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FLUID FRICTION IN PARTIALLY FILLED CIRCULAR CONDUITS

by

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Fluid Friction in Partially Filled Circular Conduits

ANALYZED

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The pressure loss due to friction was measured for conduits ranging from ¾ in. to 1¼ in. nominal diameters containing up to three insulated copper wires. The air flow rates used gave Reynolds numbers ranging from 5,000 to 50,000. It was found that all the results could be correlated in terms of an equivalent diameter, i.e., the diameter of an empty conduit which would have the same pressure-loss—mass-flow characteristics as the conduit containing wire. The ratio of equivalent diameter to conduit diameter was found to depend primarily on the ratio of wire diameter to conduit diameter and the number of wires in the conduit, and only slightly on the Reynolds number.

NOMENCLATURE

The following nomenclature is used in this paper:

A = the mean cross-sectional area of a segment of the conduit, defined by equation (1).

B_N = a constant, defined by equation (4).

C = coefficient of discharge for the flow metering orifice.

D = diameter of the flow metering orifice.

$\bar{d}$ = root mean square value of inside diameter for 10-foot section of conduit.

d_e = diameter of an empty conduit with the same pressure-loss—mass-flow characteristics as partially filled conduit.

d_w = outside diameter of the insulated layer which surrounds the electrical conductor.

f = friction factor

$$= \left(\frac{\Delta P}{i}\right) \cdot \left(\frac{(\bar{d})^5}{L \cdot D^4}\right) \cdot \left(\frac{P}{P'}\right) \cdot \frac{1}{C^2}$$

Δh_w; Δh_m = the change in height of the liquid column in the conduit and manometer respectively.

i = the differential pressure across the flow metering orifice.

L = distance between piezometer rings.

N = number of wires in the conduit.

P = the mean absolute static pressure in the test section.

P' = the mean absolute static pressure at the flow metering orifice.

ΔP = pressure drop due to friction in a length L.

$Re = \frac{V \cdot \rho \cdot \bar{d}}{\mu}$ or $\frac{V \cdot \rho \cdot d_e}{\mu}$ for empty and partially filled conduit respectively.

T = absolute temperature of the air in the conduit.

ΔW = the change in weight of the beaker of water.

X; Y = the heights of the liquid columns in the vertical manometer and micromanometer respectively when they are measuring the same differential pressure.

ρ_w; ρ_m; ρ_b = the densities of the fluids in the conduit, the vertical manometer and the micromanometer respectively.

μ = viscosity of the air in the conduit.

INTRODUCTION

Data on the pressure losses associated with fluid flow through partially filled conduits were required for the design of a pressurized electrical conduit system. A survey of the engineering literature failed to produce these data so an apparatus was set up and a series of tests carried out.

This paper deals only with straight runs of conduit where the entrance effects are negligible.

THE APPARATUS

The arrangement of the apparatus is shown in Fig. 1. The conduit was supplied in 10-ft. lengths so it was convenient to measure the pressure drop along each 10-ft. section.

Four lengths of each size of conduit were joined to form a continuous 40-ft. run of straight pipe without any irregularity at the couplings. The piezometer rings were located about 4 in. upstream from the couplings; the exact distance from the coupling varied because the piezometer rings were 10 ft. ± 1/16 in. apart whereas the lengths of the sections of conduit varied by more than 1/16 in. The inside of each section of the conduit was reamed for 6 in. at the end where the piezometer ring was fitted and 2 in. at the other end. This ensured that the internal area of the flow passage was the same at each pressure tapping and also that the inside diameter of the two sections which were brought together were the same. As shown in Fig. 1 all differential pressure measurements were made with a 400-mm. water micromanometer and the gauge pressures were measured on a 100-in. manometer with carbon tetrachloride as the manometer fluid.

The orifice tank and orifice plates which were used to meter the air flow are essentially the same as those which were calibrated by Polson[1]; the orifice coefficients deter-

1. "The Flow of Air Through Circular Orifices in Thin Plates", by J. A. Polson and J. G. Lowther, University of Illinois, Engineering Experiment Station, Bulletin No. 240, 1932.

mined by Polson were used for this test.

Before the lengths of conduit were assembled to form the continuous 40-ft. runs, the internal cross-sectional area was determined for each 10-ft. length. The auxiliary apparatus used for these measurements is shown in Fig. 2. With this arrangement water could be drawn up into the conduit from the flask: the height to which the water rose was measured with the manometer and the weight of water in the conduit determined from the change in weight of the beaker. These measurements together with the density of the manometer fluid allowed the average cross-sectional area to be calculated.

Thus
$$\overline{d^2} = \frac{4}{\pi} \cdot \frac{X}{Y} \cdot \frac{\Delta W}{\Delta h_m \cdot \rho_b} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\bar{d} = \sqrt{\frac{4}{\pi} \cdot \frac{X}{Y} \cdot \frac{\Delta W}{\Delta h_m \cdot \rho_b}} \quad . \quad . \quad . \quad . \quad (3a)$$

This method gives a root mean square value of $\bar{d}$ for each segment of the conduit as well as a root mean square value for the whole length which was measured (the 8 inches which were reamed were not measured by this method).

In all cases the variation in diameter along any 10-ft. section was so small that there was no significant difference

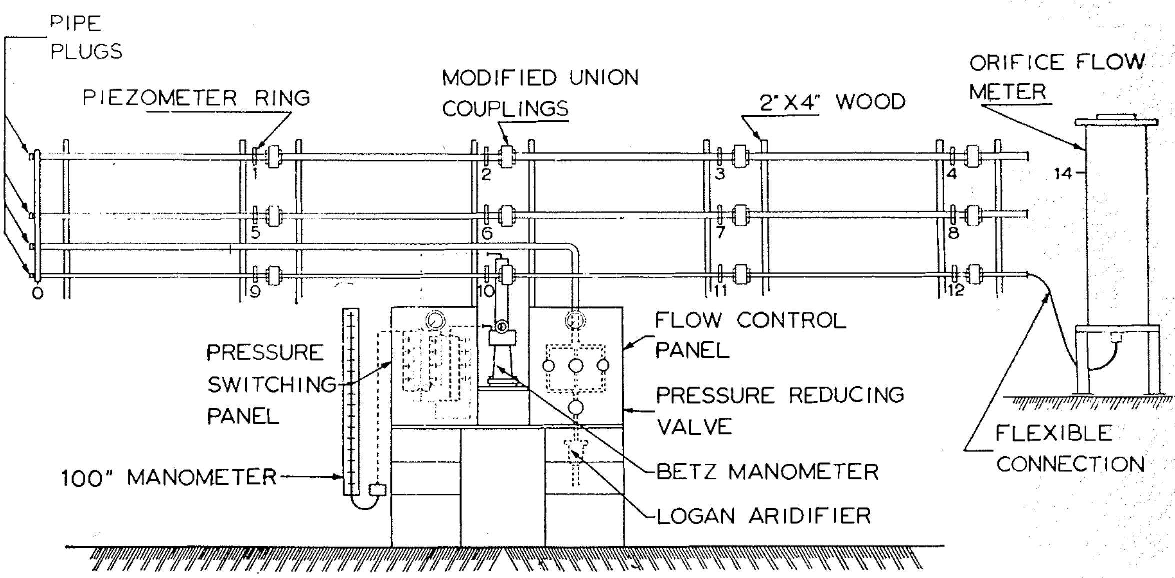

Fig. 1. Arrangement of the apparatus. Each of the numbered pressure tappings was connected to one of the points on the pressure switching panel.

TESTS

Measurement of Conduit Diameters

With a length of conduit suspended vertically as in Fig. 2 the pressure inside the conduit was lowered in steps of 8 in. of water. Each time the pressure was reduced the vertical position of the conduit was adjusted to keep the lower end 2 in. below the surface of the water and then the weight of the beaker was recorded along with the reading of the vertical manometer. Data were recorded only when water was rising in a dry conduit. The average cross-sectional area for each 8-in. segment of the pipe could be calculated from the change in weight of the beaker when the water filled that segment. The expression for the mean cross-sectional area is

$$A = \frac{\Delta W}{\Delta h_w \cdot \rho_w} = \pi \cdot (\overline{d^2})/4 \quad . \quad . \quad . \quad (1)$$

It is not necessary to know Δh_w and ρ_w to a high accuracy since the product $\Delta h_w \cdot \rho_w$ can be replaced by the equal product $\Delta h_m \cdot \rho_m$. The value of Δh_m could be measured directly, and ρ_m was checked before and after the measurements on each length of conduit by connecting the 100-in. vertical manometer in parallel with a micro-manometer and applying a pressure. Then

$$X \cdot \rho_m = Y \cdot \rho_b \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The micromanometer contained distilled water so that ρ_b could be accurately determined by measuring the temperature of the water in the micromanometer.

between the root mean square diameter $\sqrt[2]{(\overline{d^2})}$ and the root mean fifth $\sqrt[5]{(\overline{d^5})}$ so that root mean square values of $\bar{d}$ were used for calculating friction factors although the friction factor is dependent on the fifth power of the diameter.

In Appendix I the possible error in the value of $\bar{d}$ obtained by this method is shown to be 0.21 per cent.

Measurement of Fluid Friction

Initially data were obtained for the pressure loss due to friction for the empty conduits. Only the data for the sections between 20 and 30 ft. from the air supply header were used in the analysis. This ensured that entry effects were negligible.

In Appendix I, the possible error in f is shown to be approximately 2 per cent. and in Fig. 3 the deviation of the experimental results from the Moody[2] "smooth pipe" curve is less than this possible error. Thus for the range of Reynolds number covered by these tests the relative roughness of the conduit did not affect the pressure loss due to friction.

Finally the main series of tests consisted of measuring the pressure loss due to friction when the conduits were partially filled. Data were obtained for Reynolds numbers

2. "Friction Factors for Pipe Flow", by L. F. Moody, *Transactions* ASME, Vol. 66, 1944, p. 671.

ranging from approximately 5,000 to 50,000 for each of the twenty-seven combinations of the following variables:

Conduit size: ¾ in., 1 in., 1¼ in.

Wire size: 10-, 12-, 14-gauge solid copper with plastic insulation.

Arrangement: One, two, and three conductors in each conduit (not twisted).

Additional tests were made for some of these combinations with rubber insulated wire instead of plastic and with the wires twisted together. The surface of the plastic insulation was smooth enough to produce a highlight but the rubber insulation was covered with an impregnated fabric sheath which had surface irregularities which could be seen and felt without the aid of instruments.

The data obtained were used to calculate the equivalent diameter, i.e., the diameter of an empty conduit which would have the same pressure-loss—mass-flow characteristics as the conduit containing the wire. For this calculation it was assumed that the friction-factor—Reynolds-number relationship for the empty conduit was also applicable to the partially filled conduit when the Reynolds number for this latter case was based upon the equivalent diameter of the partially filled conduit. This required that the equivalent diameters be calculated by an iterative method. The values of the friction factor used for these calculations were obtained from a large-scale plot of f vs. Re which in turn was based on data obtained from the same length of conduit when empty and when the flow rate was measured with the same orifice plate. The accuracy of this calculation is discussed in Appendix I where it is shown that values of $d_e/\bar{d}$ have a possible error of 0.23 per cent. Figure 4 shows the dimensionless ratio $d_e/\bar{d}$ plotted against Reynolds number for the 1-in. conduit with the various wire fills. The data from the other sizes of conduit follow similar curves. These curves show that for the range of Reynolds numbers covered, the equivalent diameter is only slightly dependent on Reynolds number.

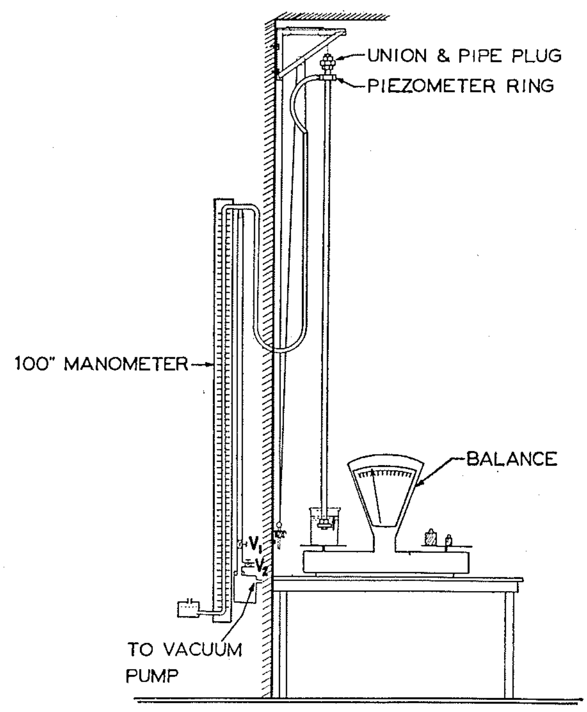

Fig. 2. Apparatus for measuring the inside diameter of conduit.

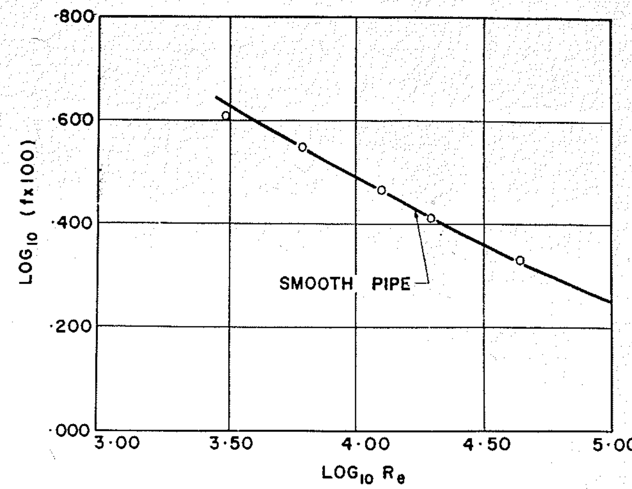

Fig. 3. Friction factor v. Reynolds number for the empty conduit compared with Moody 'smooth pipe' curve.

The values of $d_e/\bar{d}$ for a Reynolds number of 10,000 taken from Fig. 4 and the other similar graphs are shown in Fig. 5 plotted against the ratio $d_w/\bar{d}$. In this graph all of the data fall along the three straight lines

$$d_e/\bar{d} = 1 - B_N(d_w/\bar{d}) \quad \ldots \quad (4)$$

where

$$B_1 = 0.407$$
$$B_2 = 0.762$$
$$B_3 = 1.050$$

The data for the tests on rubber-insulated wires and for wires twisted three turns in each 10-ft. section fall on the same curves in Fig. 5 as all the other data. This satisfactory correlation of the results suggests that all of the significant variables have been taken into account.

CONCLUSION

To calculate the pressure losses due to friction in a partially filled conduit it is only necessary to determine an equivalent diameter using the appropriate B_N in equation (4), and then to calculate the pressure drop due to friction for this equivalent conduit in the usual way.

These results should be valid for all sizes of circular conduit containing circular cylinders as long as the ratio of fill diameter to conduit diameter does not exceed 0.3. The results are limited to smooth conduits where entry effects are negligible, and to Reynolds numbers in the range 5,000 to 50,000 but extrapolation to higher Reynolds numbers should not cause much error.

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APPENDIX I. DISCUSSION OF ERRORS

(1) Possible Error in the Mean Diameter of the Conduit

The expression used to calculate $\bar{d}$ was:

$$(\bar{d}) = \sqrt{\frac{4}{\pi}\cdot\frac{X}{Y}\cdot\frac{\Delta W}{\Delta h_m \cdot \rho^b}} \quad \ldots \quad (3a)$$

Table I. Typical Data from Pipe Diameter Measurement

Quantity	Value	Possible Error (%)
ΔW	1563 ± 2 gm.	0.13
Δh_m	$70.00 \pm .02$ in.	0.03
ρ_b	$0.9966 \pm .00005$ gm./cm.^{3}	0.005
X	$9.00 \pm .02$ in.	0.222
Y	$36.28 \pm .01$ cm.	0.03
$(\bar{d})^2$		0.417
$\bar{d}$	8.728×10^{-2} ft.	0.21

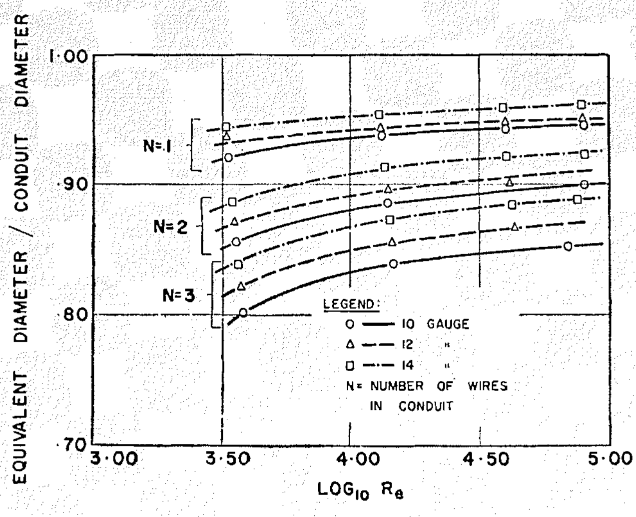

Fig. 4. Ratio of equivalent diameter to conduit diameter as function of Reynolds number for various fill.

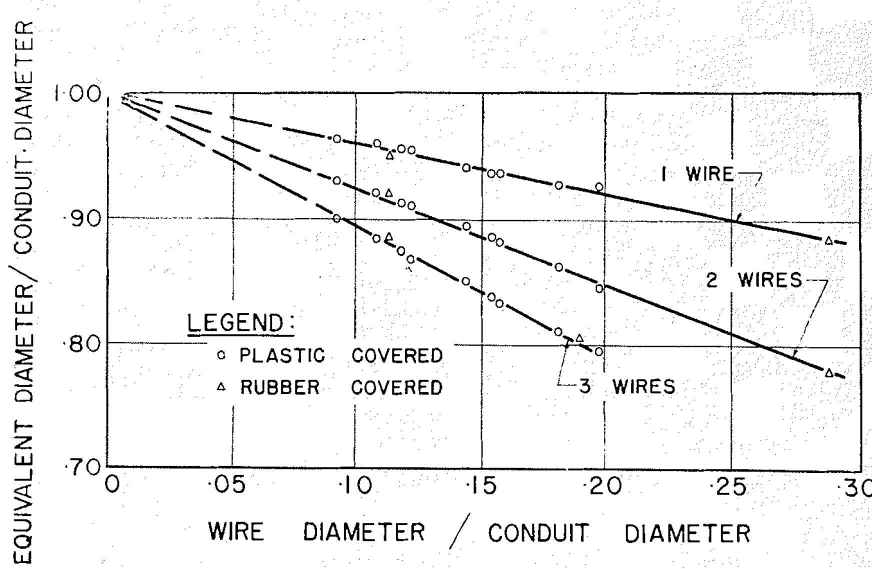

Fig. 5. Ratio of equivalent diameter to conduit diameter plotted against the ratio of fill diameter to conduit diameter for one, two, and three wire fills.

Table II. Typical Data For Determining Pipe Friction Factor

Quantity	Value	Possible Error (%)
ΔP	10.32 ± 0.05 mm. H_2O	0.50
i	160.00 ± 0.05 mm. H_2O	0.03
$(\bar{d})^5$	$(8.728 \times 10^{-2})^5$ ft.^{5}	1.05
L	10.00 ± 0.005 ft.	0.05
D^4	$(4.185 \pm 0.001)^4 \times 10^{-8}$ ft.^{4}	0.096
P	$758.2 + 2.99(4.60 \pm 0.04)$ mm. Hg	0.016
P'	$758.2 + \frac{1}{27.2}(160.0 \pm 0.05)$ mm. Hg	0.000
$*C^2$	$(0.6066 \pm ?)^2$	0.40
f	2.92×10^{-2}	2.14

* It is difficult to determine the possible error in this value since it depends on how closely the orifice used for these tests duplicates the one calibrated by Polson. A possible error of 0.2 per cent in C is thought to be a reasonable estimate since care was taken to have the orifices the same.

Table I contains representative data for measurements on a 10-ft. length of 1-in. conduit.

(2) **Possible Error in the Calculated Friction Factors**

The expression used to calculate f was:

$$f = \frac{(\Delta P)}{(i)} \cdot \frac{((\bar{d})^5)}{(L \cdot D^4)} \cdot (P/P') \cdot \frac{1}{C^2} \quad . \quad . \quad . \quad (5)$$

Table II contains representative values for a 10-ft. section of 1-in. conduit.

(3) **Possible Error in Calculated Reynolds Numbers**

The expression used to calculate Re was:

$$Re = 0.025757 \; \frac{CD^2}{\bar{d}\mu} \left(\frac{iP'}{T}\right)^{1/2} \quad . \quad . \quad . \quad (6)$$

where the quantities have the units given in Table III. Table III contains representative values for a 10-ft. section of 1-in. conduit.

(4) **Possible Error in $d_e/\bar{d}$**

The expression used to calculate d_e was:

$$(d_e)^5 = f \cdot C^2 \cdot \frac{i}{\Delta P} \cdot L \cdot D^4 \cdot P'/P \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The value of f used in this expression was taken from a plot of f vs. Re based on the data for the same section of conduit when empty, and with the same orifice plate. Thus the errors in C, L, and D, which gave rise to an error in f, will occur again and will now counteract the error in f; f is proportional to $(Re)^{-0.26}$ hence a one per cent error in Re will cause a 0.26 per cent error in f. Although the total possible error in Re is 0.62 per cent, a part of the error (0.51 per cent) is not associated with the measurements and hence will be repeated for every determination of Re. Thus the measurement errors which cause an error in d are 0.11 per cent in Re or 0.03 per cent in f.

The error in $(d_e)^5$ will be twice the sum of the errors in $i/\Delta P$, P'/P and the error in f due to part of the error in Re.

Therefore the possible error in $d_e/\bar{d}$ is 0.23 per cent.

Table III. Typical Data For Determining Reynolds Number for Flow in a Pipe

Quantity	Value	Possible Error (%)
i	160.00 ± 0.05 mm. H_2O	0.03
P'	758.2 ± 0.1 $+ \frac{1}{27.2}$ (160.0 ± 0.05) mm. Hg	0.01
T	294.2 ± 0.2° K	0.07
$(i.P'/T)^{\frac{1}{2}}$		0.06
C	0.6066 ± ?	0.20
D^2	$(4.185 \pm 0.001)^2 \times 10^{-4}$ ft.^{2}	0.05
$\bar{d}$	8.728×10^{-2} ft.	0.21
μ	see note below; slug/ft. sec.	0.10
Re	12.51×10^3	0.62

Notes to Table III

$$\mu/\mu_0 = \frac{(T_0 + K)}{(T + K)} \left(\frac{T}{T_0}\right)^{3/2} \qquad (7)$$

where K = Sutherland's Constant = 120 C°.

at T_0 = 296.16° K $\mu_0 = (1.8325 \pm 0.0010) \times 10^{-4}$ poise

$$\frac{d\mu}{\mu} = \frac{d\mu_0}{\mu_0} - \frac{dT}{T + K} + 3/2 \frac{dT}{T} \qquad (8)$$

$$\frac{d\mu_0}{\mu_0} = \frac{10}{18325} \times 100 = 0.05\%$$

$$-\frac{dT}{T + K} = \frac{0.2}{414} \times 100 = -0.05\%$$

$$3/2 \frac{dT}{T} = 3/2 \times \frac{0.2}{294} = 0.10\%$$

Therefore $\frac{d\mu}{\mu}$ = 0.10 per cent.

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