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ROOM THERMAL RESPONSE FACTORS

BY

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FACTEURS INFLUENCANT L'AMPLITUDE DES CHANGEMENTS THERMIQUES DANS UNE PIECE

SOMMAIRE

Les auteurs présentent une méthode pour le calcul à l'ordinateur des facteurs qui influencent la réponse thermique offerte par un système constitué par les éléments d'une pièce d'habitation. Cette méthode prend en considération la chaleur emmagasinée par les parois, plancher et plafond de la pièce et par son atmosphère et aussi, séparément, les transferts internes de chaleur par rayonnement et convection.

Le calcul de cette réponse thermique se base sur les équations du bilan thermique pour chacune des surfaces limitant la pièce et pour son atmosphère. La chaleur transmise aux surfaces limitant la pièce entre en ligne de compte pour le calcul de la réponse thermique du système constitué par chaque élément.

La série d'équations exprimant les températures des parois de la pièce, de l'atmosphère et des composantes de l'excitation fournie au système est écrite sous forme de matrice. La solution en est obtenue par une inversion de matrice. On calcule les facteurs de la réponse thermique offerte par le système à l'aide des éléments de la matrice inverse. Les auteurs étudient l'erreur découlant de la méthode de calcul de ces facteurs, et fournissent un exemple d'équation matricielle pour une pièce simple.

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ERRATUM SHEET

to

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(NRC 9884)

Formula (A-3) should read as follows:

$$q_{(0, n\Delta)} = \frac{\lambda L}{\alpha \Delta} \left[-\frac{1}{6} + \frac{\alpha n \Delta}{L^2} - \frac{2}{\pi^2} \sum_{m=1}^{\infty} \frac{(-1)^m \gamma_m^n}{m^2} \right]$$

Formula (A-4) should read as follows:

$$q_{(L, n\Delta)} = \frac{\lambda L}{\alpha \Delta} \left[\frac{1}{3} + \frac{\alpha n \Delta}{L^2} - \frac{2}{\pi^2} \sum_{m=1}^{\infty} \frac{\gamma_m^n}{m^2} \right]$$

ANALYZED

G. P. MITALAS

D. G. STEPHENSON

Room Thermal Response Factors

Room thermal response factors are a convenient way of presenting data on the dynamic thermal characteristics of a room for air-conditioning design calculations by digital computer. The application of these factors in cooling load and surface temperature calculations has been presented in a companion paper.[1] This paper presents a method of computing the factors for any room. It differs from the earlier work by Brisken and Reque[2] only in detail. The differences, however, lead to more accurate results.

The first difference is in the calculation of response factors for separate room components, such as walls and floor. Brisken and Reque simulate these elements by 2-lump networks and calculate the response factors for the network rather than for the actual building component; their result, therefore, has the same error as the simple 2-lump analog model.

The second difference is in the type of unit time-series that is used. The Brisken and Reque response factors are based on the response to a unit step in the excitation followed after an interval of Δ by a unit negative step, i.e., the excitation is a rectangular pulse of unit height and Δ duration. The unit time-series used in this paper is equivalent to a triangular pulse with a base width of 2Δ. A time-series based on triangular pulses represents a smooth function by a series of chords or straight-line segments, which is a better representation for a given Δ than the step type of function that is built up from rectangular pulses.

The third, and most important, difference is in the way that heat transfer between the room surfaces is represented. Brisken and Reque assume that the total heat flux at any surface is proportional to the difference between the surface temperature and the room air temperature, i.e. they use a combined coefficient to account for convection and radiation. Mitalas[3] has shown that a combined coefficient can lead to a serious error in either the surface temperatures or the cooling load. The present method accounts for the convection and radiation separately, as is shown in the following detailed discussion.

The method for obtaining response factors is based on a set of heat balance equations. The room cooling load and surface temperatures are considered as unknowns which can be found by solving the heat balance equations for all the surfaces and the room air. The temperatures and cooling load that result from a unit time-series of one component of room excitation are the response factors for that mode of excitation. For example, the excitations could be radiant energy from lights or sun that is absorbed at any surface, or sol-air temperature for the outside surface of outer wall. The non-steady-state conduction in the walls and floor is the most complex component in the various heat balance equations, so it is discussed in detail before the solution of the equations is considered.

RESPONSE FACTORS FOR A WALL

A simple application of the time-series response-factor method is in finding the heat fluxes at the two bounding surfaces of a wall. It is assumed that the heat flow can be described by the simple one-dimensional Fourier heat conduction equation, so superposition techniques are valid. The definitions of time-series and response-factors are presented in the companion paper[1] and are used here in the same sense.

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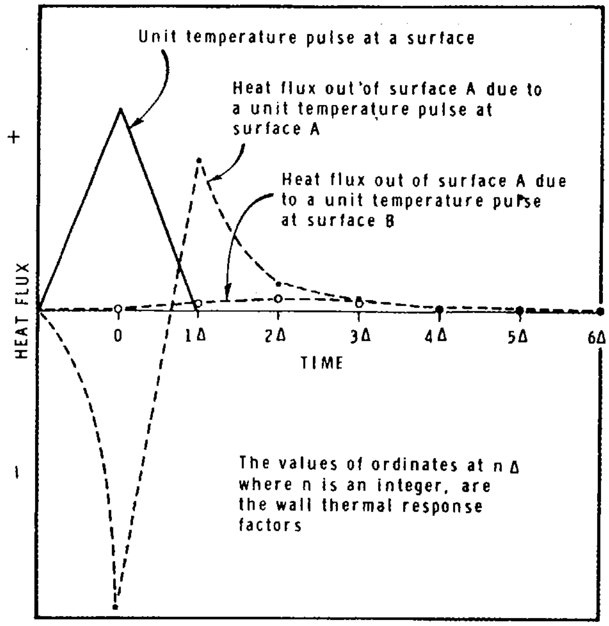

Fig. 1 Heat fluxes at surface A due to unit temperature pulse at surfaces A and B

Homogeneous Wall

If one surface of the wall is designated by subscript A and the other by subscript B, the heat fluxes, Ω, can be expressed in terms of the surface temperatures and response factors by

$$\Omega_A = \theta_A \cdot X - \theta_B \cdot Y \tag{1}$$

$$\Omega_B = \theta_A \cdot Y - \theta_B \cdot Z \tag{2}$$

where

- Ω_A and Ω_B are the time-series for the flux in to surface A and out of surface B, respectively
- θ_A and θ_B are the time-series for the temperatures at surfaces A and B, respectively
- X and Y are the time-series for the flux at surfaces A and B, respectively, due to a unit time-series of temperatures at surface A ($\theta_A = 1, 0, 0 \ldots$; $\theta_B = 0, 0, 0 \ldots$, as illustrated in Fig. 1)
- Y and Z are the time-series for the flux at surfaces A and B, respectively, due to a unit time-series of temperatures at surface B ($\theta_B = 1, 0, 0 \ldots$; $\theta_A = 0, 0, 0 \ldots$)

Because of the symmetry of the homogeneous slab, X and Z are the same. The formulae for calculating the response factors for a homogeneous slab are given in Appendix A.

Two-Layer Wall

The same basic procedure is used to determine the heat fluxes at the surfaces of a multi-layer wall. Take, for example, a wall composed of two slabs, designated by subscripts 1 and 2 on their unit response functions; outside surface of slab 1 designated A, outside surface of slab 2 designated C, and the interface between slabs 1 and 2 designated B.

Then

$$\Omega_A = \theta_A \cdot X_1 - \theta_B \cdot Y_1 \tag{3}$$

$$\Omega_B = \theta_A \cdot Y_1 - \theta_B \cdot Z_1 \tag{4}$$

$$= \theta_B \cdot X_2 - \theta_C \cdot Y_2 \tag{5}$$

$$\Omega_C = \theta_B \cdot Y_2 - \theta_C \cdot Z_2 \tag{6}$$

Thus

$$\theta_B = \frac{\theta_A \cdot Y_1 + \theta_C \cdot Y_2}{Z_1 + X_2} \tag{7}$$

and

$$\Omega_A = \theta_A \left\{ X_1 - \frac{Y_1^2}{Z_1 + X_2} \right\} - \theta_C \left\{ \frac{Y_1 \cdot Y_2}{Z_1 + X_2} \right\} \tag{8}$$

$$\Omega_C = \theta_A \left\{ \frac{Y_1 \cdot Y_2}{Z_1 + X_2} \right\} - \theta_C \left\{ Z_2 - \frac{Y_2^2}{Z_1 + X_2} \right\} \tag{9}$$

The sets in the brackets are the response factors for the composite wall. This process of combination can be repeated as many times as the number of layers in a multilayer wall.

ROOM HEAT BALANCE EQUATIONS USING THERMAL RESPONSE FACTORS

Various heat transfer processes take place inside a room. Radiant energy enters through a window and strikes the floor and other inside surfaces. Energy is released by lighting fixtures in the room, and energy is transferred through the room envelope by conduction. Energy is continually being exchanged among all the inside surfaces by radiation and between the surfaces and the room air by convection. Heat is also added or extracted from the room air by the air-conditioning system.

Because of the interdependence of the heat flows between the enclosing surfaces and the room air, it is necessary to consider the complete room as a single system. The dynamic performance of this system must be taken into account because of the continuous variation of conditions inside and outside of the room.

The equations describing the heat transfer processes can be set up by considering the heat balances at each of the several surfaces that enclose the room and for the room air. This gives the number of equations required to find the temperature of unknown surface and air temperatures.

The heat balance for any surface i at time $n\Delta$ is

$$(\chi_i + \psi_i + \omega_i + e_i)_n = 0 \tag{10}$$

where

χ_i = convection heat gain by unit area of surface i

ψ_i = net radiant heat gain by unit area of surface i, due to emission of room surfaces

ω_i = heat flux by conduction toward surface i

e_i = excitation component i, i.e., energy input to a unit area of surface i by solar radiation and/or radiant energy from lights, etc.

n = integer (for simplicity the subscript n is used instead of $n\Delta$ in the following discussion to indicate time).

The convection heat transfer at surface i is

$$\chi_i = h_i \quad (\theta_a - \theta_i) \tag{11}$$

where

h_i = convection heat transfer coefficient at surface i. (It is assumed that this coefficient is a constant.)

θ_a = air temperature

θ_i = temperature of surface i.

The net radiant heat gain by unit area of surface i can be approximated by

$$\psi_i = \sum_{j=1}^{J} g_{i,j} \quad (\theta_j - \theta_i) \tag{12}$$

where

J = number of surfaces forming the enclosure

$$g_{i,j} = f_{i,j} \quad 4\sigma \quad T^3_{avg} \tag{13}$$

$f_{i,j}$ = absorption factors for surface i (same as Hottel's script F)

σ = Stefan-Boltzmann constant

T_{avg} = time average of all absolute surface temperatures (assumed approximate value).

The approximation as given by Eq (13) is sufficiently accurate for air-conditioning design calculations[3].

Calculation of the absorption factors is quite laborious so a computer program[4] has been written to mechanize these calculations. In addition, this program calculates the distribution of shortwave radiation over the surfaces of the room.

As already demonstrated, conduction heat flux, Ω, at a wall surface can be expressed in time-series form as the sum of the products of the surface temperature time-series and the appropriate unit response functions:

$$-\Omega_i = \theta_i \ X \ - \ \theta_k Y \tag{14}$$

where the subscript k indicates the other surface of the wall. Based on Eq (14) the heat conduction through a wall surface at time n is given by

$$\omega_{i,n} = -\sum_{p=o}^{\infty} \theta_{i,\,(n-p)} \, x_p + \sum_{p=o}^{\infty} \theta_{k,\,(n-p)} \, y_p \tag{15}$$

Substitution of expressions for χ_i, ψ_i, and ω_i in Eq (10) and collection of the unknown on the left-hand side gives:

$$- \theta_{i,n} \left(h_i + \sum_{j=1}^{J} g_{i,j} + x_o\right) + \theta_{k,n} \, y_o + \sum_{j=1}^{J} g_{i,j} \, \theta_{j,n} = - e_{i,n} - \theta_{a,n} \, h_i + \sum_{p=1}^{\infty} \theta_{i,(n-p)} \, x_p - \sum_{p=1}^{\infty} \theta_{k,(n-p)} \, y_p \tag{16}$$

The heat balance for an outside surface can be simplified by the use of the sol-air temperature concept, that is,

$$H_i \ (\theta_{sa,i} - \theta_i) + \omega_i = 0 \tag{17}$$

where

H_i = appropriate surface heat transfer coefficient to be used with $\theta_{sa,i}$

$\theta_{sa,i}$ = sol-air temperature of surface i, i.e. excitation component e_i.

The heat balance for room air gives

$$\frac{d\theta_a}{dt} = \frac{(\Phi - q)_t}{B} \tag{18}$$

where

t = time

B = heat storage capacity of room air

q = rate of sensible heat removal from room air by air-conditioning system

$$\Phi = \sum_{i=1}^{J} A_i \, h_i \, (\theta_i - \theta_a) \tag{19}$$

Table I. Elements of Matrix [M] for Example Problem

$-h_1 - x_{a,o} - \sum_{j=1}^{J} g_{1,j}$	$g_{1,2} + y_{a,o}$	$g_{1,3}$	$g_{1,4}$	$g_{1,5}$	—	$g_{1,7}$	$g_{1,8}$
$\frac{1}{2}g_{2,1} + y_{a,o}$	$-\frac{1}{2}(h_2 + \sum_{j=1}^{J} g_{2,j}) - z_{a,o}$	$\frac{1}{2}g_{2,3}$	$\frac{1}{2}g_{2,4}$	$\frac{1}{2}g_{2,5}$	—	$\frac{1}{2}g_{2,7}$	$\frac{1}{2}g_{2,8}$
$g_{3,1}$	$g_{3,2}$	$-h_3 - x_{b,o} - \sum_{j=1}^{J} g_{3,j}$	$g_{3,4} + y_{b,o}$	$g_{3,5}$	—	$g_{3,7}$	$g_{3,8}$
$g_{4,1}$	$g_{4,2}$	$g_{4,3} + y_{b,o}$	$-h_4 - x_{b,o} - \sum_{j=1}^{J} g_{4,j}$	$g_{4,5}$	—	$g_{4,7}$	$g_{4,8}$
$g_{5,1}$	$g_{5,2}$	$g_{5,3}$	$g_{5,4}$	$-h_5 - x_{d,o} - \sum_{j=1}^{J} g_{5,j}$	$y_{d,o}$	$g_{5,7}$	$g_{5,8}$
—	—	—	—	$y_{d,o}$	$-H_6 - z_{d,o}$	—	—
$g_{7,1}$	$g_{7,2}$	$g_{7,3}$	$g_{7,4}$	$g_{7,5}$	—	$-h_7 - H_7 - \sum_{j=1}^{J} g_{7,j}$	$g_{7,8}$
$g_{8,1}$	$g_{8,2}$	$g_{8,3}$	$g_{8,4}$	$g_{8,5}$	—	$g_{8,7}$	$-h_8 - \sum_{j=1}^{J} g_{8,j}$

= rate of heat gain by room air from room envelope surfaces
A_i = area of surface i

As the heat storage by room air is only a small fraction of the total heat storage of the room, the rate of change of room air temperature is adequately approximated by:

$$\frac{d\,\theta_a}{dt} \approx \frac{\theta_{a,t} - \theta_{a,\,t-\Delta}}{\Delta} \tag{20}$$

Substitution of Eqs (19) and (20), with appropriate subscripts in Eq (18) and collection of unknowns on left-hand side gives

$$\sum_{i=1}^{J} A_i h_i \;\theta_{i,n} - \left(\frac{B}{\Delta} + \sum_{i=1}^{J} A_i h_i\right)\theta_{a,n} + \frac{B}{\Delta}\;\theta_{a,(n-1)} = q_n \tag{21}$$

To facilitate the manipulation of the set of surface heat balance equations [where (Eq 16) is a general equation of the set] they can be expressed by a simple matrix equation.

$$[M] \cdot [\theta]_n = [K]_n \tag{22}$$

which can be solved for the temperatures by inverting the matrix [M], i.e.,

$$[\theta]_n = [M]^{-1} \cdot [K]_n \tag{23}$$

As an example, the elements of matrix [M] and column $[K]_n$ are given in Tables I and II for a rectangular room with a single sheet of plate glass as an outside wall, and furniture covering half of the floor area. For bookkeeping purposes, the following subscript system is used:

Floor-ceiling system	a
End walls	b
Back wall	d
Ceiling surface	1
Floor surface	2

Table II. Elements of Column Matrix $[K]_n$ For Example Problem

$-e_1 - h_1\,\theta_9 + \sum_{p=1}^{\infty} (\theta_{1,n-p}\,x_{a,p} - \theta_{2,n-p}\,y_{a,p})$
$-\frac{1}{2}(e_2 + h_2\,\theta_9) + \sum_{p=1}^{\infty} (\theta_{2,n-p}\,z_{a,p} - \theta_{1,n-p}\,y_{a,p})$
$-e_3 - h_3\,\theta_9 + \sum_{p=1}^{\infty} (\theta_{3,n-p}\,x_{b,p} - \theta_{4,n-p}\,y_{b,p})$
$-e_4 - h_4\,\theta_9 + \sum_{p=1}^{\infty} (\theta_{4,n-p}\,x_{b,p} - \theta_{3,n-p}\,y_{b,p})$
$-e_5 - h_5\,\theta_9 + \sum_{p=1}^{\infty} (\theta_{5,n-p}\,x_{d,p} - \theta_{6,n-p}\,y_{d,p})$
$-H_6\,e_6 + \sum_{p=1}^{\infty} (\theta_{6,n-p}\,z_{d,p} - \theta_{5,n-p}\,y_{d,p})$
$-H_7\,e_7 - h_7\,\theta_9$
$-e_8 - h_8\,\theta_9$

Surfaces of end walls	3 and 4
Inside surface of back wall	5
Corridor surface of back wall	6
Window glass	7
Furniture surface	8
Room air	9
Corridor air	10

In setting up the matrix equation for this room, it was assumed that:

1) Thermal resistance of window pane is negligible. The heat balance, therefore, for window pane is:

$$(\chi_7 + \psi_7)_{\substack{\text{inside}\\\text{surface}}} + (\chi_7 + \psi_7)_{\substack{\text{outside}\\\text{surface}}} + e_7 = 0 \tag{24}$$

2) The lateral floor slab conduction is infinite and the furniture conduction is very low. The heat balance, therefore, is <u>for floor surface</u>

$$\tfrac{1}{2}(\chi_2 + \psi_2 + e_2) + \omega_2 = 0 \tag{25}$$

since half of floor surface is covered by furniture;
<u>for furniture surface</u>

$$\chi_8 + \psi_8 + e_8 = 0 \tag{26}$$

3) The room is surrounded by similar rooms. The surface temperature, therefore, of the floor above the ceiling, of the ceiling below the floor, and of the other side of the end walls was taken as equal to corresponding surface temperatures inside the room.

4) The convective and radiative heat transfer at outside surface of corridor wall is

$$\chi_6 + \psi_6 = H_6 (\theta_{10} - \theta_6) \tag{27}$$

where H_6 is a combined coefficient.

ROOM THERMAL RESPONSE FACTORS

The room surface temperatures and cooling load can be calculated by Eqs (21) and (23) provided the room excitation and room air temperature are known. When unit time-series excitation components are used one at a time, Eq (23) gives surface temperature response factors and Eq (21) gives cooling load response factors. The detailed procedure for evaluating these factors is given in Appendix B.

SELECTION OF TIME INTERVAL FOR ROOM THERMAL RESPONSE FACTOR CALCULATIONS

To set up Eqs (21) and (23) it is assumed that the room air and surface temperatures can be approximated by time-series. The magnitude of the error introduced in the response functions by this approximation depends on the length of the time interval Δ. It is difficult to derive a general criterion for the selection of Δ to give a specified accuracy. The error can be estimated, however, by comparison of room thermal response factors evaluated for two different values of Δ, for example, factors evaluated for time interval Δ, and the transformed factors for time interval Δ that were originally evaluated for time interval $\Delta/2$. The procedure for transforming the factor from $\Delta/2$ to Δ time intervals is given by Eq (A-12) in Appendix A. As an indication of the size of time interval to be used in practical calculations the factors were evaluated for a room, 20 by 20 by 10 ft with 200 sq ft of single plate glass as outside wall, 6-in. heavy concrete floor slab and 3-in. lightweight concrete partitions. The comparison of factors for $\Delta = 1.0$ hr and factors evaluated for $\Delta = 0.25$ hr and then transformed to $\Delta = 1.0$ hr showed only a small difference. This indicates that $\Delta = 0.25$ hr is small enough to keep the error within acceptable limits.

CONCLUSION

A set of heat balance equations has been derived to describe the dynamic thermal characteristics of a room. These equations take account of heat storage by the room envelope and room air as well as radiation and convection heat transfer within the room. This method assumes that the heat flow through building elements is one dimensional, i.e. effects of room corners or other irregularities are ignored, and that room air is at uniform temperature throughout the room.

This set of equations can be used directly to calculate surface temperatures, room air temperature and heating or cooling of room air provided the room excitation components are known. The same set of equations can also be used to calculate the room thermal response factors. These factors contain all the necessary information about the thermal characteristics of the room required for air-conditioning design calculations using digital computers. They need be calculated only once for any room, and can then be combined with any set of excitation data to give the room's response for the particular conditions.

This is a contribution from the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division.

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APPENDIX A

THERMAL RESPONSE FACTORS FOR A HOMOGENEOUS SLAB

The thermal response factors for a homogeneous slab can be expressed directly in terms of the thermal properties and thickness of the slab and the time interval.

The temperature at any point in a slab is given by the following equation (Ref 5, p. 214, Eq (2))

$$U_{(d,\tau)} = A\left[\frac{d^3 - d}{6} + d_\tau + \frac{2}{\pi^3}\sum_{m=1}^{\infty}\frac{(-1)^{m-1} e^{-m^2\pi^2\tau}}{m^3}\sin m\pi d\right] \qquad \text{(A-1)}$$

when

$$U_{(d,o)} = 0,$$

$$U_{(o,\tau)} = 0$$

and

$$U_{(1,\tau)} = A\tau$$

where

$d = \frac{1}{L}$; dimensionless distance (L = slab thickness)

$\tau = \frac{\alpha t}{L^2}$; dimensionless time

α = thermal diffusivity

t = time

A = rate of surface temperature rise, i.e.,

$$U_{1,\tau} = A\tau \text{ or } U_{(L,t)} = A\frac{\alpha t}{L^2}$$

The heat flux through any plane at the distance d is simply the product of the temperature gradient at the plane d and thermal conductivity, λ, i.e.

$$q_{(d,\tau)} = \frac{\lambda}{L} \cdot \frac{\partial U_{(d,\tau)}}{\partial d} \qquad \text{(A-2)}$$

Selection of A so that the surface temperature is one unit at $t = \Delta$, i.e. $A = \frac{L^2}{\alpha\Delta}$, and substituting $t = n\Delta$ and $\tau = \frac{\alpha n\Delta}{L^2}$ gives

$$q_{(0,n\Delta)} = \frac{\lambda L}{\alpha \Delta}\left[-\frac{1}{6} + \frac{\alpha n\Delta}{L^2} + \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^m \gamma_m^n}{m^2}\right] \qquad \text{(A-3)}$$

$$q_{(L,n\Delta)} = \frac{\lambda L}{\alpha \Delta}\left[\frac{1}{3} + \frac{\alpha n\Delta}{L^2} + \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^m \gamma_m^n}{m^2}\right] \qquad \text{(A-4)}$$

where

$$\gamma_m = \exp\left(-(m)^2 \cdot \pi^2 \cdot \frac{\alpha\Delta}{L^2}\right) \qquad \text{(A-5)}$$

Thus the surface heat fluxes through the surfaces due to the triangle surface temperature variation, i.e., thermal response factors for a homogeneous slab, are:

$$x_o = -\frac{\lambda}{L}\frac{L^2}{\alpha\Delta}\left[-\frac{1}{3} - \frac{\alpha\Delta}{L^2} + \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{\gamma_m}{(m)^2}\right] \qquad \text{(A-6)}$$

$$x_1 = -\frac{\lambda}{L}\frac{L^2}{\alpha\Delta}\left[\frac{1}{3} + \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{\gamma_m^2 - 2\gamma_m}{(m)^2}\right] \qquad \text{(A-7)}$$

$$x_n = -\frac{\lambda}{L}\frac{L^2}{\alpha\Delta}\frac{2}{\pi^2}\sum_{m-1}^{\infty}\frac{\gamma_m^{n+1} - 2\gamma_m^n + \gamma_m^{n-1}}{(m)^2} \qquad \text{(A-8)}$$

where $n \geq 2$;

$$y_o = -\frac{\lambda}{L}\frac{L^2}{\alpha\Delta}\left[\frac{1}{6} - \frac{\alpha\Delta}{L^2} + \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^m \gamma_m}{(m)^2}\right] \qquad \text{(A-9)}$$

$$y_1 = -\frac{\lambda}{L}\frac{L^2}{\alpha\Delta}\left[-\frac{1}{6}\right.$$

$$\left. + \frac{2}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^m(\gamma_m^2 - 2\gamma_m)}{(m)^2}\right] \quad \text{(A-10)}$$

$$y_n = -\frac{\lambda}{L}\frac{L^2}{\alpha\Delta}\frac{2}{\pi^2} \quad \text{(A-11)}$$

$$\sum_{m=1}^{\infty}\frac{(-1)^m(\gamma_m^{n+1} - 2\gamma_m^n + \gamma_m^{n-1})}{(m)^2}$$

where $n \geq 2$.

The factors calculated for time interval Δ can be used to calculate factors for 2Δ time interval by

$$\rho_n = 0.5r_{(2n-1)} + r_{2n} + 0.5r_{(2n+1)} \quad \text{(A-12)}$$

where

r_n = response factor for time interval Δ

ρ_n = response factor for time interval 2Δ

A computer program has been prepared [6] to carry out the calculations as defined by Eqs (A-6) to (A-12) as well as to combine response factors of homogeneous layers to give the factors for multi-layer wall.

APPENDIX B

PROCEDURE FOR CALCULATING ROOM THERMAL RESPONSE FACTORS

The surface temperature and cooling load response factors for any one excitation component and for room air temperature can be calculated by Eqs (23) and (21), i.e.,

$$[\theta]_n = [M]^{-1} \quad [K]_n \quad \text{(B-1)}$$

and

$$q_n = \sum_{i=1}^{J} A_i h_i \theta_{i,n} - \left(\frac{B}{\Delta} + \sum_{i=1}^{J} A_i h_i\right) \quad \text{(B-2)}$$

$$\theta_{a,n} + \frac{B}{\Delta}\theta_{a(n-1)}$$

The elements in the $[\theta]_n$ column matrix are the values of the surface temperatures of all the surfaces at time n. This column is obtained by post multiplication of the $[M]^{-1}$ matrix by the $[K]_n$ column matrix. It should be noted that the $[M]^{-1}$ matrix is a constant while the elements of $[K]_n$ column matrix depend on the excitation components at time n, room air temperature at time n and the complete temperature histories of the surfaces as shown by the formulae given in Table I for the example.

The surface temperature history is generated as the solution proceeds from one time step to the next. The history is all zero for n = 0; for n = 1 the history is the temperatures calculated for n = 0, and so on.

The surface temperature response factors for any excitation component, e_j for example, are evaluated by letting e_j assume a unit time-series variation (i.e. 1, 0, 0, 0, ...) while all the other excitation components and room air temperature are zero. For response to room air temperature variation, all the excitation components are zero and room air temperature has unit time-series variation.

The calculations must start, of course, at n = o, in which case the elements of $[K]_o$ are very simple. In case of response factor calculations for excitation component e_j, all the elements are zero except the element $K_{j,o}$, which equals one. In case of room air temperature all the terms of the elements $K_{i,o}$, are zero except the term $h_i\,\theta_{a,o}$, i.e. $h_i\,\theta_{a,o} = h_i$

Using $[K]_o$ the temperatures for n = o are calculated by post multiplication of $[M]^{-1}$ by $[K]_o$; then using the temperatures calculated for n = o the elements of column $[K]_1$ are calculated. The calculations are repeated for successive values of n until the ratio of successive terms in each of the temperature time-series becomes constant. From then on the rest of the terms can be evaluated by continuing the geometric progression.

The temperatures, $\theta_{i,n}$ calculated for excitation component e_j, or room air temperature are the surface temperature response factors, $s_{i,j,n}$

The cooling load response factors for the surface excitations e_j can be obtained directly from the surface temperature response factors i.e.,

$$r_{j,n} = \sum_{i=1}^{J} A_i h_i s_{i,j,n} \quad \text{(B-3)}$$

This gives the heat that is transferred to the room air from the surfaces when $\theta_a = 0$ and hence this is the amount that must be removed from the room air to maintain $\theta_a = 0$. It is necessary to use Eq (B-2), however, to get the cooling load response factors for a unit pulse variation in θ_a.

There are, obviously, as many sets of room response factors as there are excitation components. The whole procedure outlined above has to be repeated in turn, therefore, with a unit time-series for each excitation component and once for room air temperature.

DISCUSSION

KEN-ICHI KIMURA, Tokyo, Japan (Written): 1) I should think Cooling Load Calculations by Thermal Response Factor Method is an extensively organized study on cooling load calculation method. The fundamental concept is similar to the method discussed among our Japanese investigators using weighting function expressed in exponential polynomials as room thermal response against a unit excitation pulse. I rather prefer the finite difference method because the procedure is simple, it can be treated in completely arithmetic expression, and temperature and load values come out simultaneously. I am currently trying to have it computerized and obtaining similar results to those of this paper. In addition I have been making some experiments with a small test room of 2 x 1 x 1 (m) fixed to the west exposure and with the recently completed semi-full size test room of 4 x 4 x 2 (m) which can be revolved to be adjusted to any orientation. With these, similar cooling load profile have been obtained, except a slightly longer time lag in peak load, to the curve of this paper in spite of different environmental conditions. In this connection, a presentation of cooling load curve versus incident solar radiation as Carrier's storage factor curve would seem to have been a clearer reference to be easily compared to others with different conditions.

2) Judging from the cooling load profile and temperature variation, the assumption that furniture has no heat storage capacity seems to have greatly effected the cooling load due to solar radiation and temperature of furniture. I should think furniture with a certain amount of heat capacity, say, 0.2 Btu/F against one ft^2 of floor area including various kinds of paper, would have resulted in less peak load. Furniture covering one-half floor area assumed here is felt somewhat ambiguous because it is not clear if furniture is assumed to be fixed on the floor, and whether or not convective and radiative heat transfer from or to furniture surface is assumed to occur on both upper and lower surfaces of furniture as buoyant in the room space.

With regard to Room Thermal Response Factors: 1) would it be impossible or very difficult to calculate one overall room thermal response which could be combined with one excitation function as an integration of total transmitted solar radiation and outside air temperature so that total cooling load could be directly obtained instead of summing up the load components due to floor, ceiling, etc. ? If it were to be realized and expressed in terms per unit floor area, I would think it very easy to compare the thermal characteristics of different rooms with several standardized cases if prepared as criteria.

2) Would not the radiation exchange among inside surfaces except against glass be negligible compared to the convection component? If it were to be neglected the procedure would be considerably reduced.

3) What value would have been taken as the convective heat transfer coefficient at room surfaces?

4) How would the reflection component of the solar radiation upon the room surfaces have been treated?

AUTHOR STEPHENSON (Written): With regard to Cooling Load Calculation By Thermal Response Factor Method, the authors have used the finite difference method mentioned by Dr. Kimura but have discarded it in favor of the method described in this paper. The main reason being that the response factor method is much cheaper in terms of computer time. The only lengthy calculation is finding the response factors for a room. These need be calculated only once and then they can be combined very easily with any set of driving functions to get the loads that would result for that particular circumstance. With the finite difference method every run is equivalent to finding the response factors.

Dr. Kimura also expressed the opinion that the Carrier storage method of relating cooling load to incident solar radiation is preferable to the method given in the paper. The authors agree that the Carrier method is very convenient but we think that it oversimplifies the problem. We hope that future research into methods of predicting loads and energy requirements will show whether the simple methods are adequate for modern engineering analysis and whether the more complex methods are really any better.

Dr. Kimura's second question dealing with furniture representation has been dealt with in reply to Mr. Priester. We recognize that this method of representing furniture is quite crude but we think that more accurate representations of furniture must be based on experimental studies in a calorimeter room. We have plans to do some work on this during the coming summer.

With respect to Room Thermal Response Factors, it is quite easy to combine the cooling load response factors for the room surfaces and various excitations into a single set of factors relating cooling load to solar intensity. It is important to remember, however, that the combined factors are valid only for the one situation for which they were calculated. The main point of the method described in this paper is that it keeps the factors that characterize the room separate from the excitation data. The room response factors, as given in the paper, being independent of the excitations, can be combined with any set of excitation data to give the cooling load for that specific case.

The radiation exchange among the surfaces that enclose a room has been studied (Ref 1) and found to be quite important. A recent paper by Muncey and Spencer (Journal of IHVE, April 1966, p. 35) suggests that radiant energy transfer may be allowed for by a simpler method than the one given in this paper, but it would certainly be a mistake to neglect it entirely.

The convection coefficients at all the room surfaces were taken as 0.8 Btu/ft^2, hr, F for the example problem. The inter-reflection of solar radiation between the walls, floor and ceiling surfaces is taken into account when calculating the solar absorption factors for the various surfaces. The point is discussed fully in Ref 4.

G. B. PRIESTER, Baltimore, Md. (Written): The paper Cooling Load Calculations by Thermal Response Factor Method by Stephenson and Mitalas was read with considerable interest and should be of help to many engineers.

To make the paper more beneficial, would the authors please extend their example to show exactly how specific values in Tables I and II were obtained. In other words, extend the example on the floor surface load by showing which values were used in what equations in Ref 6 of the paper to obtain the response factor 17.7. Also, show how to obtain the excitation factor 80.75 with specific values in a specific equation.

AUTHOR STEPHENSON (Written): It is always difficult to decide what should be included in a paper and what should be left out. In this paper we deliberately left out the numerical work that was done by computer, and only indicated the operations that were required. It is quite impractical to present in this closure all the calculations involved in obtaining any specific response factor. It may help, however, to summarize the steps that are involved:

1) Calculate the thermal response factors for outside wall, floor-ceiling and partition walls by the procedure given in appendix A of the second paper.
2) Calculate the radiant energy absorption factors for the room (these are the factors designated by a script F in most heat transfer books, e.g., McAdams).
3) Calculate the numerical values of the elements of matrix [M] and column [K] using the previously calculated thermal response factors and radiation absorption factors along with the surface heat transfer coefficients. (The expressions for the elements of these matrices for the example problem are given in Tables I and II of the second paper.) At this stage it must be decided how the furniture is to be represented in the mathematical model. For the example problem it was assumed that the furniture covered half of the floor surface, i.e., that perfect insulation covered half of the floor area.
4) Invert matrix [M], i.e., calculate $[M]^{-1}$.
5) Calculate the surface temperature response factors using $[M]^{-1}$ and column [K] following the procedure outlined in appendix B of the second paper.
6) Calculate the cooling load response factors by Eq B-3 using the surface temperature response factors as data.

The excitation factor 80.75 in the fourth column of Table I is a more manageable thing to discuss; it is the sum of three components:

a) radiant energy from lights absorbed at the surface
b) diffuse solar radiation absorbed at the surface
c) direct solar radiation absorbed at the surface

The direct and diffuse solar radiation incident on the floor were calculated using the procedure in Equations for Computer Calculation of Solar Heat Gain Factors in the ASHRAE Handbook of Fundamentals (1967), p. 479. The data were:

West Orientation
40 deg North Latitude
18 deg North solar declination
Apparent solar constant, $A = 368$ Btu/ft^2/hr
Atmospheric extinction coefficient, $B = 0.223$
Sky diffuse factor, $C = 0.10$
Ground reflectivity, $\rho_g = 0.20$
Window glass parameter, $Kl = 0.15$

The values of the three components are:

a	=	7.50	Btu/ft^2/hr
b	=	3.80	"
c	=	69.45	"
Total	=	80.75	"

It was assumed that the transmitted solar radiation is uniformly distributed over all the floor surface. Also it should be noted that only 94% of the transmitted solar radiation is absorbed by the floor and furniture surfaces. The other 6% is absorbed by the other room surfaces or is reflected back to the outside through the window.

G. V. PARMALEE, Cleveland, Ohio: It is very encouraging to see this paper by Dr. Stephenson and Mr. Mitalas on this important subject, one that has concerned air-conditioning engineers for a good many years. I think most of you recall the early work of Professor Mackey and Charles Leopold in pointing out the magnitude of the effect of heat storage on air-conditioning equipment loads.

I have one comment on the mathematical model that the authors have used. It is a model which most of us have used, that is, treating radiation separately from convection because they follow different laws and the exchanges by the two modes may have opposite signs. However, I wonder if the model for convection has been somewhat oversimplified. Conceivably, the convection currents that arise in the room due to heat from lighting, solar radiation, and other sources are not necessarily immediately and entirely picked up by the air cooling system; perhaps a portion redistributes some of the heat input to other surfaces in the room where that energy is stored. The result would be to increase the heat storage effect and further modify the disparity between instantaneous gain and system load. I should like to ask the authors if they see this effect as having some importance, and if so, do they plan to modify their mathematical model?

I would like to compliment the authors on producing a very workable type of calculation method. It should be especially adaptable to correlating field studies, which I understand are being planned, with the mathematical methods, because their method does not require having a steady periodic heat input function. Weather and load conditions vary considerably from day to day and this presents real problems on trying to correlate analytical load calculations to actual performance.

AUTHOR STEPHENSON (Written): Professor Parmelee's comment really questions the validity of our assumption that the air in a room is so well stirred that it is all at the same temperature. Obviously there are air temperature differences within most rooms, so the mathematical model that ignores this has some error. We think that this error is probably less important than the error associated with the representation of furniture. We also think that the adequacy of our mathematical model should be checked by a carefully conducted experimental determination of cooling load for a full-sized room. We plan to make such a test before we do any further development of the mathematical model.

D. R. BAHNFLETH, Chicago, Ill.: This is for Dr. Stephenson. I believe this is another contribution that will help unleash the computer power available to us which has been used only in a limited way in system design. I have not read your paper and it may answer this question which was prompted by Fig. 5. The curves seem to indicate that the room temperature rise was coincident with the point where the cooling load exceeded the system capacity. This seems to deny the existence of heat storage in the system and I wonder if this is the case?

AUTHOR STEPHENSON: The method, of course, does not neglect this heat storage but because of the rather light model we used, and the scale of this graph being what it is, you do not see at first glance that the air temperature is above room temperature after the cooling load has fallen within the capacity of the unit again.

If we had been dealing with a much more massive room, the time shift would have been more apparent. The result you question is a consequence of the particular example that we have used. The rise in air temperature starts when the cooling load exceeds the system capacity, and the period of off-control extends a short while after the juncture where the plant is again able to cope with the cooling load.

A. E. GALSON, Syracuse, N.Y.: It was a very interesting paper to me, as a consulting engineer. The consulting engineer is interested in the most economical selection of the terminal unit in the room. This selection involves two questions:

1) What is the correct room temperature? As mentioned in the first paper, it is not necessarily 75 or 72 degrees. It is subject to variations in room surface temperature; and, 2) What is the permissible room temperature swing? The occupants would probably be quite satisfied with a limited temperature swing. How would this calculation consider the effect of room surface temperature on comfort or effective temperature and the permissible temperature swing for the most economic terminal unit load?

AUTHOR STEPHENSON: If you can decide on what you need in the room for comfort, the computer can be used to calculate the cooling capacity that is required to meet the specification.

E. L. GALSON, Syracuse, N.Y.: We felt by the references that the Canadian National Research Council has quite an ongoing program on computer methods of calculating heat loads. I know that the Society is about to embark on a program to study energy consumption in buildings using computer techniques similar to what you are doing now. I wonder if you contemplate any active participation in this program, or how you coordinate the work of the Canadian National Research Council with the Society.

This question may not be pertinent to this specific paper, but I think it is pertinent to the entire research effort.

CHAIRMAN STOECKER: The author has asked me to answer.

The Task Group on the Requirements for Heating and Cooling Energy is represented by Dr. Stephenson, and the National Research Council is represented by Dr. Stephenson, so certainly there will be a very close exchange of information and techniques.

G. S. CAMPBELL, Chattanooga, Tenn.: Assuming this procedure is developed into the basic industry calculation methods, and the type of computer program is based on something like the St. Louis APC proposed computer center, what is the outlook in terms of getting a practical heat load?

AUTHOR STEPHENSON: I can only answer that by telling you what computer facilities we have used; and let me start by saying we have restricted ourselves to looking at one room of a large building. This is the smallest module you might reasonably start to study. If the problem gets bigger, the computer requirements will get bigger in proportion.

We have been able to handle the simple office module on the 1620 that was available to us a few years ago. You can break the calculation down into pieces, and do it a piece at a time. You evaluate the response factors in stages if you can't do it all at the same time. As Mr. Mitalas pointed out, you have to work out the radiation form factors, before you calculate response factors. This can be a completely separate program.

Finally, if you have the data for your building in the form of response factors, it requires only a very simple machine to put those together with the data on the environment, and finally come out with the cooling loads.

It is our thought that while this method is being evaluated, the consultant wouldn't necessarily calculate from scratch his own building response factor; these might be pre-calculated by a group working in connection with the Task Group on Energy Requirements or, in the case of Canada, we might be able to assist them in this pre-calculation of response factors, and then let the consultants use them for standard types of buildings in the multitude of situations where they might be interested in having an estimate of cooling load.

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