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THERMAL DIFFUSION OF WATER VAPOUR THROUGH GLASS FIBRE INSULATION

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ABSTRACT

A set of published results of thermal tests on glass fibre insulation (when wet and dry) have been analyzed to derive the thermal diffusion coefficient for water vapour through glass fibre insulation. The hypothesis is that the vapour flux is related to the gradients of vapour concentration and temperature by an extended form of Fick's Law: viz.

$$\frac{JV}{\rho_v} = -\alpha\left(\frac{1}{\rho_v}\frac{d\rho_v}{dx}\right) - \beta\left(\frac{1}{T}\frac{dT}{dx}\right)$$

The tests on the dry specimen were simulated to obtain the coefficients that relate the flux by conduction and radiation to the temperature profile. These coefficients were used to calculate the conduction and radiation components of the total energy flux for the wet tests, but based on the temperature profile that obtained for these tests. The vapour flux (and associated energy) was calculated using the extended version of Fick's Law. For both the wet and dry condition simulations the temperature profile was defined at a set of evenly spaced nodal points through the specimen. The temperature at each nodal point was adjusted by a relaxation process to make the heat flux into the warm boundary match the test result.

For the wet tests the value of α was based on the vapour permeance of the specimen that had been determined by a standard permeance test under isothermal conditions. The value of β was found by a regula-falsy process that made the calculated heat flux through the warm (wet) boundary match the test result. It was found that the same value for β was appropriate for all of the tests with different boundary temperatures. Thus, this appeared to be the correct value for the thermal diffusion coefficient for vapour diffusion through this specimen.

KEY WORDS: water vapour diffusion, heat-vapour transfer, thermal diffusion

INTRODUCTION

Under isothermal conditions, as usually obtain when vapour permeance is determined, the flux of water vapour, JV, through a layer of insulation is proportional to the gradient of the partial pressure of the water vapour, *viz.*

$$JV = -\delta\frac{dP_v}{dx} \qquad (1)$$

P_v is the partial pressure of water vapour
δ is the permeability for water vapour

But for most situations in buildings, where water vapour migrates through insulation, there is a temperature gradient as well as a gradient of vapour pressure. The temperature gradient causes the vapour flux to be greater than it would be under isothermal conditions. Regardless of which potential function is causing the vapour to migrate it is a diffusive phenomenon, which should be characterized by diffusion coefficients. Therefore, the hypothesis for this study is that the vapour flux is related to the gradients of the vapour density and the temperature by an extended form of Fick's Law, *viz.*

$$\frac{JV}{\rho_v} = -\alpha\left(\frac{1}{\rho_v}\frac{d\rho_v}{dx}\right) - \beta\left(\frac{1}{T}\frac{dT}{dx}\right) \qquad (2)$$

where α and β are diffusion coefficients that are specific to the material that the vapour diffuses through. The objective of this paper is to test this hypothesis.

It is difficult to measure directly the rate of vapour migration through a material such as glass fibre when there is a temperature gradient through the material. Kumaran (1987) adopted an indirect way: he measured the total heat flux through a layer of wet insulation and obtained the component that was due to the vapour flux by subtracting the components due to conduction and radiation from the total heat flux.

Kumaran's paper gives the heat flux into a specimen of low density glass fibre insulation under steady state conditions; first when the material is wet and then when it is dry. The tests cover a range of temperature from 285 K to 329 K. And in addition to these thermal data he also included the isothermal permeance of the specimen. These data are just what is required to test whether the extended form of Fick's Law is valid for vapour diffusion through glass fibre insulation.

THE TESTS

Kumaran used a slab of glass fibre insulation enclosed in a sealed polyethylene envelope with some water in the envelope. The specimen was installed in a horizontal heat flow meter apparatus of the type that is commonly used to measure the thermal resistance of dry specimens. The heat flux into the specimen was recorded for several hours for selected pairs of hot and cold surface temperatures. Initially all the water was at the hot surface (at the bottom). The heat flux started high and soon settled to a steady value, Q_{WET}, and remained at this value for a few hours. It then started to decrease and eventually reached another steady value, Q_{DRY}. Figure 1 shows the result from one of these tests.

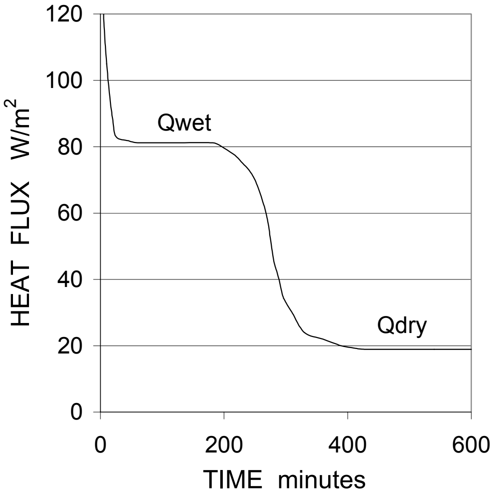

FIGURE 1. Typical Test Result

THE SIMULATION OF THE TESTS

The Kumaran tests have been simulated using Equation 2 to calculate the vapour flux (and the associated energy flux) at a set of evenly spaced nodal points between the hot and cold boundaries. This required determining the temperature at each of these nodal points as well as the heat flux due to conduction and radiation. The temperature profile was obtained by a relaxation process that made the calculated heat flux into the specimen under steady state conditions match Q_{WET} when there was vapour migration and Q_{DRY} when there was no vapour flux.

This procedure requires having an algorithm relating each component of heat flux to the temperature profile.

DRY PHASE

The conduction is primarily by the air in the space between the fibres, thus:

$$QC_j = -\lambda_j \left(\frac{dT}{dx} \right)_j \qquad (3)$$

λ_j is the thermal conductivity of air at temperature T_j

j is the nodal number

The component due to radiation, QR_j, is related to the temperature profile by a matrix of coefficients, *viz.*

$$\begin{bmatrix} QR_0 \\ QR_1 \\ \cdot \\ \cdot \\ \cdot \\ QR_N \end{bmatrix} = \begin{bmatrix} \\ \\ & CM & \\ \\ \\ \end{bmatrix} \bullet \begin{bmatrix} T_0^4 \\ T_1^4 \\ \cdot \\ \cdot \\ \cdot \\ T_N^4 \end{bmatrix} \qquad (4)$$

The coefficients in the CM matrix are functions of the emissivity, e, of the boundary surfaces; the emissivity, B, of the glass fibres; and the extinction coefficient, E. The extinction coefficient indicates the opacity of the material for thermal radiation. Reference 2 outlines a procedure for calculating the coefficients that obtain for specific values of E, B, e and N, the number of nodal points. These radiation properties of the specimen were established by simulating the dry phase of each test and systematically varying the value of E to obtain a calculated value of QC plus QR that is the same at every nodal point, and is equal to Q_{DRY}.

Table 1. Test and Simulation Results – Dry

Test #	T_{HOT} K	T_{COLD} K	Q_{DRY} W/m²	Q_{CALC} W/m²
1	312.5	284.7	12.6	12.64
2	312.6	271.6	18.1	18.02
3	323.8	285.2	18.1	18.12
4	329.2	290.0	18.9	18.90

Table 1 gives the results of four tests that Kumaran made on an 85.5 mm thick slab of low density glass fibre insulation (17 kg/m^3). The values shown for Q_{CALC} were obtained using CM that was based on $E = 940\text{m}^{-1}$, $B = 0.90$, e_0, $e_N = 0.90$ and $N = 125$.

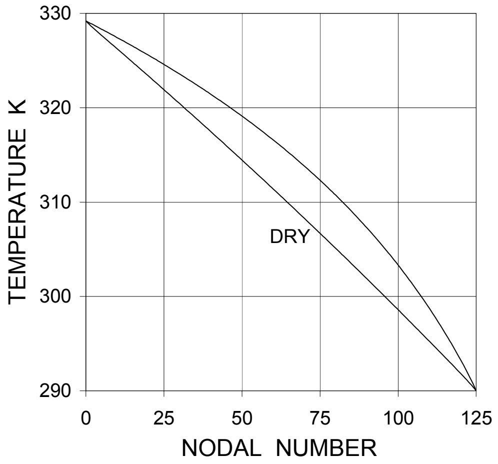

FIGURE 2. Temperature Profiles

Figure 2 shows the calculated temperature profile through the specimen for test number 4. Figure 3 shows QC, QR and their sum Q for locations through the specimen for this test. These values were calculated using N = 125. The calculated value of Q is 18.904 W/m^2. With N = 115, Q = 18.903 W/m^2, and with N = 105, Q = 18.901 W/m^2. These small differences are due to the use of finite difference approximations for the derivatives of T and the components of QR. These approximations are four-point central difference expressions for all nodal points more than one interval from the boundaries.

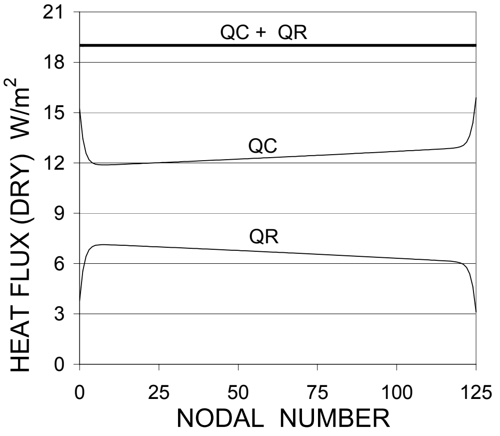

FIGURE 3. Heat Flux (Dry)

WET PHASE

The energy flux, QV_j, associated with the moisture flux, JV_j, is

$$QV_j = h_j JV_j \tag{5}$$

h_j is the enthalpy of saturated water vapour at temperature T_j

This analysis of Kumaran's test results is based on the assumption that

$$\left(\frac{JV}{\rho_v}\right)_j = -\alpha\left(\frac{1}{\rho_v}\frac{d\rho_v}{dx}\right)_j - \beta\left(\frac{1}{T}\frac{dT}{dx}\right)_j \tag{6}$$

and
$$\frac{1}{\rho_v}\frac{d\rho_v}{dx} = \frac{1}{P_v}\frac{dP_v}{dx} - \frac{1}{T}\frac{dT}{dx} \tag{7}$$

Thus

$$JV_j = \frac{-\alpha}{R_v T_j}\left(\frac{dP_v}{dx}\right)_j - \frac{(\beta-\alpha)(P_v)_j}{R_v T_j^2}\left(\frac{dT}{dx}\right)_j \tag{8}$$

and
$$\alpha = R_v T_j \delta \tag{9}$$

R_v is the gas constant for water vapour

The objective of this analysis of the results of the tests is to find a value for β that will give calculated values for the heat flux into the hot side of the specimen that match the measured values of Q_{WET} for all of the tests.

DETERMINATION OF VAPOUR FLUX AND THE THERMAL VAPOUR DIFFUSION COEFFICIENT

The value of the thermal vapour diffusion coefficient, β, can be determined by a similar type of relaxation process as used to find the extinction coefficient. Five assumptions are involved:

1. The vapour pressure at each nodal point is the saturation vapour pressure over water at the temperature at that nodal point.
2. The condensate that forms at each nodal point remains at that location.
3. The radiation coefficient matrix, CM, that was determined from the simulation of the

dry phase of the test is valid for the phase with vapour flux.

4. The thermal conductivity of saturated air (100% RH) is the same as for the much lower humidity that obtained for the final dry phase of the test.

5. The water vapour in the saturated air-vapour mixture acted like a perfect gas, i.e.

$$\rho_v = \frac{P_v}{R_v T}$$

On the basis of these assumptions QC_j, QR_j, JV_j and QV_j were calculated using equations (3), (4), (5), (8) and (9). The simulation started with $\alpha = 1.85\times10^{-5} m^2/s$, which corresponded with the higher of two values of permeance of the specimen reported by Kumaran, and an initial estimate for β of $10^{-4} m^2/s$.

The relaxation process started from a linear temperature profile between the hot and cold boundary temperatures and incremented the temperature at each internal nodal point, (i.e. $1 \le j \le N-1$, T_0 and T_N being the fixed boundary temperatures).

$$T_j = T_j - k\left(\frac{dQ}{dx} - H\cdot\frac{dJV}{dx}\right)_j$$

H is the enthalpy of liquid water at T

k is a relaxation factor, that must be small enough that the relaxation process converges

This relaxation continued until a temperature profile was achieved that satisfied the requirement

$$\left(\frac{dQ}{dx}\right)_j = H_j\left(\frac{dJV}{dx}\right)_j$$

at every internal nodal point.

The heat flux into the hot side of the specimen is

$$Q_{IN} = Q_0 - H_0 JV_0 \tag{10}$$

and the heat flux out at the cold boundary is

$$Q_{OUT} = Q_N - H_N JV_N \tag{11}$$

When the initial calculated value for Q_{IN} does not match the test value of Q_{WET}, the process is repeated with different values for β until the calculated Q_{IN} is the same as Q_{WET}. This process was repeated for each of the four tests.

Kumaran reported two different values for the permeance of the specimen so values of β were determined for three different values of α. The results are given in Table 2. The temperature profile for the wet phase of test 4 is shown in Figure 2. Figure 4 shows the vapour pressure through the specimen that corresponds with this temperature profile.

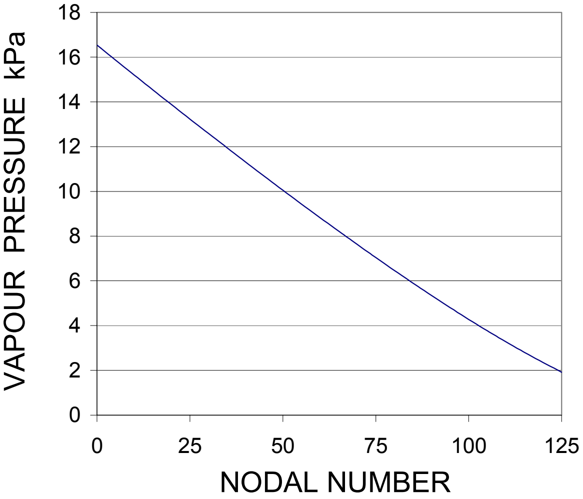

FIGURE 4 Vapour Pressure Profile

Figure 5 shows the vapour flux at each nodal point.

At all of the nodal points from j = 1 to j = 122 the values of $\left(\frac{d\rho_v}{dx}\right)_j$ and $\left(\frac{\rho_v}{T}\frac{dT}{dx}\right)_j$ lead to a negative value for $\left(\frac{dJV}{dx}\right)_j$ indicating that some of the vapour must condense.

But adjacent to the cold boundary these gradients were sufficient to cause $\left(\frac{dJV}{dx}\right)_j$ to be positive. This could not happen as there would not be any water there to evaporate. Thus the vapour flux was constant for the final three nodes.

TABLE 2. Test and Simulation Results – Wet

Test Values			Simulation Results					
T_{HOT} K	T_{COLD} K	Q_{WET} W/m^2	α mm^2/s	β mm^2/s	Q_{IN} W/m^2	JV_0 mg/m^{2}s	Q_{OUT} W/m^2	JV_N mg/m^{2}s
312.5	284.7	38.9	17.5 18.0 18.5	110 102 95	38.62 38.69 38.72	13.16 13.19 13.20	39.85 39.93 39.96	8.59 8.63 8.64
312.6	271.6	48.0	17.5 18.0 18.5	110 102 95	47.94 48.03 48.07	16.35 16.39 16.40	52.38 52.49 52.54	7.45 7.49 7.51
323.8	285.2	67.5	17.5 18.0 18.5	110 102 95	67.40 67.49 67.52	24.92 24.96 24.97	70.64 70.74 70.77	15.67 15.72 15.75
329.2	290.0	81.2	17.5 18.0 18.5	110 102 95	81.38 81.34 81.47	31.01 30.99 31.04	85.61 85.58 85.72	21.07 21.07 21.14

$E = 940\ m^{-1}$; $B = 0.90$; e_0, $e_N = .90$

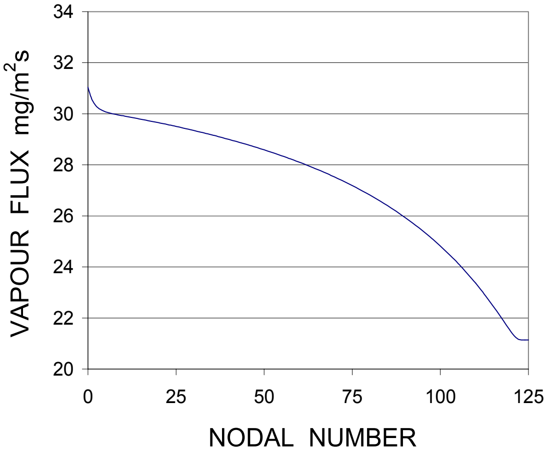

FIGURE 5 Vapour Flux

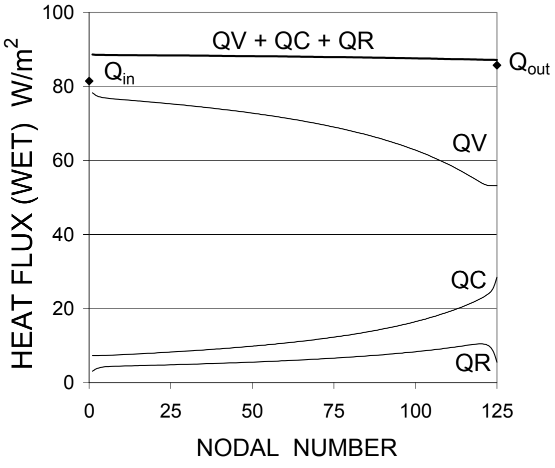

FIGURE 6 Heat Flux (Wet)

Figure 6 shows the three components of the energy flux and their sum for test number 4. The total flux at the hot side of the specimen Q_0 is 88.75 W/m^2. The Q_{IN} is 81.47 W/m^2, the difference being the enthalpy of the water that evaporated. At the cold boundary the total energy flux Q_{125} is 87.21 W/m^2, being less than Q_0 due to the enthalpy of the condensate that remained in the specimen. The Q_{OUT} is 85.72 W/m^2 . This is greater than the Q_{IN} because all of the condensate is at a lower temperature than when it was at the high temperature side of the specimen.

The Kumaran paper includes a graph of Q_{IN} vs time for test 1, where the specimen contained 91 g of water. The specimen was 60 cm by 60 cm, so this corresponds to 0.25 kg/m^2 of water. The calculated rate of evaporation at $j = 0$ for this test is 13.20 mg/m^{2}s so the initial steady state should last for 5.3 hours. In the paper it is reported that the initial steady state lasts about 5½ hours. There is a report on another test with the same conditions as test 1 but with only 57 g of water in the specimen, and in this case the initial steady state is reported to last for 3 hours.. With an evaporation rate of 13.20 mg/m^{2}s it would require 3.3 hours to evaporate the 57 g of water at the hot side.

These reported times for the start of dry-out provide a valuable check on the calculated rate of evaporation.

Figure 7 shows the value of $-\left(\frac{dJV}{dx}\right)_j$, which is the rate at which vapour condenses at each nodal location. As there is no condensation at nodal points 123 or 124, this small region adjacent to the cold boundary is dry. Figure 7 does not show the rate of condensation at the cold boundary ($j = 125$) as it has the dimension mg/m^{2}s. The amount of condensate at each nodal point has a profile similar to the rate of condensation until all of the water has evaporated from the hot boundary: after that the condensate in the specimen evaporates progressively from the hot side until all the water is at the cold boundary.

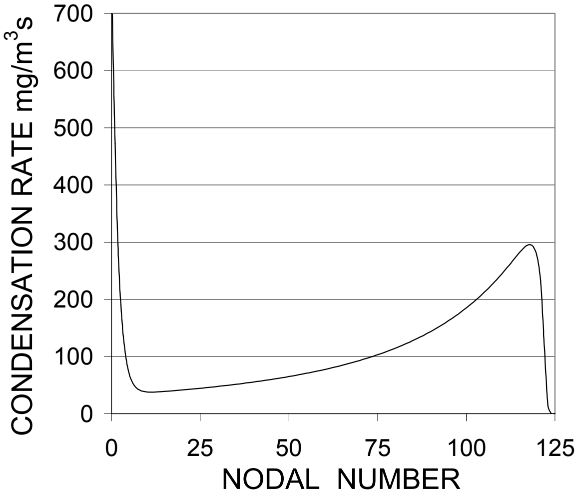

FIGURE 7 RATE OF CONDENSATION

DISCUSSION

The very good agreement between the calculated values for Q and the measured values of Q_{DRY} confirm that the algorithms for calculating QC and QR are appropriate for dry porous materials like glass fibre insulation. They take into account the “edge effect” due to the lower value of QR close to the boundaries. The concept of an “apparent thermal conductivity” to account for energy transport by conduction and radiation combined fails to allow for this edge effect (a.k.a. thickness effect).

The simulation of the wet phase of the tests has shown that the extended form of Fick’s Law is valid for water vapour diffusion through glass fibre insulation when there is a temperature gradient through the material.

The simulation of situations like these tests makes it possible to see if vapour is condensing and where the condensate is located.

The idea expressed in Kumaran’s paper that “*It may be possible to formulate a test method based on these experiments, applicable to wet glass-fibre insulation in which all the moisture is initially at the hot surface*” is valid. But the results of such a series of tests have to be analyzed using the correct algorithms to calculate each component of the energy flux.. The tests should measure the heat flux at both boundaries of the specimen. It is also necessary to measure the vapour permeance of the specimen

CONCLUSION

The hypothesis that the flux of water vapour through stagnant air in glass fibre insulation can be related to the gradients of vapour density and temperature by an extended form of Fick’s Law has been shown to give values in good agreement with experimental results. The diffusion coefficient associated with the gradient of the vapour density is proportional to the isothermal permeability of the material. The thermal vapour diffusion coefficient has been found to be approximately five times the coefficient for the diffusion due to the gradient of the vapour density.

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REFERENCES

1. Kumaran, K.K., Moisture Transport Through Glass-Fibre Insulation in the Presence of a thermal Gradient, Journal of Thermal Insulation, Vol. 10, April 1987, pp 243-255
2. Stephenson, D.G., Thermal Diffusion of Water Vapour Through Glass Fibre Insulation, Journal of Thermal Envelope and Building Science, Vol. 27, No. 1 July 2003, pp 31-48