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SOLAR TRANSMISSION THROUGH WINDOWS WITH VENETIAN BLINDS

by

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TRANSMISSION DU RAYONNEMENT SOLAIRE AU TRAVERS DES FENETRES MUNIES DE STORES VENITIENS

SOMMAIRE

L'auteur présente dans cet exposé une méthode de calcul de la luminosité d'un store vénitien. La méthode tient compte de la variation de luminosité qui s'opère selon la largeur de la lame, et prend également en considération la réflexion qui se produit à la surface de la vitre. On admet que les lames de la jalousie sont plates et qu'elles reflètent la lumière incidente de façon diffuse.

La répartition de l'éclairement est utilisée pour calculer les fonctions de luminance d'une jalousie illuminée par le rayonnement direct du soleil, le rayonnement diffusé réfléchi par le sol et le rayonnement diffusé provenant du ciel. On intègre ensuite les fonctions de luminance pour obtenir le facteur de transmission d'éclairement. Ce facteur est égal au rapport entre le coefficient d'éclairement en un point d'une surface à l'intérieur de la pièce et le coefficient d'éclairement en un point de la face externe de la fenêtre. La luminosité est également utilisée pour déterminer le facteur d'absorption lumineuse des jalousies.

L'auteur présente les données relatives à un store vénitien normal de couleur claire, dont les lames sont inclinées à un angle de 45°, et qui est placé derrière une vitre en verre ordinaire. Les facteurs de transmission et de réflexion du verre sont donnés sous la forme de polynômes à 5 termes, où le cosinus de l'angle d'incidence est la variable principale.

Les résultats expérimentaux ne sont pas donnés.

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11 SOLAR TRANSMISSION THROUGH WINDOWS WITH VENETIAN BLINDS

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11.1 Introduction

Building designers are often surprised when they discover that a square foot of unshaded window can transmit solar radiation that is equivalent in heating effect to a square foot of steam radiator. In addition to causing uncomfortably high temperatures in summer, the direct solar beam falling on light-coloured surfaces inside a room can also cause very severe glare. There are, therefore, quite compelling reasons for restricting the entry of solar radiation into most buildings.

There is, however, a conflicting requirement that the shading system must allow at least partial vision through the window and enough light transmission to light the room adequately. A venetian blind comes close to satisfying all the requirements for a good shading system and consequently is widely used. The particular feature of a horizontal slat-type of shade that makes it so useful is that it can be adjusted so that its brightness in the direction of the ceiling is several times greater than that to the occupants of the room. A quantitative knowledge of this directional effect is necessary for both lighting and air conditioning design calculations. The work presented in this paper was undertaken to provide this information when no adequate data were found in the literature.

The literature on the absorption and transmission characteristics of venetian blinds is surprisingly small. The earliest and most extensive studies were carried out at the laboratory of the American Society of Heating and Ventilating Engineers in Cleveland, and were reported in the Society's transactions for 1952 and 1953 (1, 2, 3,). The primary purpose of this work was to determine the shading factor for a blind, so the results did not include any directional transmission or brightness data. A paper by Moon & Spencer (4) published in 1962 gave directional transmission values but it is of little practical value because reflection from the blind slats was neglected, i.e. it was assumed that the slats were perfectly black. O'Brien (5) presented a paper to the Illuminating Engineering Society (U.S.A.) in 1962, which outlined a new method of calculating the luminance of the surface of a louver slat. He did not, however, use the luminances of the slats to evaluate a directional transmission function for a venetian blind. The most recent paper is by Nicol (6), which includes some curves showing the variation of blind brightness with viewing angle but it seems deficient in two respects: it completely neglects the effect of reflection from the window glass, and the variation of luminance over the inward-facing surface of the slat is ignored.

The present paper uses essentially the O'Brien method to calculate the luminance of a venetian blind, but takes account of the fact that the blind is adjacent to a window. The luminance of the slat is used to evaluate directional transmission and brightness functions for the blind and these functions are integrated to give illumination factors for surfaces inside the room. The illumination factor is just the ratio of the illumination at some point on a surface in the room to the illumination at the outside of the window.

The luminance of the blind slats also permits an accurate evaluation of the absorption factor for the blind, which is of particular interest to the air-conditioning designer.

11.2 O'Briens' method

The fundamentals of O'Brien's method of computing slat luminance can be seen by considering the case shown in figure 11.1. Each surface of a blind slat is subdivided into several strips (in this case, seven), each of which is assumed to have a uniform luminance. The validity of this assumption can be checked by changing the number of strips; if the luminance remains unchanged, it means that an adequate number of strips is being used. Seven is an adequate number by this test. The relationship between luminance of the strips, the geometry of the blind cavity and the incident radiation can be expressed as follows:

$$\frac{A_n L_n}{\rho_n} = A_n E_n + A_1 L_1 f_{1,n} + A_2 L_2 f_{2,n} + \quad \ldots \tag{1}$$

where

L_n	is the luminance of strip n
A_n	is the area of strip n
E_n	is the illumination on strip n coming directly from outside the cavity
$f_{m,n}$	is the fraction of the radiation emanating from strip m that falls on strip n
ρ_n	is the reflectance of strip n.

As

$$A_n f_{n,m} = A_m f_{m,n} \tag{2}$$

if follows that:

$$L_n\left(\frac{1}{\rho_n} - f_{n,n}\right) - f_{n,1}L_1 - f_{n,2}L_2 \ldots = E_n. \tag{3}$$

This statement implies that the surfaces are diffuse reflectors. The ASHVE research indicated this is a valid assumption for painted blinds. The set of equations for all the strips can be arranged in the following matrix form:

$$[M] \cdot [L] = [E] \tag{4}$$

where

$$[M] = \begin{bmatrix} \frac{1}{\rho_1} & -f_{1,1}, & -f_{1,2}, & -f_{1,3}, & \ldots & -f_{1,n} \\ -f_{2,1}, & \frac{1}{\rho_2} & -f_{2,2}, & -f_{2,3}, & \ldots & \\ -f_{3,1}, & -f_{3,2}, & \frac{1}{\rho_3} & -f_{3,3}, & \ldots & \\ -f_{n,1}, & \ldots & & & & \end{bmatrix} \tag{5}$$

$$[L] = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ L_n \end{bmatrix} \tag{6}$$

$$[E] = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{bmatrix} \qquad (7)$$

Thus

$$[L] = [M]^{-1} \quad [E] \qquad (8)$$

11.3 Extension of O'Brien's Method to Include Reflection from Window Glass

The factors $f_{m,n}$ for a blind alone are the ordinary form factors, and can be calculated by the simple crossed string formula given in appendix 11.A. When the blind is behind a sheet of glass, however, there is an additional component related to the form factor from m' to n, where m' is the image of strip m. If glass had the same reflectivity for all angles of incidence the augmenting component of the form factor would be $f_{m',n}$ times the reflectivity of glass. The reflectivity of glass is not constant for all angles of incidence, however, so the variation has to be taken into account. The details of how this is done are given in appendix 11.A. The luminance of the slat surfaces can be calculated in the same way as if there was no glass, except that the form factors are the sum of the direct and specularly reflected components. The radiation that is absorbed by the blind and the overall transmission to the room can be calculated more readily if the matrix equation is in terms of the absorptance, B, instead of the luminance

$$B_n = \frac{1 - \rho_n}{\rho_n} \cdot L_n \qquad (9)$$

so

$$[B] = [M^*]^{-1} \cdot [E] \qquad (10)$$

where

$$[M^*] = \begin{bmatrix} \frac{1-\rho_1 f_{1,1}}{1-\rho_1}, & \frac{-\rho_2 f_{1,2}}{1-\rho_2}, & \frac{-\rho_3 f_{1,3}}{1-\rho_3} & \cdots \\ \frac{-\rho_1 f_{2,1}}{1-\rho_1}, & \frac{1-\rho_2 f_{2,2}}{1-\rho_2}, & \frac{-\rho_3 f_{2,3}}{1-\rho_3} & \cdots \\ \frac{-\rho_1 f_{3,1}}{1-\rho_1}, & \frac{-\rho_2 f_{3,2}}{1-\rho_2}, & \frac{1-\rho_3 f_{3,3}}{1-\rho_3} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \qquad (11)$$

The opening at the room side of the blind cavity (strip 8 in figure 11.1) has zero reflectivity, so the corresponding column in M* has all the elements zero except on the diagonal, which is one.

The transmission factor for a blind is equal to the absorptance of the inter-slat opening on the room side when there is unit illumination on the opening at the window side of the blind. Similarly, the absorptance of the slats gives the absorption factor for the blind. The factors evaluated in this way include the effect of reflection by the window pane, so the illumination on the blind should be just the illumination on the outside of the window times the transmittance of the glass.

The values of the elements in the E matrix depend on the direction of the incident radiation. If all the radiation entering the space between the slats falls on strip 1, $E_1 \times \delta = 1 \times S$, i.e., $E_1 = S/\delta$, and all the other elements are zero. At a lower shadow angle the incident radiation can fall on strips 1 and 2, in which case $E_1 = E_2 = \frac{S}{2\delta}$ and the others are zero, and so on.

Each set of values for the E elements should add up to S/δ; each set corresponds to radiation falling on the window at a particular vertical shadow angle. For the diffuse radiation from the sky (and ground) the values of E_n have been taken as twice the form factor from strip n to the sky (or ground). This distribution is appropriate if the sky (or ground) is uniformly bright when viewed through a window pane. Other brightness distributions can be handled by the matrix method but the uniform brightness is much the simplest.

11.4 Directional Transmission Function

The radiation entering a room from a window shaded by a venetian blind can come into the room directly from the blind surface, or it can be reflected into the room by the window glass; in the latter case it appears to come from the image of the blind. Formulae are derived in appendix 11 B for the reflection factor $R(\theta)$ that appears in Equation (12). Figure 11.5 shows the angle co-ordinates that are convenient to describe the reflection from the glass. The radiant flux leaving a blind cavity in the directions characterized by $-\pi/2 \leq \gamma \leq \pi/2$ and θ = constant is

$$T(\theta) = \frac{\sin(\theta - \varphi)}{2} \int\limits_{\substack{\text{Area of slat visible from} \\ \text{room, when viewed at} \\ \text{angle } \theta}} L\, dA \tag{12}$$

$$+\, R(\theta) = \frac{\sin(\theta + \varphi)}{2} \int\limits_{\substack{\text{Area of slat that can be} \\ \text{viewed by reflection at} \\ \text{angle } \theta}} L\, dA$$

When the luminances of the various segments of the slat are based on unit illumination at the opening on the window side of the blind, the $T(\theta)$ function becomes a directional transmission function. The integral of this function with respect to θ for the range $-\pi/2 \leq \theta \leq \pi/2$ should equal the absorptance of the opening on the room side of the blind. Thus the evaluation of the integral provides a useful check on the accuracy of the directional transmission function. The $R(\theta)$ function takes account of the reflectivity of the glass, including the variation with incident angle.

11.5 Blind Luminance Function

The luminance of a light source in a particular direction is defined as the luminance of a perfectly diffuse source of equal size that would produce the same intensity in that direction. Thus the luminance in the direction θ of a long blind is radiating $T(\theta)$ lumens/(radian sq ft) is

$$2 \times \frac{T(\theta)}{\cos\theta} \quad \text{ft-lamberts}$$

The function $J(\theta) = T(\theta)/\cos\theta$ is the luminance function. The luminance of a blind in the direction θ is equal to $2 \times I \times \tau \times J(\theta)$, where I is the illumination on the outside of the window and τ is the transmittance of the window pane. The values of $J(\theta)$ are different for illumination from the sky, from the ground, and from the direct solar beam. Thus the three components of the blind luminance have to be computed separately and added to get the total blind luminance.

The illumination in lumens/sq ft^2 on a vertical surface due to diffuse radiation from a sky of uniform luminance is equal to half the value of the sky luminance in foot lamberts. Thus,

Sky component of blind luminance = $J_{sky}(\theta) \times \tau_{diffuse}$ x sky luminance.

The ground component can be calculated in the same way.

11.6 Illumination Factors

The illumination factor, K, is the ratio of the illumination at a point on a surface inside a room to the illumination on the outside surface of the window. Here, too, the three components of illumination (sky, ground, and direct beam) have to be computed separately.
The method of evaluating the direct* illumination from a window with a venetian blind is quite similar to finding the daylight factors for an unshaded window.

$$K = \tau \times F \times G$$

The transmittance, τ, of the window pane is a function of the incident angle. It has been approximated by a fifth order polynomial of the cosine of the incident angle, i.e.,

$$\tau(i) = \sum_{j=o}^{5} t_j \cos^j(i)$$

The values of the t_j coefficients for several types of glass are given in appendix 11.C.
The factor, F, is related to the blind luminance and the orientation of the surface that receives the light. There are three cases of special interest, viz:

1 A wall parallel to the window wall,

$$F_1 = \int_{\theta_1}^{\theta_2} J(\theta)\cos\theta \, d\theta = \Phi_1(\theta_2) - \Phi_1(\theta_1)$$

where the function $\Phi_1(\theta)$ is the integral from zero to θ

2 A wall perpendicular to the window wall,

$$F_2 = \int_{\theta_1}^{\theta_2} J(\theta)\, d\theta = \Phi_2(\theta_2) - \Phi_2(\theta_1)$$

3 A horizontal surface,

$$F_3 = \int_{\theta_1}^{\theta_2} J(\theta)\sin\theta \, d\theta = \Phi_3(\theta_2) - \Phi_3(\theta_1)$$

The limits θ_1 and θ_2 for these integrals are shown on the vertical section sketch in figure 11.3.
The functions Φ_1, Φ_2, and Φ_3 are called illumination integrals.
The factor, G, allows for the finite length of the window just as the factor obtained from the auxiliary protractor does in the BRS method for daylight factor calculation. In fact, the BRS protractor No. 2 gives the G factor for cases 1 and 3. They are:

* "direct" means that interreflections within the room are not taken into account.

$$G_1 = G_3 = \frac{2}{\pi} \int_{\gamma^*_1}^{\gamma^*_2} \cos^2 \gamma \ d\gamma$$

and for case 2

$$G_2 = \frac{2}{\pi} \int_{\gamma^*_1}^{\gamma^*_2} \cos\gamma \ \sin\gamma \ d\gamma \ .$$

The angles γ^*_1, and γ^*_2 are related to γ_1, γ_2, θ_1, and θ_2 by the expressions

$$\tan \gamma^*_1 = \tan \gamma_1 \cos \frac{\theta_1 + \theta_2}{2}$$

$$\tan \gamma^*_2 = \tan \gamma_2 \cos \frac{\theta_1 + \theta_2}{2}$$

where γ_1 and γ_2 are the angles on a plan drawing that are analogous to θ_1 and θ_2 on a vertical section drawing.

Since the limits on the G integral are functions of θ, this function should really be evaluated for all values of θ between θ_1 and θ_2 and then be included with the other functions of θ when evaluating the F functions. For most purposes, however, this complication is not justified and it is sufficient to use the mean values γ^*_1 and γ^*_2 given by the expression above.

There is another small error inherent in these G functions. There is no allowance for the fact that the radiation reflected from the glass has not quite the same angular distribution in the γ- plane as the direct radiation from the blind slat. But here too the error is so small that it does not justify further complicating the calculation of illumination factors.

11.7 Results

This paper has presented data for only one blind arrangement. The results given are for a standard light-coloured blind behind a sheet of ordinary window glass. (W/S = 1.2, absorptivity of slat = 0.4, absorption parameter, $k\ell$, of glass = 0.1, slat angle = 45 degrees.)

Figure 11.2 shows the blind luminance factors for illumination from a sky of uniform luminance, uniformly bright ground surface, and direct solar irradiation at vertical shadow angles of 10, 33 and 60 degrees. The 33-degree shadow angle is the condition for maximum insolation on a vertical window; the 10-degree case is the minimum shadow angle for no direct solar transmission through the blind, and 60 degrees is representative of the midday shadow angle that occurs for south windows in summer.

The following example illustrates the use of these data.

Problem : Find the maximum luminance of a standard venetian blind as seen by the occupants of a room with a western exposure.

Data :	Maximum insolation occurs at ψ	= 33°
	Direct solar insolation	= 75 watts/ft^2
		= 6750 lumens/ft^2
	Sky luminance	= 800 ft-lamberts
	Ground luminance (ρ = 0.2)	= 1000 ft-lamberts
	Transmittance of glass	= 0.82 for direct beam
		= 0.75 for diffuse radiation

Figure 11.2 shows maximum value of J (θ) for negative values of θ at $\theta = -10^{\circ}$

Luminance factors for θ	$= -10^{\circ}$
J_{sky}	$= 0.050$
J_{ground}	$= 0.070$
J_{direct} for $\psi = 33$	$= 0.051$

Blind luminance:

Sky component	= 800 x 0.75 x 0.050	= 30 ft-lamberts
Ground component	= 1000 x 0.75 x 0.070	= 52 ft-lamberts
Direct beam component	= 2 x 6750 x 0.82 x 0.051	= 565 ft-lamberts
	Total	= 647 ft-lamberts

The illumination integrals Φ_1, Φ_2, and Φ_3 are presented in tables 12.1 and 12.2. The application of these data is illustrated in the following extension of the previous example.

Problem: Find the "direct" illumination at a point on the ceiling of a room where the values of θ_1 and θ_2 are 40 and 60 degrees respectively.

The room is sufficiently long to take G = 1.0.

Data: Φ_3 is the function that is appropriate for a ceiling.

$F_{3, sky}$	= 0.0373 - 0.0085 = .029
$F_{3, ground}$	= 0.3580 - 0.1470 = .211
$F_{3, direct}$ for $\varphi = 33$	= 0.0350 - 0.0098 = .025

Ceiling Illumination:

Sky component	$= \frac{800}{2}$ x 0.75 x 0.029	= 9 lumens/sq ft
Ground component	$= \frac{1000}{2}$ x 0.75 x 0.211	= 79 lumens/sq ft
Direct beam component	= 6750 x 0.82 x 0.025	=138 lumens/sq ft
	Total	= 226 lumens/sq ft

The blind luminance curves show that a large part of the radiation that is transmitted through a venetian blind falls on the ceiling. It is important, therefore, to have a light-coloured ceiling so that as much as possible of this light will be reflected downward as useful illumination. These results also suggest that there should be some provision for heat extraction from the ceiling since even a light-coloured surface absorbs about half of the solar radiation falling on it.

11.8 Conclusion

Venetian blinds are a useful type of window shade because they can intercept the direct rays of the sun and still allow a partial view through the window. They have a sufficiently low luminance when viewed from the normal position of the occupants that they do not cause glare; but, at the same time, they transmit enough daylight toward the ceiling to provide a reasonable standard of illumination throughout the room.

The computations that have been made during this study have provided data that are needed for calculating blind luminance and the luminance of surfaces in a room with venetian blinds. The computations have included the reflection by the window pane and the variation of luminance over the width of the blind slats. It has been assumed that the slats are flat and that they reflect diffusely.

There has been no experimental confirmation of the results.

11.9 Acknowledgements

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Fig. 11.1
Vertical section through typical blind cavity.

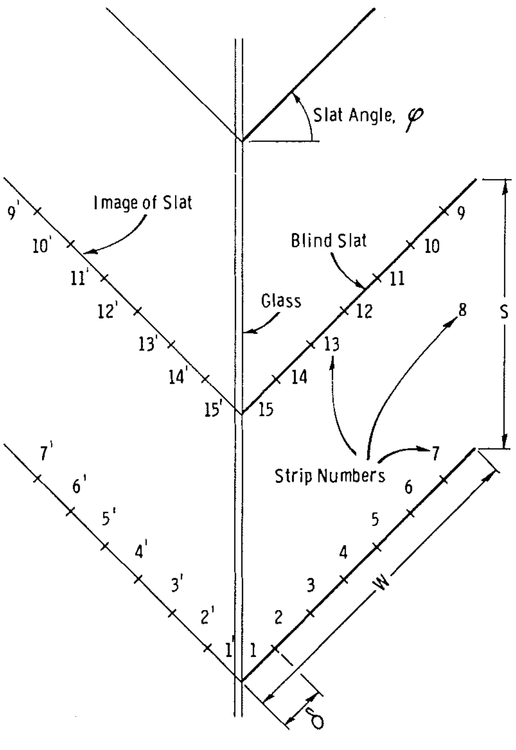

Fig. 11.2

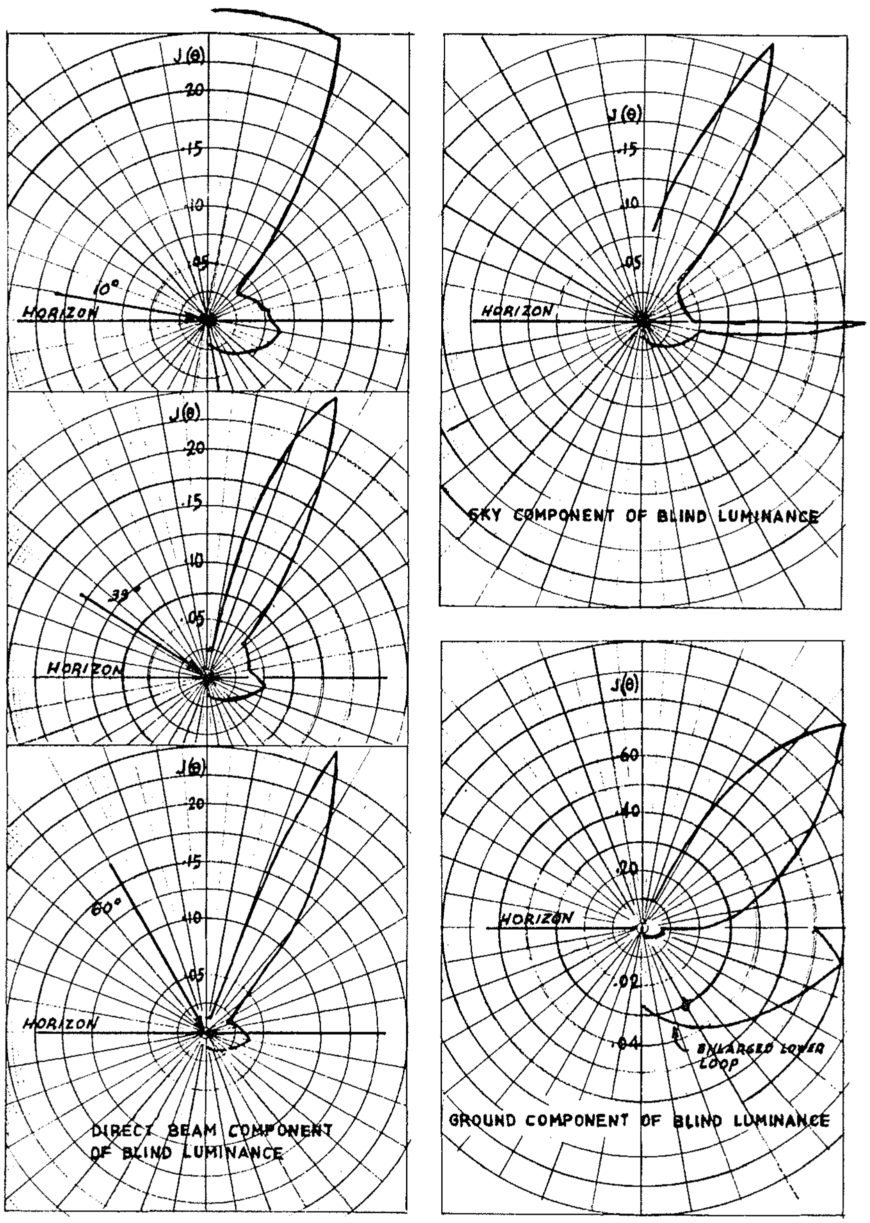

Fig. 11.3
Vertical section

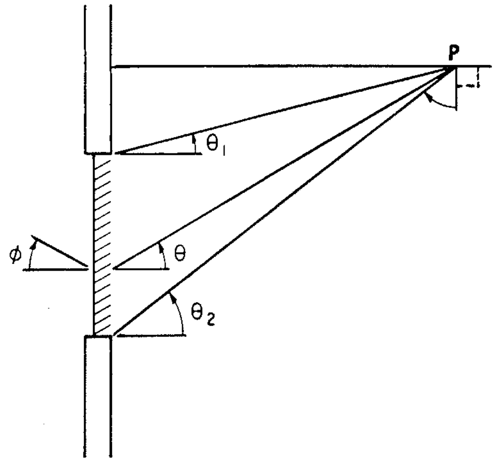

Fig. 11.4
Enlarged view of cross section through window and blind

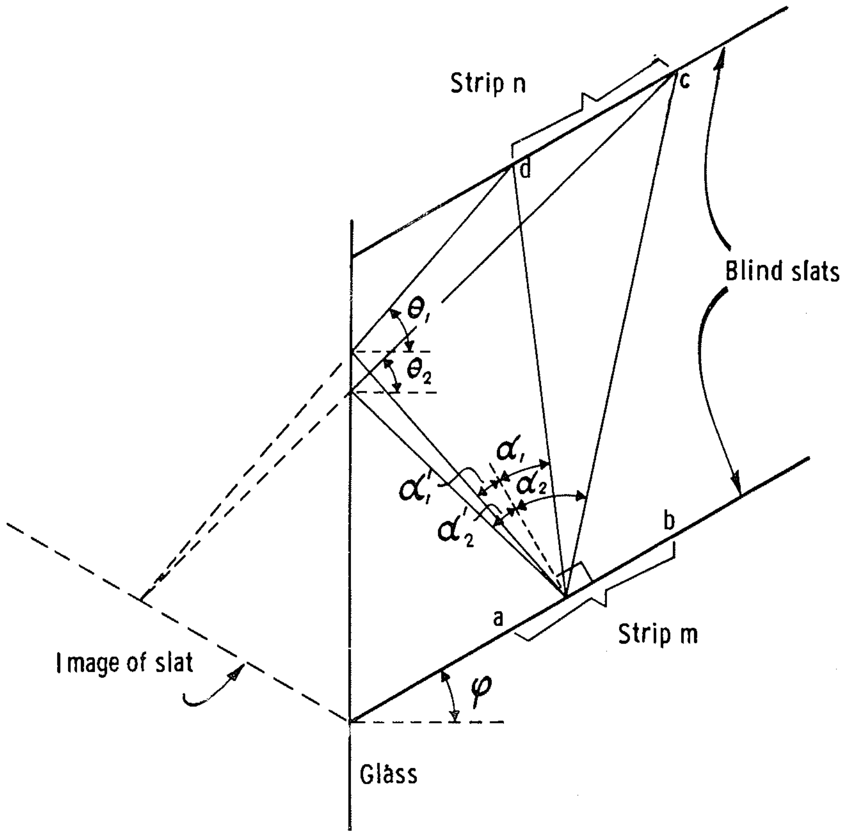

Fig. 11.5
Angular co-ordinates for light ray incident on window glass

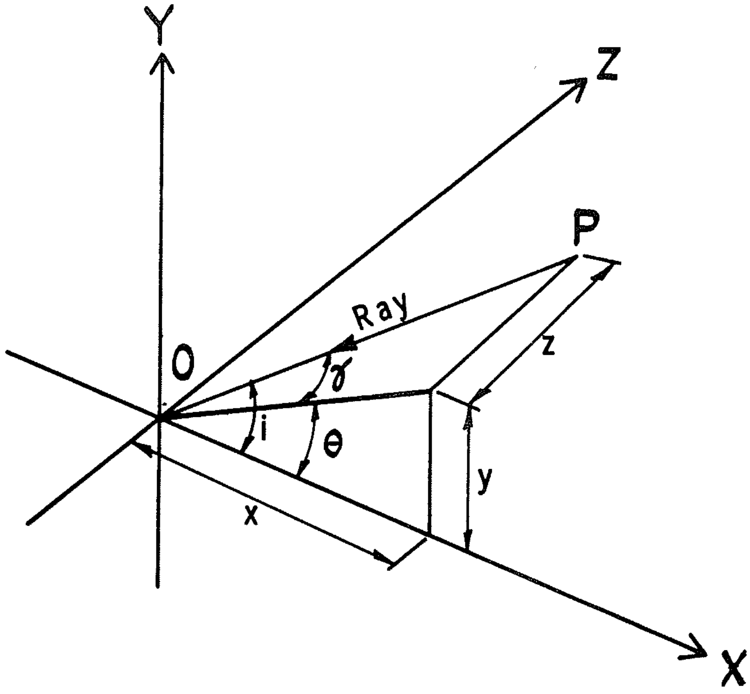

Table 11.1 Illumination integrals for the sky-diffuse and ground - reflected solar radiation

	Sky			Ground		
θ	Φ_1	Φ_2	Φ_3	Φ_1	Φ_2	Φ_3
90	.0705	.1450	.1123	.5260	.7143	.3820
80	.0690	.1265	.0980	.5243	.6953	.3800
70	.0620	.1025	.0710	.5211	.6763	.3790
60	.0450	.0615	.0373	.5053	.6433	.3580
50	.0307	.0355	.0150	.4363	.5233	.2600
40	.0245	.0263	.0085	.3233	.3583	.1470
30	.0190	.0197	.0050	.2133	.2273	.0680
20	.0135	.0137	.0025	.1223	.1233	.0240
10	.0070	.0072	.0005	.0483	.0483	.0040
0	.0000	.0000	.0000	.0000	.0000	.0000
-10	-.0207	-.0208	-.0011	-.0100	-.0100	-.0015
-20	-.0297	-.0305	-.0033	-.0220	-.0230	-.0040
-30	-.0363	-.0377	-.0066	-.0330	-.0340	-.0080
-40	-.0417	-.0440	-.0103	-.0400	-.0430	-.0140
-50	-.0455	-.0493	-.0140	-.0450	-.0510	-.0190
-60	-.0482	-.0540	-.0180	-.0495	-.0580	-.0250
-70	-.0500	-.0578	-.0213	-.0520	-.0650	-.0310
-80	-.0510	-.0612	-.0246	-.0535	-.0700	-.0370
-90	-.0512	-.0645	-.0277	-.0540	-.0760	-.0430

Table 11.2 Illumination integrals for the direct beam

ψ	10			33			60		
θ	Φ_1	Φ_2	Φ_3	Φ_1	Φ_2	Φ_3	Φ_1	Φ_2	Φ_3
90	.0845	.2107	.1670	.0700	.1395	.1023	.0600	.1005	.0790
80	.0781	.1550	.1210	.0697	.1290	.1007	.0593	.1002	.0768
70	.0660	.1090	.0755	.0635	.1045	.0735	.0583	.0980	.0720
60	.0487	.0625	.0336	.0460	.0625	.0350	.0415	.0600	.0390
50	.0350	.0400	.0156	.0320	.0370	.0170	.0262	.0310	.0150
40	.0300	.0323	.0100	.0257	.0277	.0098	.0180	.0198	.0068
30	.0245	.0260	.0062	.0200	.0206	.0053	.0130	.0138	.0035
20	.0177	.0187	.0029	.0133	.0135	.0020	.0094	.0097	.0018
10	.0095	.0100	.0010	.0070	.0070	.0004	.0056	.0056	.0005
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-10	-.0100	-.0100	-.0010	-.0080	-.0080	-.0007	-.0061	-.0065	-.0007
-20	-.0203	-.0215	-.0033	-.0165	-.0168	-.0027	-.0121	-.0126	-.0022
-30	-.0290	-.0313	-.0072	-.0225	-.0240	-.0055	-.0170	-.0180	-.0041
-40	-.0362	-.0390	-.0121	-.0277	-.0300	-.0091	-.0210	-.0227	-.0067
-50	-.0415	-.0465	-.0174	-.0315	-.0350	-.0130	-.0238	-.0262	-.0096
-60	-.0450	-.0530	-.0226	-.0341	-.0394	-.0165	-.0255	-.0295	-.0127
-70	-.0475	-.0585	-.0280	-.0355	-.0433	-.0198	-.0268	-.0325	-.0152
-80	-.0490	-.0633	-.0323	-.0363	-.0465	-.0227	-.0275	-.0350	-.0175
-90	-.0491	-.0662	-.0360	-.0367	-.0493	-.0255	-.0278	-.0375	-.0194

Appendix 11.A

Form factor between a strip on a slat and the image of another strip

The form factor $f_{m,n}$ for radiant energy exchange between any two areas. A_m and A_n, is the ratio of the radiation emanating from A_m that falls on A_n, to the total radiation emanating from A_m. When energy is transferred between the two surfaces due to specular reflection at some third surface it is convenient to include the specularly reflected energy with the direct radiant interchange. In this case the form factor is the sum of the ordinary (direct) form factor and a supplementary factor that takes account of the specularly reflected energy.
Figure 11.4 is an enlargement of part of figure 11.1. The form factor from a strip of infinitesimal width, dm, on strip m to all of strip n is

$$f_{dm,n} = \int_{\alpha_1}^{\alpha_2} \frac{\cos\alpha \; d\alpha}{2} \qquad \ldots \text{A 1}$$

When this is integrated over all of strip m it yields the usual crossed string formula

$$f_{m,n} = \frac{ac + bd - ad - bc}{2ab} \qquad \ldots \text{A 2}$$

The supplementary form factor from a strip of infinitesimal width dm on strip m to all of strip n is

$$f'_{dm,n} = \int_{\alpha'_1}^{\alpha'_2} \frac{\cos\alpha \; d\alpha}{2} \cdot R(\theta) \qquad \ldots \text{A 3}$$

where R (θ) is the reflection function that is derived in appendix 11.B
The angles α , θ and the slat angle φ are related by the expression

$$\alpha + \theta + \varphi = \pi/2 \qquad \ldots \text{A 4}$$

Thus $\cos\alpha = \sin\theta \, \sin\varphi + \cos\theta \, \sin\varphi$ $\qquad \ldots$ A 5

and $d\alpha = -d\theta$. $\qquad \ldots$ A 6

Therefore

$$f'_{dm,n} = -\frac{\cos\varphi}{2} \left\{ C_o \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta + C_1 \int_{\theta_1}^{\theta_2} \sin\theta \, \cos\theta \, d\theta + \ldots \int_{\theta_1}^{\theta_2} \sin\theta \, \cos^5\theta \, d\theta \right\}$$

$$- \frac{\sin\varphi}{2} \left\{ C_o \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta + C_1 \int_{\theta_1}^{\theta_2} \cos^2\theta \, d\theta + \ldots C_5 \int_{\theta_1}^{\theta_2} \cos^6\theta \, d\theta \right\} \qquad \ldots \text{A 7}$$

$$= \frac{\cos\varphi}{2} \left\{ C_o \cos\theta + \frac{C_1}{2} \cos^2\theta + \ldots\ldots\ldots\ldots \frac{C_5}{6} \cos^6\theta \right\}_{\theta_1}^{\theta_2}$$

$$- \frac{\sin\varphi}{2} \left\{ \theta\left(\frac{C_1}{2} + \frac{3C_3}{8} + \frac{5C_5}{16}\right) + \sin\theta \left(C_o + \frac{2C_2}{3} + \frac{3C_4}{15}\right) \right.$$

$$+ \sin\theta \cos\theta \left(\frac{C_1}{2} + \frac{3C_3}{8} + \frac{5C_5}{16} \right) + \sin\theta \cos^2\theta \left(\frac{C_2}{3} + \frac{4C_4}{15} \right)$$
$$+ \sin\theta \cos^3\theta \left(\frac{C_3}{4} + \frac{5C_5}{24} \right) + \sin\theta \cos^4\theta \left(\frac{C_4}{5} \right)$$
$$+ \sin\theta \cos^5\theta \left(\frac{C_5}{6} \right) \Bigg|_{\theta_1}^{\theta_2} \qquad \ldots \text{A 8}$$

The inclusion of the R (θ) function so complicates the expression for $f'_{dm,n}$ that it is impractical to try to integrate it over strip m in the usual way. It is much more convenient to evaluate $f'_{dm,n}$ at several points across strip m and then integrate numerically to get $f'_{m,n}$.

Appendix 11.B

Radiation from a blind that is reflected by the window pane

Much of the radiation that falls on a venetian blind is reflected back toward the window pane. When this reflected radiation strikes the surface of the glass part of it is again reflected, and this reflection by the window glass is specular. The reflectivity of glass depends on the angle of incidence of the light ray so the luminance of the image of a blind slat depends on the direction from which it is viewed.

Figure 11.5 shows a light ray incident on the YZ plane from the direction (θ , γ) where θ and γ are angular co-ordinates of the ray direction measured on orthogonal planes. If the YZ plane is taken as the plane of the window glass and P as a point on the surface of a blind slat, the flux density incident at point O on the glass is

$$\text{Flux density} = \iint_{\text{slat}} L \cos i \; dw$$

Where L is the luminance of the slat at point P in the direction toward O. The reflected flux density is

$$\text{Reflected flux density} = \iint_{\text{slat}} L \cos i \;\; \rho(i) \;\; dw$$

where (i) is the reflectivity of the glass at O for radiation incident on it at an angle i.

Thus the mean reflectivity of the surface at O is

$$\bar{\rho} = \frac{\iint_{\text{slat}} L \cos i \;\; \rho(i)\; dw}{\iint_{\text{slat}} L \cos i \;\; dw}$$

The reflectivity of a glass surface can be expressed as

$$\rho(i) = \sum_{j=o}^{5} r_j \cos^j i.$$

The coefficients r_j, for different types of clear glass are given in appendix 11.C.

$$\cos i = \cos\theta \cdot \cos\gamma$$

$$dw = \cos\gamma \; d\gamma \; d\theta$$

Thus if the blind slat luminance is independent of the Z co-ordinate and the slat goes from Z = $-\infty$ to Z = $+\infty$ (i.e. $\gamma = -\pi/2$ to $\gamma = +\pi/2$), the glass reflectivity can be expressed as a function of

$$R(\theta) = \frac{\sum_{j=o}^{5} r_j \cos^{j+1}\theta \int_{-\pi/2}^{\pi/2} \cos^{j+2}\gamma \;\; d\gamma}{\cos\theta \int_{-\pi/2}^{\pi/2} \cos^2\gamma \; d\gamma}$$

$$= \sum_{j=o}^{5} C_j \cos^j\theta$$

$$\text{where } C_j = \frac{2r_j}{\pi} \int_{-\pi/2}^{\pi/2} \cos^{j+2}\gamma \;\; d\gamma$$

i.e.,

$$
\begin{aligned}
C_o &= r_o \\
C_1 &= \frac{8}{3\pi} r_1 \\
C_2 &= \frac{3}{4} r_2 \\
C_3 &= \frac{32}{15\pi} r_3 \\
C_4 &= \frac{5}{8} r_4 \\
C_5 &= \frac{64}{35\pi} r_5 .
\end{aligned}
$$

Appendix 11.C

Reflection, absorption and transmission coefficients for a single sheet of glass

The reflectance, absorptance and transmittance of a single sheet of glass depend on the thickness and type of glass, and on the angle at which the radiation strikes the surface. The parameters are:

$K\ell$ — the product of extinction coefficient of the glass and the thickness of the sheet;
i — the angle between the incident ray and the normal to the surface;
η — the index of refraction of the glass.

The reflectance, absorptance and transmittance have been evaluated by Fresnel's relationships for many combinations of K_l and i for glass with refractive index of 1.52. The results of these calculations have been approximated by polynomial expressions using cos i as the argument : i.e.,

reflectance, $\rho(i) = \sum_{i=o}^{5} r_j \cos^j i$

absorptance, $\alpha(i) = \sum_{j=o}^{5} a_j \cos^j i$

transmittance, $\gamma(i) = \sum_{j=o}^{5} t_j \cos^j i$

Since $\rho + \alpha + \gamma = 1$, it follows that $r_o + a_o + t_o = 1$

and $r_j + a_j + t_j = 0$ for $j > 0$

Table 11.3 gives the values of r_j and t_j for the range of K_l that includes ordinary window glass and clear plate glass. If the a_j coefficients are required they can be evaluated from the values of r_j and t_j.

Table 11.3 Reflection and transmission coefficients for clear window glasses

$K\ell$ / j	.05		.10		.15		.20	
	r_j	t_j	r_j	t_j	r_j	t_j	r_j	t_j
0	0.99732	-0.00886	0.99478	-0.01114	0.99363	-0.01201	0.99316	-0.01218
1	-3.48910	2.71236	-3.80155	2.39372	-4.05533	2.13037	-4.26368	1.90950
2	4.56728	-0.62063	6.36051	0.42978	7.75296	1.13834	8.85765	1.61391
3	-1.50588	-7.07329	-5.39114	-8.98263	-8.32266	-10.07925	-10.59465	-10.64873
4	-1.37813	9.75996	2.31557	11.51799	5.04483	12.44162	7.12296	12.83699
5	0.88713	-3.89922	-0.40373	-4.52065	-1.34259	-4.83285	-2.04771	-4.95199