b$ and $c$ coefficients rather than just $\lambda$. When the $a$, $b$, $c$ values are known they can be used to evaluate both $\lambda$ and $d\lambda/dT$ for any $T$.

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## BIOGRAPHIES

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