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by

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Surface Temperatures and Heat Fluxes for Flat Roofs

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The results of calculated and measured surface temperatures and heat fluxes for a ventilated flat roof are compared and discussed. The thermal response factor method of calculating heat fluxes and temperatures gives values in good agreement with measured values when the sol–air temperature (evaluated from several derived formulae) includes the effect of long-wave radiation exchange between the roof and sky.

1. INTRODUCTION

THE RANGE of temperatures that can occur within the walls and roof of a building must be known before these elements can be designed properly. The rate of heat transfer through the inner surface of the walls and roof must also be known in order that the heating and cooling loads may be determined. It is important, therefore, that a designer be able to calculate the temperatures and heat fluxes for any combination of materials exposed to any inside and outside conditions.

As every calculation procedure involves some assumptions, the accuracy of any method must ultimately be established by comparing the results of calculations with experimental results. This paper compares the results of calculations made at the Division of Building Research, National Research Council, Ottawa, with results obtained experimentally for a flat roof and calculated by an RC-network method at the Royal Institute of Technology, Division of Building Technology, Stockholm.

The non-steady, non-periodic heat flux through a new roof construction was calculated first using measured values of the temperatures at the inner and outer surfaces of a roof as data. The results were compared with the measured values of the heat flow at the inner surface. The calculations were then repeated using sol–air temperature (evaluated in various ways) and inside air temperature instead of surface temperatures. This latter case corresponds to the normal design heat loss calculation.

2. EXPERIMENTAL STUDY

2.1 *Data used for calculations*

For the calculation of surface temperature and heat flux the roof (figure 1) was approximated by three homogeneous layers (figure 2):

1. 7·5 cm of cellular concrete (representing the top slab and the topping),
2. 7 cm dead air space (ventilation and spacer blocks ignored),
3. 20 cm of cellular concrete.

The moisture content of the cellular concrete was 6·5 per cent (by weight) for layer 1 and 5 per cent for layer 3.

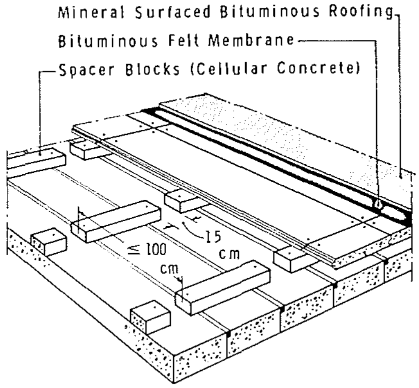

Fig. 1. Isometric figure of the flat roof construction.

The thermal properties listed in Table 1 take account of these moisture contents, conductivity being based on the simple linear relationship[1]:

$$k_m \simeq k_d + \frac{U}{300} \qquad (1)$$

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Table C-1. Coefficients and properties

Layer	Solar absorption coefficient	Conductivity (W/m degC)	Density (kg/m^3)	Specific heat (J/kg degC 10^2)	Thickness (m)	Resistance (m^2 degC/W)	Heat capacity (J/degC 10^3)
Outside surface	0·9	—	—	—	—	0·05	—
I	—	0·15	500	10·88	0·075	0·50	40·6
II	—	—	—	—	0·07	0·17	—
III	—	0·13	500	10·47	0·20	1·57	104·7
Inside surface	—	—	—	—	—	0·12	—

where

k_m = thermal conductivity in the moist state, kcal/mh degC,

k_d = thermal conductivity in the dry state, kcal/mh degC,

U = per cent moisture by weight, dry basis.

The solar absorption coefficient, a, for black bituminous roofing is assumed to be 0·9; and the resistances at the outside and inside surfaces equal 0·05 and 0·12 m^2 deg C/W, respectively.

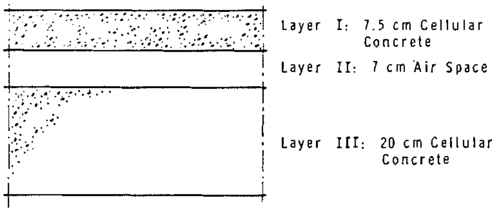

Fig. 2. Simplified construction divided into three layers for the calculations.

2.2 *Measurements*

Measurements were taken at different periods from December 1963 to April 1965[2].

Heat flow through the roof system was measured with thermo-electric heat flow meters at four points on the bottom, warm surface.

Temperature (air and surface) was measured by means of copper–constantan thermocouples. These and the heat flow meters were connected to potentiometric recorders (Honeywell–Brown). The thermocouples on the top of the roof were covered with the roof surfacing material.

Solar radiation was measured by means of Kipp and Zonen solarimeters.

Moisture content of the two cellular concrete layers was determined at different times by removing cores.

2.3 *Conditions during the period used for comparison*

Heat flows and surface temperatures were calculated for a period of seven days in April 1965. During this time the weather varied from completely overcast to very clear conditions. The daily total solar radiation incident on a horizontal surface varied from 4 to 15 MJ/m^2 and the outside air temperature from −12 to +13°C. There was no snow on the roof at any time during the week. The inside air temperature was steady, ranging between +23 and +24°C.

3. RESPONSE FACTOR METHOD FOR CALCULATING HEAT FLUX

The temperatures and heat fluxes at the surface of a roof are related by the response factor sets:

$$Q_o = T_o \,.\, X - T \,.\, Y \tag{2}$$

$$Q_i = T_o \,.\, Y - T_i \,.\, Z \tag{3}$$

where

Q = heat flux (heat flow towards inside is positive),

T = temperature,

subscripts o and i refer to outside and inside surface respectively.

X, Y and Z are the three sets of response factors that characterize the heat flow through the roof. All of these quantities are in time-series form, i.e. each symbol represents a set of values.

The response factor method of computing heat flux through an element of a building has been described by Mitalas and Stephenson[3, 4] in papers that also give a method for computing the factors for a multilayer element such as this roof. The response factor method is particularly well suited for these calculations because it does not require the assumption of periodic conditions, nor any complex analysis of the driving temperatures. It is necessary to know only the values of the driving temperatures at regular intervals.

The calculation of heat fluxes using this technique involves the multiplication of the driving temperatures by the appropriate response factors and the summation of a series of such products. A desk calculator may be used, but it is preferable to prepare a simple program to enable a digital computer to do the arithmetic.

The method is applicable only to situations where the thermal conductivity and specific heat of the materials are independent of time. If the coefficients of heat transfer at the surfaces are constant, they can be combined with the roof response factors to give a set of over-all factors that relate surface fluxes to the outside and inside air temperatures. Table 2 gives the factors for the roof alone that must be used with roof surface temperatures.

Table C-2. Surface to surface response factors for the flat roof

Time	X	Y	Z
1	4·612847	0·000001	3·925070
2	−2·764944	0·000202	−2·299253
3	−0·476747	0·002001	−0·378288
4	−0·245092	0·007297	−0·195861
5	−0·153975	0·014755	−0·125306
6	−0·106497	0·021362	−0·089228
7	−0·078391	0·025619	−0·067830
8	−0·060378	0·027510	−0·053850
9	−0·048125	0·027614	−0·044057
10	−0·039375	0·026562	−0·036824
11	−0·032864	0·024853	−0·031253
12	−0·027845	0·022833	−0·026820
13	−0·023858	0·020719	−0·023201
14	−0·020612	0·018644	−0·020186
15	−0·017916	0·016680	−0·017636
16	−0·015641	0·014863	−0·015453
17	−0·013698	0·013206	−0·013570
18	−0·012024	0·011712	−0·011934
19	−0·010571	0·010372	−0·010507
20	−0·009305	0·009177	−0·009257
21	−0·008197	0·008114	−0·008160
22	−0·007225	0·007170	−0·007196
23	−0·006371	0·006334	−0·006348
24	−0·005620	0·005595	−0·005601
25	−0·004958	0·004940	−0·004942
26	−0·004375	0·004362	−0·004362
27	−0·003861	0·003851	−0·003849
28	−0·003407	0·003400	−0·003397
29	−0·003007	0·003001	−0·002999
30	−0·002654	0·002650	−0·002647
31	−0·002343	0·002339	−0·002336
32	−0·002068	0·002065	−0·002062
33	−0·001825	0·001823	−0·001820
34	−0·001611	0·001609	−0·001607
35	−0·001422	0·001420	−0·001418
36	−0·001255	0·001254	−0·001252
37	−0·001108	0·001106	−0·001105
38	−0·000978	0·000977	−0·000975

$$\frac{X_n}{X_{n-1}} = \frac{Y_n}{Y_{n-1}} = \frac{Z_n}{Z_{n-1}} = 0{\cdot}8827$$

4. RESULTS OF CALCULATIONS

4.1 *Heat flow based on measured surface temperatures*

The heat fluxes at both the inner and outer surfaces of the roof were calculated using the experimentally measured values of the roof surface temperatures. The calculated fluxes are shown in Table 3, column 3, along with experimentally measured values. The good agreement between calculated and measured values shows that the surface temperature and heat flux measurements are self-consistent, that the values chosen for thermal properties are appropriate, and that the air space ventilation and the spacer blocks have a negligible effect on the measured heat flux.

4.2 *Mean heat flow based on mean inside and outside air temperatures*

The daily average heat flow is sometimes calculated by multiplying the average inside-to-outside air temperature difference by the over-all transmittance for a building element (Table 3, column 4). This simple method gives very large errors because it neglects solar radiation entirely. For the 7th and 8th of April (clear days) the average heat flow through the test roof calculated in this way is about 50 per cent higher than the average of the measured values. If the thermal resistance of the outer layer is discounted because the air space is vented, the discrepancy is even higher.

4.3 *Heat flow based on simple sol–air temperatures*

The simplest expression for sol–air temperature first given by Mackay and Wright[5] is

$$\theta_{SA} = \theta_{oa} + aIR_o \tag{4}$$

where

θ_{SA} = sol–air temperature,
θ_{oa} = outside air temperature,
a = absorptivity of surface for short-wave radiation,

Table C-3. Measured and calculated heat flow, daily mean values (W/m^2)

Time	Measured heat flow q_o	Computed heat flow based on the temperature difference $\theta_{si}-\theta_{so}$ q_1	$\theta_{ia}-\theta_{oa}$ q_2	$\theta_{ia}-\theta_{SA}$ q_3	$\theta_{ia}-\theta_{SA}$ q_4	$\theta_{ia}-\theta'_{SA}$ q_5	$\theta_{ia}-\theta''_{SA}$ q_6	Difference per cent $\frac{q_1-q_o}{q_o}$	$\frac{q_2-q_o}{q_o}$	$\frac{q_3-q_o}{q_o}$	$\frac{q_4-q_o}{q_o}$	$\frac{q_5-q_o}{q_o}$	$\frac{q_6-q_o}{q_o}$
Apr. 2	6·7	(7·8)	8·9	(6·8)	(6·5)	(7·4)	(7·6)	(+16)	+33	(+ 2)	(− 3)	(+10)	(+13)
3	7·4	7·8	8·9	7·2	7·1	7·4	7·6	+ 5	+20	− 3	− 5	0	+ 3
4	7·3	7·7	7·3	7·2	6·9	7·5	7·7	+ 5	0	− 2	− 6	+ 3	+ 5
5	6·2	6·3	6·8	5·6	5·1	6·5	6·6	+ 2	+10	−11	−17	+ 6	+ 6
6	6·2	6·2	7·4	5·0	4·8	5·9	6·0	0	+19	−19	−23	− 5	− 3
7	6·0	6·0	9·1	5·8	5·7	6·3	6·4	0	+52	− 3	− 6	+ 5	+ 6
8	7·0	7·3	10·4	5·8	5·7	7·3	7·6	+ 4	+49	−17	−18	+ 4	+ 8

Notes:

All days *before* 2 April are assumed to have the same conditions as 2 April.
q_1, q_3, q_5 and q_6 are computed by means of the response factor method.
q_4 is computed by means of RC-network.
q_2 is computed by multiplying the inside-to-outside mean air temperature difference by the air-to-air transmittance.

I = short-wave radiation incident on unit area of roof,

R_o = resistance to heat transfer at outside surface.

The daily mean values of heat flow based on hourly values of θ_{SA}, obtained by both the response factor and an RC-network method, are shown in Table 3, columns 5 and 6, respectively; values of θ_{SA} were calculated from measured values of θ_{oa} and I, and an assumed average R_o of 0·05 m^2 deg C/W. This average value of R_0 is in accordance with the measurements of Brown[6].

The daily mean heat flows calculated by both methods, using θ_{SA}, are in good agreement with each other, but agreement with measured values is not as satisfactory as was obtained with measured roof surface temperatures. A comparison of hourly values, given in figures 3 and 4 for cloudy and clear days, respectively, indicates a greater discrepancy for clear days.

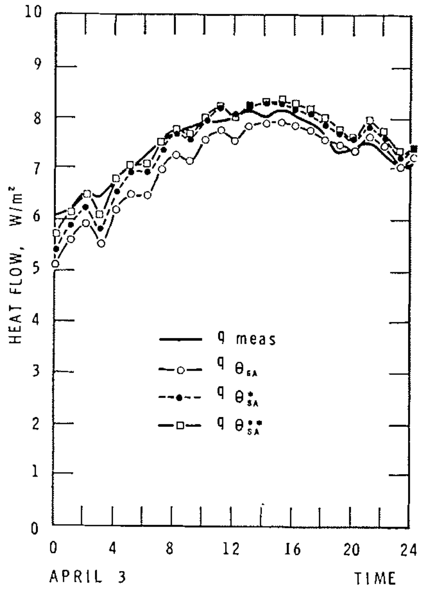

Fig. 3. *Measured and calculated heat flow for a cloudy day (3 April), following a day with fairly cloudy conditions.*

Similarly, calculated outside surface temperatures using θ_{SA}, were in reasonably good agreement with measured values on a cloudy day and during daylight hours, but the measured temperatures on clear nights were lower than the calculated ones. In fact, the surface temperature falls well below the outside air temperature on clear nights. This indicates that θ_{SA} as given by equation (4) is too high, especially at night. Equation (4) is based on the assumption that the sky radiates as a black body at outside air temperature, but the results in Table 3 and figures 3 and 4 show that this is not valid. Mackay and Wright[5] recognized this fact but used the simple form for cooling calculations as this gave conservative results. In 1952 Mackay suggested a modified formula that included a term for long-wave radiation [discussion of reference 7].

4.4 *Heat flow calculation based on modified sol-air temperature θ'_{SA}*

In Appendix A a modified sol-air temperature based on Brunt's[8] sky radiation formulae is given by:

$$\theta'_{SA} = \frac{1}{H_o}\,[aI + \epsilon R - \epsilon q + h_c\,\theta_{oa}] \qquad (5)$$

where

H_o = outside surface heat transfer coefficient,

a = solar radiation absorption factor,

I = solar radiation incident on the surface,

ϵ = emissivity of the surface,

R = long-wave radiation incident on the surface,

q = constant (in a linear approximation of the σT^4 curve),

h_c = outside surface convection heat transfer coefficient,

θ_{oa} = outside air dry-bulb temperature.

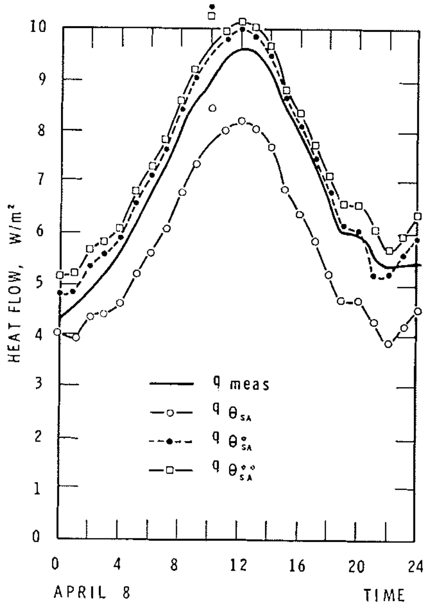

Fig. 4. *Measured and calculated heat flow for a very clear day (8 April), following a day with clear condition.*

Calculations based on this sol-air temperature give good agreement for both heat flow and surface temperatures. As shown in Table 3, the biggest discrepancy between calculated and measured daily mean values of heat flow is only +6 per cent. The calculated heat flow for each hour of the day is very close to that measured on both clear and cloudy days, as is shown in figures 3 and 4.

The comparison of calculated and measured values indicates that the modified sol-air temperature θ'_{SA} adequately accounts for both short- and long-wave exchange at the exterior surface.

4.5 *Heat flow based on modified sol-air temperature θ''_{SA}*

In Appendix B a modified sol-air temperature based on the equivalent radiant temperature of sky and ground is given by:

$$\theta''_{SA\,(\text{day})} = \theta_{oa} + aIR_o - \frac{9-m}{9}\,(4{\cdot}2 - 0{\cdot}06\;\theta_{oa}) \qquad (6\text{a})$$

$$\theta''_{SA\,(\text{night})} = \theta_{oa} - \frac{9-m}{9}\,(5{\cdot}6 - 0{\cdot}08\;\theta_{oa}) \qquad (6\text{b})$$

where

m = sky clearness in oktas and all other symbols have same meaning as in equation (4).

The last term in equation (6) takes account of the fact that the long-wave radiation from the sky is less than that from a black body at outside air temperature.

The inside surface heat flux calculated using θ''_{SA} is shown in Table 3 and in figures 3 and 4. Values are in good agreement with measurements and with values obtained with θ'_{SA} for all conditions

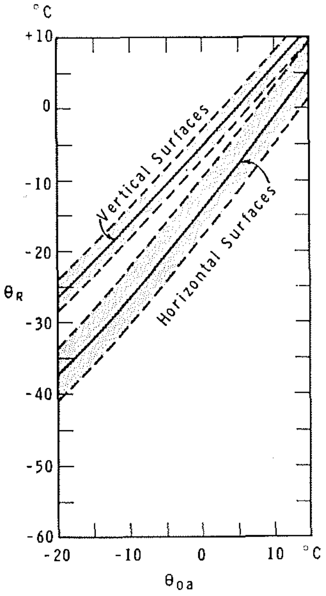

Fig. 5. The equivalent radiant temperature of sky and ground (θ_R) (effective re-radiation temperature) as a function of the outside air dry-bulb temperature (θ_{oa}) during completely clear nights, vertical and horizontal surfaces[6].

4.6 *Time delay and decrement factor method*

Calculation of the actual heat flow for each time of day by the approximate ' time delay and decrement factor ' method for periodic heat flow as given in the I.H.V.E. Guide gives poor agreement with the measurements in this case, since the actual heat flow is non-periodic.

4.7 *Some thermal characteristics of the roof*

The heat flow, as mentioned before, is determined on the under surface. During a 24-h period the maximum heat flow (at noon) can be about 40 per cent higher and the minimum (at midnight) about 30 per cent lower than the mean value. This variation, on the other hand, is very small compared with the variation on the top surface, where the extreme values can be nine times lower or higher than the mean value.

The temperature fluctuation is effectively damped out by the roof construction. For example, the variation between maximum and minimum temperature on the outside surface can be more than 50 deg C, while it is only about 2 deg C on the inside surface.

Steady-state heat flow seldom occurs in the construction with such a high storage and the rapid fluctuations of air temperature, solar heating and radiation cooling. The temperature gradient is not linear, even for rather cloudy conditions, contrary to some common design assumptions.

The time lag is found to be 10–12 h; the heat loss was maximum at about mid-day and minimum during the night.

5. CONCLUSION

Non-steady-state heat flow through a roof can be calculated quite accurately by the response factor method, even when the conditions are not periodic. It is necessary, however, that the sol-air temperature allow for the fact that clear sky emissivity is less than unity. Results based on two different sol–air temperature formulae are in equally good agreement with the measured values.

Even an RC-network is fairly well suited for such calculations, but it is more laborious (for non-periodic conditions) because of practical difficulties in producing the driving functions by function generators.

Calculation of the inside surface heat flux by the ' time delay and decrement factor ' method gives poor agreement with experimental results because the assumption of periodic conditions was not satisfied in the experimental situation.

The results confirm that the effect of free ventilation of the air space and the cold bridge effects of spacer blocks are negligible on the measured heat flux, and also confirm that the assumed properties are appropriate for this roof. The material above the air space has a significant effect on the heat loss through the roof.

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APPENDIX A

Modified sol–air temperature based on Brunt's sky radiation formulae

A heat balance for an outside horizontal surface gives:

$$aI+\epsilon R-\epsilon R_s+h_c(\theta_{oa}-\theta_s) = Q \qquad \text{(A-1)}$$

where

a = solar radiation absorption factor,
I = solar radiation incident on the surface,
ϵ = emissivity of the surface,
R = long-wave radiation incident on the surface,
ϵR_s = long-wave radiation emitted by the surface,
h_c = surface convection heat transfer coefficient,
θ_s = surface temperature,
θ_{oa} = air dry-bulb temperature.

The heat flux Q can also be expressed using sol–air temperature

$$H_o(\theta'_{SA}-\theta_s) = Q \qquad \text{(A-2)}$$

where

H_o = surface heat transfer coefficient
θ'_{SA} = sol–air temperature.

Elimination of Q in equations (A-1) and (A-2) gives

$$\theta'_{SA} = \frac{1}{H_o}(aI+\epsilon R-\epsilon R_s+h_c\,\theta_{oa}-h_c\,\theta_s)+\theta_s \qquad \text{(A-3)}$$

The long-wave radiation incident on a horizontal surface is the radiation emitted by the sky. This emission is a function of the sky cloud conditions. Assuming that the form factor from a horizontal surface to the part of the sky that is cloudy is directly proportional to the sky clearness number m (oktas)*, then

$$\epsilon_a = \frac{m}{8}\,\epsilon_c+\frac{8-m}{8}\,\epsilon_{cs} \qquad \text{(A-4)}$$

where

ϵ_a = average sky emissivity,
ϵ_c = clouded sky emissivity,
ϵ_{cs} = clear sky emissivity.

The emissivity of the clear sky is given in reference [7] [see also reference 8]

$$\epsilon_{cs} = 0{\cdot}55+0{\cdot}33\,\sqrt{(P_w)} \qquad \text{(A-5)}$$

where

P_w = water vapour pressure at ground level, inches mercury,

and the emissivity of the cloudy sky is 0·96[7].

Both emissivity values (i.e. cloudy and clear sky) are based on the assumption that the sky is at the ground level air dry-bulb temperature therefore

$$R = \epsilon_a\,\sigma\,T_{oa}^4 \qquad \text{(A-6)}$$

where

T_{oa} = air dry-bulb absolute temperature,
σ = Stefan–Boltzmann constant.

The black body emission can be approximated sufficiently accurately by a linear equation when the temperature undergoes moderate variation, i.e.

$$\sigma T^4 \approx q+p\,\theta \qquad \text{(A-7)}$$

where

p = slope of the σT^4 curve at θ_n, i.e., $p = 4\sigma T_n^3$,
θ_n = time average of the surface temperature,
$q = \sigma T_n^4-4\theta_n\,\sigma T_n^3$ (A-8)

Substitution of the linear expression for R_s in equation (A-3) and collection of terms gives

$$\theta'_{SA} = \frac{1}{H_o}[aI+\epsilon R-\epsilon q+h_c\,\theta_{oa}-(\epsilon p+h_c)\,\theta_s]+\theta_s \qquad \text{(A-9)}$$

The surface temperature, θ_s, is eliminated from equation (A-9) by making $H_0 = \epsilon p+h_c$, i.e.

$$\theta'_{SA} = \frac{1}{H_o}[aI+\epsilon R-\epsilon q+h_c\,\theta_{oa}] \qquad \text{(A-10)}$$

This is then a modified sol–air temperature to be used in equation (A-2) to calculate surface heat flux.

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*m ranges from 0 to 8,
m = 0 denotes clear sky,
m = 8 denotes completely overcast sky.

APPENDIX B

Modified sol–air temperature based on the equivalent temperature of sky and ground

The net outgoing long-wave radiation from a surface can be given by

$$Q_r = \sigma\epsilon\,(T_s^4 - T_R^4). \qquad \text{(B-1)}$$

Writing this equation

$$Q_r = h_r\,(\theta_s - \theta_R)$$

then

$$h_r = \sigma\epsilon\,(T_s + T_R)\cdot(T_s^2 + T_R^2) \sim 4\sigma\epsilon T_{\text{avg}}^3 \sim 4\sigma\epsilon Toa_{\text{avg}}^3 \qquad \text{(B-2)}$$

where

Q_r = long-wave radiation heat flow,

σ = Stefan–Boltzmann's constant,

ϵ = emissivity of the surface for long-wave radiation,

T_s = absolute temperature of the surface,

T_R = absolute equivalent radiant temperature of sky and ground,

h_r = outside surface radiation heat transfer coefficient,

T_{avg} = time average of T_s and T_R

Toa_{avg} = time average of absolute outside air dry-bulb temperature.

The heat balance at the roof surface can now be written [compare reference 9]:

$$aI - h_c\,(\theta_s - \theta_{oa}) - h_r\,(\theta_s - \theta_R) = -k\,\frac{\partial\theta_s}{\partial x} \qquad \text{(B-3)}$$

$$aI - h_r\,(\theta_{oa} - \theta_R) - (h_r + h_c)\,(\theta_s - \theta_{oa}) = -k\,\frac{\partial\theta_s}{x\partial} \qquad \text{(B-4)}$$

i.e.

$$\frac{aI}{h_r + h_c} - \frac{h_r}{h_r + h_c}\,(\theta_{oa} - \theta_R) - \theta_s + \theta_{oa} = -\frac{k}{h_r + h_c}\,\frac{\partial\theta_s}{\partial x} \qquad \text{(B-5)}$$

where

a = absorptivity of surface for short-wave radiation,

I = intensity of short-wave solar radiation incident on the surface,

h_c = outside convection heat transfer coefficient,

k = thermal conductivity,

$\dfrac{\partial\theta_s}{\partial x}$ = temperature gradient at the surface,

θ_R = the equivalent radiant temperature of sky and ground.

Using the sol–air temperature θ_{SA}'' the heat balance can be expressed

$$(h_r + h_c)\,(\theta_{SA}'' - \theta_s) = h_o\,(\theta_{SA}'' - \theta_s) = -k\,\frac{\partial\theta_s}{\partial x} \qquad \text{(B-6)}$$

$$\therefore \theta_{SA}'' - \theta_s = \frac{aI}{h_o} - \frac{h_r}{h_o}\,(\theta_{oa} - \theta_R) - (\theta_s - \theta_{oa}) \qquad \text{(B-7)}$$

giving

$$\theta_{SA}'' = \theta_{oa} + \frac{aI}{h_o} - \frac{h_r}{h_o}(\theta_{oa} - \theta_R) \qquad \text{(B-8)}$$

where

h_o = outside heat transfer coefficient.

Brown[6] has given θ_R as a function of the air temperature θ_{oa} from values observed in Stockholm during *completely cloudless nights* (figure 5).

An approximation gives for a *horizontal* surface:

$$\theta_R \sim 1{\cdot}2\theta_{oa} - 14 \qquad \text{(B-9)}$$

for a *vertical* surface:

$$\theta_R \sim 1{\cdot}1_{\theta oa} - 5$$

This is a reasonable approximation, especially when the deviation in the observed values is relatively great. Brown's measured values for horizontal surfaces are in good agreement with calculated ones[10].

During the daytime the net heat loss from the ground to the sky is practically the same as it would be at night with the same atmospheric conditions of temperature and humidity[8].

Thus for clear conditions

$$\theta_{SA}'' = \theta_{oa} + \frac{aI}{h_o} - \frac{h_r}{h_o}(14 - 0{\cdot}2\theta_{oa}) \qquad \text{(B-10)}$$

where

$\dfrac{h_r}{h_o} \sim$ 0·4 during the night and $\sim$ 0·3 during the day for a horizontal surface.

For a sky of m oktas cloud the last term in equation (B-10) is to be multiplied by the ratio (9-m)/9 following a proposal by Ångström[11].

Equation (B-10) can now be separated into two parts

$$\text{(1)}\ \theta_{SA(\text{day})}'' = \theta_{oa} + aIR_o - \left(\frac{9-m}{9}\right)(4{\cdot}2 - 0{\cdot}06\,\theta_{oa}) \qquad \text{(B-11)}$$

where

R_o = outside surface resistance

$$\text{(2)}\ \theta_{SA(\text{night})}'' = \theta_{oa} - \left(\frac{9-m}{9}\right)(5{\cdot}6 - 0{\cdot}08\,\theta_{oa}) \qquad \text{(B-12)}$$

(at night I is zero).

APPENDIX C

Conversion factors

1. *Linear measure*
   1m = 100 cm = 39·3701 in
2. *Bulk density*
   1 kg/m^3 = 0·06244 lb/ft^3
3. *Temperature*
   °C = (°F − 32)/1·8
   °F = 1·8°C + 32
4. *Specific heat*
   1 J/kg degC = 0·2388 . 10^{-3} B.t.u./lb degF
5. *Heat capacity*
   1 J/degC = 0·5266 . 10^{-3} B.t.u./degF
6. *Heat flow*
   1 W/m^2 = 0·3170 B.t.u./ft^2 h
7. *Thermal conductivity*

$$1\ \text{W/m degC} = \begin{cases} 0{\cdot}5778 & \text{B.t.u./ft h degF} \\ 6{\cdot}933 & \text{B.t.u. in/ft}^2\ \text{h degF} \end{cases}$$

8. *Thermal resistance*
   1 m^2 degC/W = 5·678 ft^2 h degF/B.t.u.
9. *Surface coefficient*
   1 W/m^2 degC = 0·1761 B.t.u./ft^2 h degF

Les résultats de calcul et de mesurage de température de surface ainsi que de flux thermiques sont revus et comparés. La méthode de réponse du coefficient thermique, appliquée pour le calcul des flux de chaleur et des températures donnent les valeurs étant en conformité satisfaisante avec les valeurs mésurées, sous la condition toutefois, que la température de solarisation (evaluée suivant plusieurs formules dérivées) comprend les effets d'échange de la radiation d'ondes longues entre le toit et le ciel.

Die Ergebnisse der berechneten und gemessenen Flächentemperaturen und Wärmeströmungen für ein ventiliertes flaches Dach wurden verglichen und besprochen. Die Methode des thermischen Reaktionskoeffizienten für die Berechnung von Wärmeströmungen und Temperaturen ergibt Werte, die mit den gemessenen Werten gut übereinstimmen, wenn die solare Temperatur (an Hand mehrerer abgeleiteter Formeln abgeschätzt) die Wirkung des Austausches der langwelligen Strahlung zwischen Dach und Himmel einschließt.