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DYNAMIC CHARACTERISTICS OF A MULTI-STOREY CONCRETE BUILDING

BY

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WITH DISCUSSION

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Dynamic characteristics of a multi-storey concrete building

H. S. WARD, BSc, PhD, MICE*

The first four lateral modes and frequencies of vibration of a 15-storey reinforced concrete building were obtained from an analysis of wind-induced vibration measurements. The building consists of flat slabs supported on columns and shear walls. Theoretical values of the vibrational modes are calculated using different structural models and considering upper and lower bounds of floor weights and material properties. For some select models, the influence of foundation compliance in the rocking and sliding mode is also investigated theoretically. Results indicate that changes in modal frequencies may be substantial for practically realizable variations in material properties. Measured frequencies of the building fall within the calculated frequency bounds for one particular mathematical model. It is also demonstrated that the frequencies of the lower modes of vibration may be decreased significantly by foundation rocking.

Introduction

Mechanical and aeronautical engineers traditionally have been concerned with evaluating the dynamic characteristics of structures in order to determine the effect of dynamic loads on such components as turbine blades and aircraft wings. The civil engineer has not usually been concerned with such considerations, because experience has shown that the characteristics of most of the dynamic loads with which he has to contend almost preclude the occurrence of resonance, and the dynamic loads can be safely replaced by equivalent static loads.

2. There are exceptions to this rule, however, and it is well known that the interaction of wind loading and the flutter characteristics of a suspension bridge is a major consideration in the design of such a structure. Another example is provided by the response of structures subjected to earthquake motions. Studies have shown that the modes and frequencies of vibration of a structure, together with its damping, have an important influence on the induced loads.[1–3]

3. Earthquakes are a sufficiently common occurrence in many parts of the world to justify a better understanding of the dynamic characteristics of tall buildings, since any damage to this type of structure represents a potential cause of considerable loss, both in terms of human life and financial investment. Some building codes already specify earthquake loads in terms of the natural frequencies of vibration of the structure,[4] and this approach will become more common as more knowledge is obtained about these frequencies. Such a process will lead ultimately to a better definition of earthquake loads,

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which in turn will lead to a more economical solution to the design of earthquake-resistant structures than is presently available.

4. One of the prerequisites for this is the development of reliable methods for predicting the modes and frequencies of vibration of the complex system represented by a multi-storey building. The only way that confidence can be established in such theoretical methods is to compare their predictions with measurements obtained from existing buildings. There is already a considerable amount of information available on this subject.[5-10]

5. The Author has been associated with such a programme of research at the Division of Building Research of the National Research Council of Canada. Two previous papers[11,12] contain the results for the frequencies and modes of vibration of four buildings, obtained from the measurements of wind-induced vibrations. Theoretical values for the frequencies of vibration of these buildings were obtained by assuming that they were shear-type frame structures and that all floors were of equal weight. In some cases the agreement with the measured values was reasonable, but the theoretical model was too restrictive to provide many useful conclusions.

6. The object of this Paper is to consider a number of different simple theoretical models for a 15-storey building and to compare the predicted modes and frequencies of vibration with the measured values. The factors investigated are the distribution of stiffness and mass in the structure and the influence provided by some assumed restraints of the foundation soil.

7. The first three lateral modes and frequencies of vibration about each of the two main axes of the building were obtained from the analysis of wind-induced vibrations. Theoretical values of the first four lateral modes of vibration of each of the models were calculated, and the results show that some of these models give a reasonable estimate of the dynamic characteristics of the type of building considered in this study.

8. The results also show that factors on which there is little available information can influence the vibrations of tall buildings. A great deal of research will be required in order to provide the necessary information on these factors. It is shown that one feature that does have an appreciable effect on the dynamic characteristics of a building is the restraint provided by the foundation.

The building

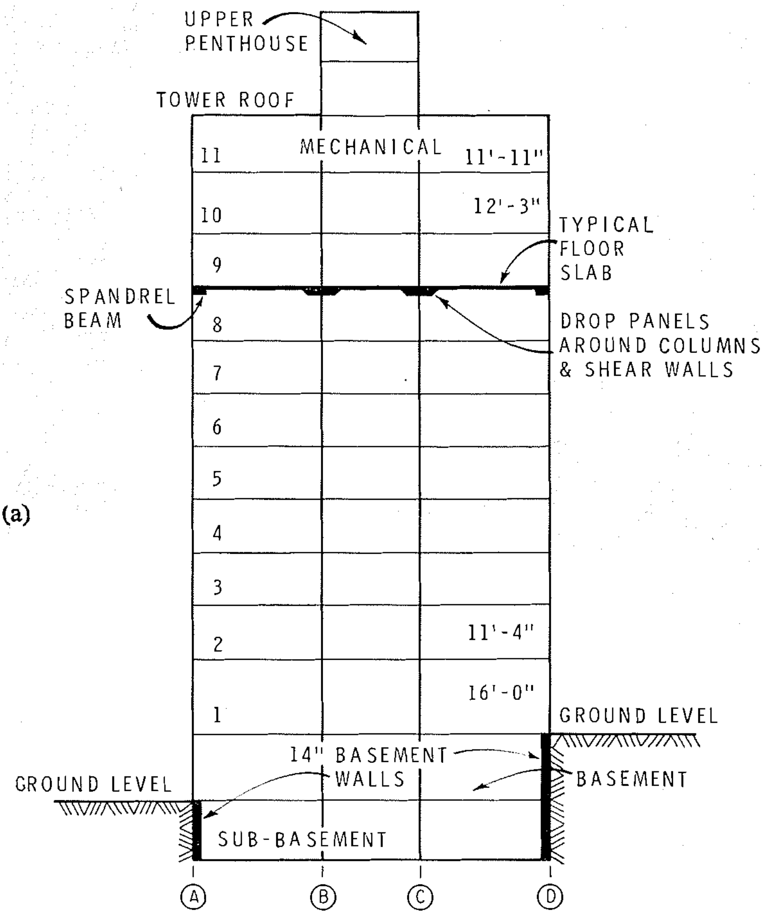
9. The building in which the measurements were taken is the Administration Building of the Canadian Department of Agriculture, located in the City of Ottawa. The principal features of the building are as follows.

10. The dimensions of the building, together with a schematic representation of the main structural details, are shown in Figs 1 (a) and (b). The building is oriented so that its longest axis runs approximately east–west; in the north–south direction there are three bays, the central one being 19 ft 6 in. wide and the other two 27 ft $4\frac{1}{2}$ in. wide. The main tower of the building is made up of 13 storeys, together with a 2-storey penthouse that occupies the central bay in the north–south direction and lies between lines 3 and 13 in the east–west direction.

Fig. 1 (a) (top right) Side elevation of the building; (b) (right) plan view of the building

CHARACTERISTICS OF A MULTI-STOREY CONCRETE BUILDING

(a)



(b)

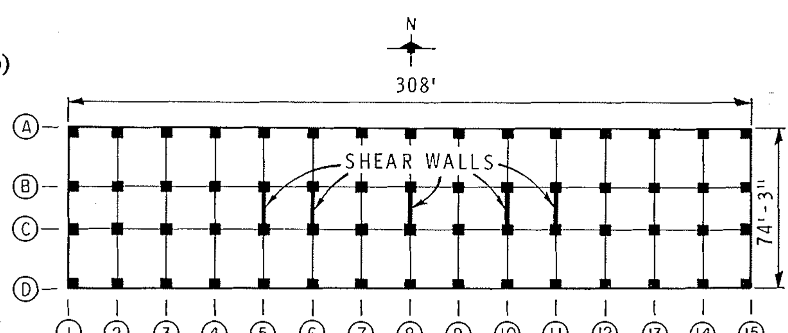


Fig. 2. View of south face of building at the time vibration measurements were taken

11. The main tower is connected to two adjacent wings, which run north–south, on its east and west sides. These wings extend from the sub-basement to the second floor level. The connexions between the tower and the two wings consist of a 7 ft 4 in. projection of the tower floors at the west end of the building, and a projection of 21 ft 2 in. at the east end. At the junction of each of these projections with the adjoining wing there is a $1\frac{1}{2}$ in. wide expansion joint, made up of asphalt-impregnated fibreboard. In the calculations that follow, it is assumed that the expansion joints are sufficient to isolate the tower structurally from the two wings.

12. Along the south side of the building the ground surface is at first floor level and a 14 in. thick reinforced concrete wall extends two floors down to the sub-basement. On the north side the ground surface is at basement floor level, and there is only a one-storey basement wall. There are no basement walls along the east and west faces of the tower, because there is direct access to the adjacent wings.

13. Soil studies made by the Department of Public Works indicated that the overburden consists of 0–22 ft of stiff to highly plastic clay overlying 14–40 ft of firm glacial till. The bedrock is limestone with interbedded thin shale layers. The foundations of the columns and shear walls of the tower are carried by groups of Franki piles with 6–10 piles in each group. Each pile has a working load capacity of 100 tons and is at least 8 ft long. Where the sub-basement slab rests directly on the soil, the specifications required that the soil should be capable of sustaining 1000 lb/sq. ft.

14. The structural skeleton consists of a combined system of a frame and shear walls, both constructed in reinforced concrete. The exterior frames are made up of columns and spandrel beams, but the interior frames consist of flat slab construction (the slab is generally 8½ in. thick with drop panels 7 ft 6 in. by 8 ft 6 in. by 14 in. around the columns). As shown in Fig. 1(b), the shear walls, occupying the central core of the tower, run from the sub-basement to the floor of the upper penthouse. Two extra shear walls are located in the central bay along lines 2 and 13, but these occupy only the area from the sub-basement to the first floor.

15. The shear walls provide a symmetrical distribution of stiffness in the structure; there are minor deviations from symmetry in the frames, but in the theoretical analysis it has been assumed that there is perfect symmetry of stiffness about the two geometrical axes of symmetry. Based on the cross-sectional area of the concrete sections, the ratio of second moment of area of shear wall to that of an individual column is of the order of 1500/1 about the long axis of the building, and 50/1 about the short axis. The stiffnesses of the spandrel beams and exterior columns are approximately equal.

16. On the few occasions when a detailed correlation has been obtained between the measured and predicted stiffness of a multi-storey building,[13] it has been found that the building is much stiffer than was predicted. Internal partition and load-bearing walls can provide considerable stiffness in a building and, in order to eliminate these factors from the analysis, the measurements were taken before any work had been started on this type of construction. The exterior curtain wall had been completed, however, except for one bay on the south wall (Fig. 2).

Vibration measurements

17. The methods of measuring the modes and frequencies of vibration have been described in some detail in previous papers,[11,12] but for the sake of completeness the basic details are briefly described in the following paragraphs.

18. Six Willmore Mark II seismometers were used to record the wind-induced vibrations of the building. These are sensitive electromagnetic transducers with a fixed coil and a heavy magnet that acts as the moving mass. The natural periods of the transducers were set at 2 s, and electrical damping was provided equivalent to 0·65 of the critical value. Thus, the transducer response could be expected to be flat over the frequency range of the building's vibrations. The electrical outputs from the seismometers were fed into a mobile laboratory where the signals were passed through d.c. amplifiers, and the amplified signals were recorded on a seven-channel f.m. tape recorder.

19. The measurements were taken in the following manner. The transducers were placed on floors 2, 3, 4, 5, 6 and 7, in the centre of the building to reduce the signal that might arise from torsional vibrations of the building. Records were taken for ¾ h for the lateral vibrations about one of the main axes of the building; the transducers were then rotated through 90° and records taken for another ¾ h for vibrations about the other main axis. The transducer on floor 7 was then left in place, and the other transducers moved up to floors 8, 9, 10, 11 and the roof of the tower. Records were again taken as for the lower floors.

20. The frequencies and modes of vibration of the building were obtained from an analysis of the data stored on the magnetic tape. The tape was made into continuous loops and a Honeywell–Brown analyser was used to make a Fourier analysis of the records. Peaks in the Fourier analysis indicated the natural frequencies and the amplitude of the vibration. In order to draw the mode shapes, the phase relations between the vibrations of the different floors were determined by comparing the results of first adding and then subtracting the vibrations recorded on two different floors.

21. By repeating this process for all the records obtained on the different floors it was possible to determine the first three lateral modes of vibration of the building about each of the main axes. The results of this process are shown in Figs 3 (a) and (b), where the mode shapes are drawn with respect to arbitrary scales. The absolute value of the displacements will obviously depend on the applied force, but the relative relation between amplitudes in any of the modes will remain the same as that shown in Figs 3 (a) and (b). Although it was not possible to define the fourth mode shape, some of the records contained frequency peaks that could have represented this mode. The measured values of frequencies for the first four modes are given in Tables 1 and 2.

22. The maximum displacements that were measured during this work were the displacements of the tower roof in the fundamental mode for lateral motion about the two axes. These displacements were of the order of 0·005 in. for a wind velocity of approximately 15 mile/h. For motion perpendicular to the long axis the ratios of the maximum measured amplitudes in the first, second and third mode were respectively 1:0·4:0·1. The corresponding ratios for the other axis were 1:0·5:0·25. It can be seen therefore that the wind predominantly excited the fundamental modes of the building.

Calculation of modes and frequencies of vibration

23. In a multi-storey building most of the weight of the structure is concentrated at each floor level, and the free vibrations of the structure can be obtained by considering the behaviour of a concentrated mass system. Hence, for lateral, torsional or vertical modes of vibrations there are as many degrees of freedom as there are floors. Fig. 4 shows a diagrammatic representation of a concentrated mass system and the parameters required to determine the free vibrations of such a system. When the lateral vibrations of a tall building are being considered the spring connexions of interest provide resistance to horizontal motion.

24. The equations of motion of any structure can be expressed in terms of its flexibility or stiffness matrix. A computer program based on the Stodola method,[14] was used to calculate the modes and frequencies of vibration. With this method the lowest modes of vibration are obtained initially if the flexibility matrix is used, whereas the highest modes are obtained with the stiffness matrix. As it was the lowest modes of vibration that were recorded (and in general these modes are the most important) it was necessary to calculate the flexibility matrices of the theoretical models.

25. The structural analysis involved the determination of the deflexions at all floor levels when a unit horizontal force was acting on one of them; in this way it was possible to determine one column of the flexibility matrix. The process is demonstrated in Fig. 4(b). In some cases it was easier to calculate

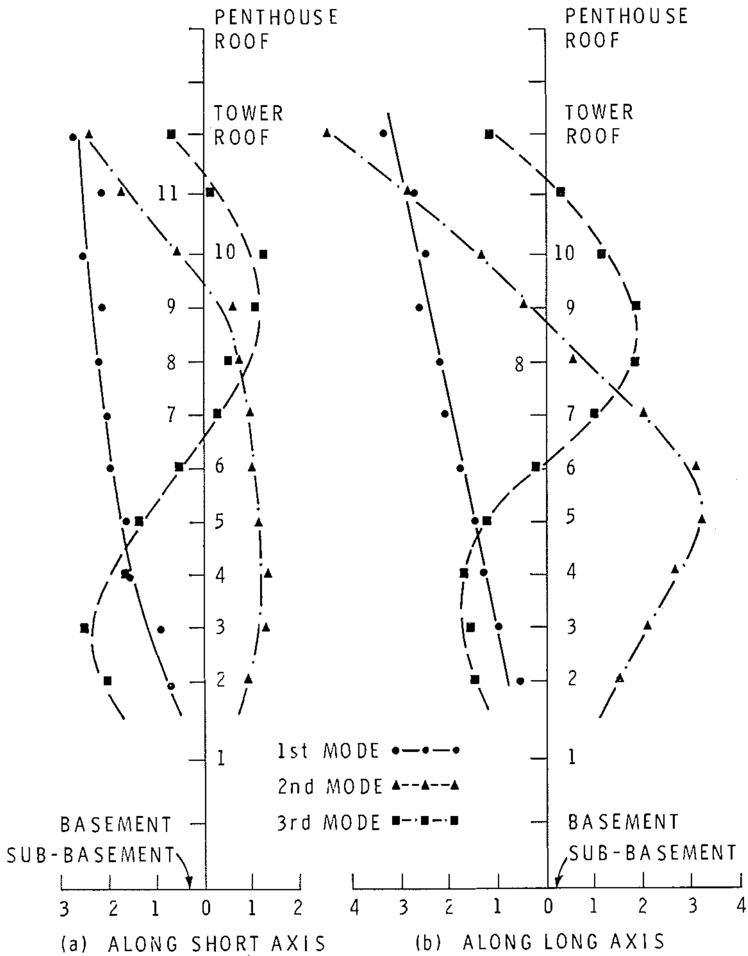


Fig. 3. Measured modal deflexions

the stiffness matrix, and the flexibility matrix was obtained by matrix inversion. From the limited amount of information available[15,16] it appears that damping in modern buildings is small, and this was assumed to be the case in the subsequent theoretical analyses.

Theoretical models and methods of analysis

26. The lateral stiffness of the building under consideration is provided by two distinct structural systems: the shear walls and the framed part of the building. In the different models that were investigated it was assumed that these two systems either acted independently, or were interconnected in such

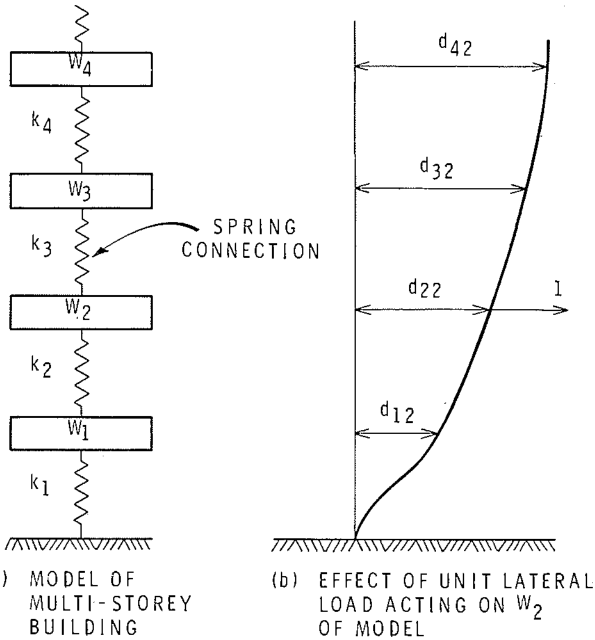


Fig. 4. Concentrated mass representation of a multi-storey building

a manner that for any given floor level the displacements of the two systems were equal. In effect this latter condition assumes that the in-plane stiffness of the floor system is high.

27. In the majority of the calculations it was assumed that the structure was founded on a rigid base so that there was no lateral motion at the sub-basement level. The effect on one of the models when this restraint is removed was also investigated, and will be described later. In most cases it was assumed that the weights of all the floors up to the roof of the tower were equal, and the weights of the penthouse floors were 18·7% of this weight (the weight was taken to be proportional to the floor plan area). In the structural analysis the stiffness of any concrete member was based simply on its cross-sectional area.

Shear wall models

28. Since the shear walls are apparently much stiffer than the columns of the frame for lateral motion along the short axis, the first model that suggested itself was a consideration of the shear walls acting on their own. In this case it was assumed that the shear walls could be represented by a number of free-standing cantilevers connected by rigid diaphragms at each floor level. Because the second moments of area of the individual cantilevers were similar, it was possible to add together the individual moments of area between each

floor and analyse an equivalent single cantilever, which had a varying second moment of area along its height. The flexibility matrix of the cantilever was obtained by means of the moment-area method.

29. For vibrations along the long axis a model consisting of shear walls acting as cantilevers did not seem reasonable and therefore was not considered.

Shear frame models

30. Results from previous work[7-11] have shown that, for framed structures up to 20 storeys, good agreement can be obtained between theoretical and measured values of vibration characteristics when a shear frame model is assumed. In this model the flexural rigidity of the beams or slab system is assumed to be much greater than that of the columns, so that there can be no rotation of the joints in the frame. The agreement of computed results appears to be good, even when the stiffnesses are of the same order of magnitude. The structural analysis is very simple in this instance because the model can be represented by a series of masses with spring connexions only between adjacent floors (Fig. 4(a)). It must be remembered, however, that the springs provide resistance to lateral motion rather than the vertical motion implied in Fig. 4(a).

31. The masses in the model are equal to the masses of each floor, and the spring value, k, between any two adjacent floors is given by

$$k = \frac{12E}{L^3} \sum I_{col} \qquad (1)$$

where E is Young's modulus
L is the distance between the two floors, and
$\sum I_{col}$ is the summation of the values of the second moment of area of the columns between the two floors.

32. Although this method of analysis provides the elements of the stiffness matrix, there is no need in this case to resort to the process of matrix inversion in order to calculate the flexibility matrix. When the values of k_i for the building have been obtained from equation (1), it is possible to calculate directly the flexibility matrix. This is achieved by considering the representation in Fig. 4(a), and using the fact that vertical loads and displacements represent the lateral loads and displacements. Thus suppose the displacements d_{12}, d_{22}, d_{32} and d_{42} in Fig. 4(b) are required. A unit force acting upwards on W_2 would displace W_1, W_2, W_3 and W_4, respectively, by the amounts $1/k_1$, $(1/k_1+1/k_2)$, $(1/k_1+1/k_2)$ and $(1/k_1+1/k_2)$, and these represent a column of the flexibility matrix. If this process is repeated for all the floors it is possible to calculate the full flexibility matrix of the system.

33. The independent shear frame models investigated in this study were obtained by ignoring all parts of the shear walls, except those portions on the frame centrelines with the same cross-sectional areas as the columns in the framed part of the building. Both fixed and pinned conditions were considered for the degree of fixation of the columns at sub-basement level.

Combined models

34. The most realistic model to represent the structure consists of a system that incorporates the combined action of both the shear walls and the framing. In the analysis of this type of model it was assumed that the frames and shear

walls were linked at each floor by a rigid diaphragm, so that all components were subjected to the same displacement at any floor level. It was also assumed that only horizontal reactions between the frame and the shear wall system were mobilized. In other words, the effects of vertical shear forces and bending moments in the beams connecting the frames to the shear walls were ignored. The effects of shear deformations and rotary inertia were also omitted.

35. The combined models considered here can be represented by a number of parallel systems each consisting of interconnected masses and springs. Within the assumptions described, it is possible to calculate the stiffness matrix of the combined system merely by adding together the stiffness matrices of the separate systems. The method of analysis thus consisted of evaluating the combined stiffness matrix, then inverting this matrix to obtain the flexibility matrix of the combined system.

36. The first type of combined model to be considered was the combination of shear walls and shear frames. This particular model was investigated for lateral motion about the two main axes of the building. Finally, for lateral motion perpendicular to the long axis, an attempt was made to formulate a more realistic model by assuming that the shear walls were combined with frames in which joint rotation occurred. In this final model it was assumed that the second moment of area of each horizontal member in the frame was 50 000 $in.^4$; this was of the same order as the values for the majority of columns in the building.

37. To derive the combined stiffness matrix of this final model it was necessary to calculate the stiffness matrices of the multi-storey frames along the lines 1, 2, 3, 4 and 7 in Fig. 1(b). Because of symmetry, only half the structure had to be analysed. The analysis for the different frames was performed by the moment distribution method. In all the combined models it was assumed that the frames had fixed-ended supports at the sub-basement level.

Results for rigid base models

38. In the structural analysis of the building, all flexibility coefficients were expressed in terms of the constant (H^3/EI) so that it was possible to calculate the natural frequencies of vibration, f, of the building from an equation of the form

$$f = \frac{(n)^{1/2}}{2\pi} \sqrt{\left(\frac{EIg}{WH^3}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here n is a computer-derived number related to the flexibility coefficients
W is the weight of a typical floor in the tower section
H is the height of the building
E is Young's modulus of concrete
g is the acceleration due to gravity
and I is a value for the second moment of area of a characteristic section in the theoretical model.

39. Values for I and H were known with a fair degree of accuracy, but it was more difficult to be precise about E and W. Consequently, two values for the frequencies of vibration of the models were calculated by using upper

and lower estimates of the ratio E/W. The upper and lower values used for E were 4×10^6 and 2×10^6 lb/sq. in. and for W were $2{\cdot}8\times10^6$ and $2{\cdot}5\times10^6$ lb. These yield 1·6 and 0·714/sq. in. for the two values of E/W. The value of g was taken to be 32 ft/s^2 and H was 178 ft. The natural frequencies of the various models calculated with these values are shown in Tables 1 and 2. Two values are given for each modal frequency. The first value refers to the case when $E/W=1{\cdot}6$/sq. in. and the second value to the case when $E/W=0{\cdot}714$/sq. in. The computed mode shapes are not affected by changes in the values of E/W. The last set of figures in Table 1 (Model 3*c*) was obtained by assuming the 11th floor was 30% heavier than the other typical floors since it was designed for and carried the mechanical equipment in the building.

40. It is not practical to present all the calculated mode shapes, but in Fig. 5 a few of them are plotted for models that appear to give the best overall agreement with the measured frequencies. For motion along the short axis, the model chosen is the shear frame with pinned connexions at the foundation; for the other axis the model is the shear frame model with fixed-ended basement columns. The respective mode shapes are shown in Figs 5 (a) and (b).

41. The mode shapes are again plotted in terms of arbitrary scales, since the relative values of the amplitudes remain constant. A comparison of Figs 5 and 3 shows that the theoretical mode shapes are similar to the measured ones. Measurements were not made on the penthouse, so the amplitudes in this section of the building cannot be compared.

Table 1. Modal frequencies for motion along short axis

Modes of lateral vibration	1st	2nd	3rd	4th
Measured values of frequency, *Hz*	0·89	3·13	4·90	6·94*
1. Shear wall model (cantilever)	0·51† 0·76	2·96 4·42	6·70 10·02	9·56 14·30
2. Shear frame model				
(*a*) Fixed ended sub-basement columns	1·02 1·53	2·81 4·20	3·99 5·97	5·38 8·05
(*b*) Pin ended sub-basement columns	0·90 1·34	2·64 3·94	3·86 5·78	4·94 7·39
3. Combined model				
(*a*) Shear wall and frame	1·16 1·73	3·82 5·72	8·00 12·00	10·91 16·33
(*b*) Shear wall and frame with joint rotation	0·75 1·12	3·27 4·89	7·51 11·26	10·34 15·47
(*c*) Shear wall and frame with joint rotation plus effect of mechanical floor	0·73 1·09	3·25 4·87	7·50 11·22	10·30 15·47

* There is some uncertainty in the identification of this mode.
† The two values for the frequencies are for $E/W=1{\cdot}6$ and 0·714, respectively.

Table 2. Modal frequencies for motion along long axis

Modes of lateral vibration	1st	2nd	3rd	4th
Measured values of frequency, *Hz*. . . .	0·93	3·04	5·38	7·25*
1. Shear frame model				
(*a*) Fixed ended sub-basement columns.	0·73†	2·18	3·46	4·36
	1·09	3·26	5·19	6·52
(*b*) Pin ended sub-basement columns .	0·63	2·02	3·23	4·02
	0·94	3·02	4·84	6·01
2. Combined model				
(*a*) Shear wall (without basement walls) and shear frame	0·73	2·17	3·77	5·30
	1·09	3·25	5·64	7·92

* There is some uncertainty in the identification of this mode.
† The two values for the frequencies are for $E/W = 1{\cdot}6$ and 0·714, respectively.

Effect of the restraint of the foundation

42. All the results described so far involved the assumption that the building rests on a rigid base. Previous experience of the Author with measurements on other buildings has shown that it is difficult to interpret the measurements of the vibrations of the foundation slab and so directly test the degree to which this assumption is valid. Although these previous attempts were not investigated in any detail, one of the major difficulties was the presence in the records of comparatively high levels of vibration at frequencies of the order of 15 Hz. These vibrations, which are probably caused by traffic movement, effectively mask the vibrations of interest.

43. The two simple forms of restraint that were investigated are shown in diagrammatic form in Figs 6 (a) and (b). Fig. 6(a) represents a system in which the weight of the foundation slab, W_s, is assumed to move laterally against a spring with a total stiffness k_T. In Fig. 6(b) lateral motion of the slab is prevented, but the foundation can rotate about a horizontal axis through the centre of the slab. In this latter case the factors that enter into the equations of motion are the polar moment of inertia of the slabs, J_s, about this axis, and the rotational stiffness, k_R, of the spring.

44. The model could be made more complicated by considering a combination of the two models, such that translation and rocking can occur simultaneously. But in view of the lack of any experimental evidence, it was considered that there was no justification for making this refinement in the analysis at this time.

45. Both the foundation restraint models introduce an extra degree of freedom compared with the rigid base models. In Fig. 6(a) the extra co-ordinate is X_s, the lateral motion of the slab, and in Fig. 6(b) it is θ, the rotation of the slab. The flexibility coefficients of both systems can be easily calculated from a knowledge of the flexibility matrix of the building founded on a rigid base, and the values of k_T or k_R. In this instance the rigid base model and the foundation restraint are two systems in series, in which case the

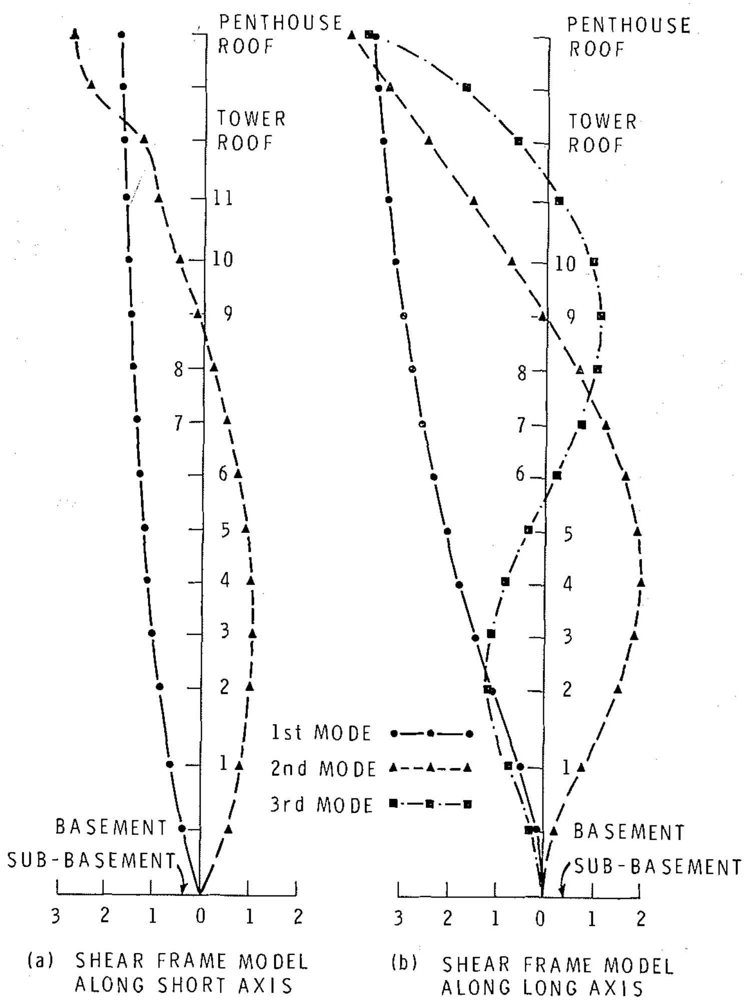


Fig. 5. Computed modal deflexions for rigid base model

flexibility matrix of the combined system is obtained by adding the flexibility matrices of the separate systems.

46. Thus suppose d_{42} represents the deflexion of the fourth floor of the rigid base model due to a unit load acting at the second floor. When the restraint in Fig. 6(a) is introduced, the new coefficient in the flexibility matrix becomes t_{42}, where

$$t_{42} = d_{42} + \frac{1}{k_{\mathrm{T}}} \quad \quad (3)$$

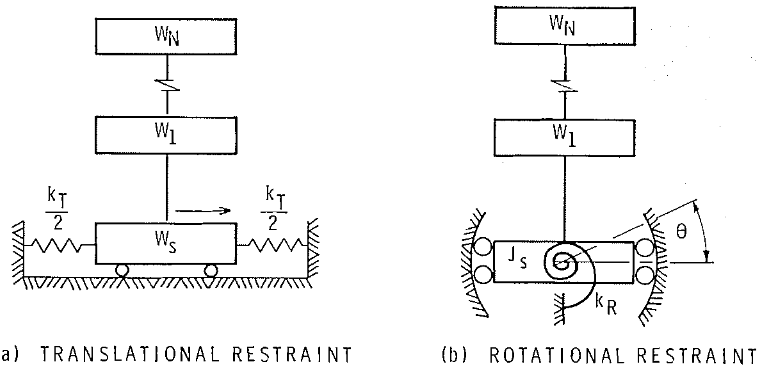


Fig. 6. Foundation restraints

Similarly the restraint of Fig. 6(b) leads to a new coefficient, r_{42}, where

$$r_{42} = d_{42} + \frac{h_2 h_4}{k_R} \qquad (4)$$

In equation (4), h_i is the distance from the foundation to the ith floor, and k_R has units of torque/unit radian.

47. The equation for lateral motion of the foundation slab is in the same form as the equations of motion for the floors of the building. This is not the case for the rocking motion of the slab because the equations are expressed in terms of polar moments of inertia and rotations, rather than weights and deflexions. When the latter equation of motion is multiplied by the height of the building, H, however, it can be re-adjusted to have the same form as the equations for lateral motion.

48. As a consequence there was no need to make any significant alterations to the computer program in order to investigate the foundation restraints shown in Fig. 6. For the translational restraint the extra data required were the weight W_s and the stiffness k_T. These values were expressed in the following forms:

$$W_s = aW$$

$$k_T = b\frac{EI}{H^3} \qquad (5)$$

where a and b are the numbers read in as data by the computer, and the other symbols are as previously defined. For the rotational restraint the extra data were the polar moment of inertia, J_s, and the stiffness, k_R. These values were expressed in the form

$$J_s = pH^2W$$

$$k_R = q\frac{EI}{H} \qquad (6)$$

The data to the computer consisted of the values for p and q.

Table 3. Effect of foundation translation on modal frequencies. (Model 3*b*—shear wall and frame with joint rotation)

Foundation parameters	Mode 1	Mode 2	Mode 3	Mode 4
	(Modal frequency, *Hz*)			
1. $W_s = W$				
$k_T = 1{\cdot}29 \times 10^9$ lb/in.	0·75	3·26	7·49	10·27
$= 8{\cdot}6 \times 10^6$ lb/in.	0·68	2·05	4·61	8·21
$= 8{\cdot}6 \times 10^4$ lb/in.	0·13	1·20	4·25	8·14
2. $W_s = 2W$				
$k_T = 1{\cdot}29 \times 10^9$ lb/in.	0·75	3·26	7·49	10·27
$= 8{\cdot}6 \times 10^6$ lb/in.	0·67	1·95	4·32	8·04
$= 8{\cdot}6 \times 10^4$ lb/in.	0·12	1·15	4·06	8·00
Same model on rigid base	0·75	3·27	7·52	10·34

Table 4. Effect of foundation rotation on modal frequencies for motion perpendicular to long axis. (Model 3*b*—shear wall and frame with joint rotation)

Foundation parameters	Mode 1	Mode 2	Mode 3	Mode 4
	(Modal frequency, *Hz*)			
1. $J_s = 0{\cdot}016\ WH^2$ (Inertia of foundation only)				
$k_R = 2{\cdot}92 \times 10^{12}$ lb ft/rad	0·71	3·21	7·42	10·20
$= 2{\cdot}92 \times 10^{11}$ lb ft/rad	0·52	2·98	6·98	9·72
$= 2{\cdot}92 \times 10^{10}$ lb ft/rad	0·22	2·81	6·64	9·45
2. $J_s = 0{\cdot}032\ WH^2$				
$k_R = 2{\cdot}92 \times 10^{12}$ lb ft/rad	0·71	3·21	7·42	10·19
$= 2{\cdot}92 \times 10^{11}$ lb ft/rad	0·52	2·98	6·89	9·55
$= 2{\cdot}92 \times 10^{10}$ lb ft/rad	0·21	2·79	6·46	9·18
3. $J_s = 0{\cdot}23\ WH^2$ (Inertia of all floors)				
$k_R = 2{\cdot}92 \times 10^{12}$ lb ft/rad	0·71	3·21	7·34	9·79
$= 2{\cdot}92 \times 10^{11}$ lb ft/rad	0·51	2·86	4·99	7·83
$= 2{\cdot}92 \times 10^{10}$ lb ft/rad	0·21	2·53	4·37	7·78
Same model on rigid base	0·75	3·27	7·52	10·34

49. The frequencies arising from different values for the translational parameters are shown in Table 3. Corresponding values for the effect of the rotational restraint are given in Table 4. The values of k_T and k_R in Tables 3 and 4 were calculated by substituting the value $E=2{\cdot}0\times10^6$ lb/sq. in. in equations (5) and (6) respectively, and the values of the frequency are based on the assumption that $E/W=0{\cdot}714$/sq. in. The effect of other values of E and the ratio E/W on the values of k_T, k_R and the frequencies can be obtained from equations (5), (6) and (2), respectively.

50. The range of values for k_T and k_R was selected as follows. The highest values of k_T and k_R were chosen so that the displacement of the basement floor due to a unit load at that floor was twice the value for the assumed condition of a rigid base. Similarly, the lowest values of k_T and k_R were chosen so that the deflexion of the penthouse roof due to a unit load at that floor was also double the value for the rigid base model.

51. It can be seen in Table 3 that a 100% increase in W_s produces only a second-order effect on the frequencies for the whole range of values of k_T that were investigated. These results also show that as k_T decreases from ∞ to $8{\cdot}6\times10^6$ lb/in., there is a decrease in all the first four frequencies of vibration. As k_T is reduced further, however, the fundamental mode frequency decreases, but there is very little change in the higher mode frequencies.

52. The frequencies in Table 4 were calculated by using three different values for J_s. The smallest value for J_s was obtained by assuming that the rotary inertias of all floors except the basement were zero, whereas the largest value for J_s was calculated by including the rotary inertias of all floors.

53. The results when $J_s=0{\cdot}0162\ WH^2$ and $0{\cdot}0324\ WH^2$ show that only the fundamental mode frequency is appreciably influenced by the variation of k_R. When the rotary inertias of all floors are included, however, all four frequencies are affected as k_R decreases from ∞ to $2{\cdot}921\times10^{11}$ lb ft/rad, but any further reduction of k_R has a major influence on only the fundamental mode frequency.

54. The results in Tables 3 and 4 show that with consideration of some foundation movement a reasonable fit with the measured results may be obtained. For $E/W=0{\cdot}714$/sq. in. the translational restraint parameters $W_s=W$, $k_T=8{\cdot}6\times10^6$ lb/in. provide reasonable agreement with the measured frequencies of the first, third, and fourth modes, but give a poor estimate for the second mode. The rotational restraint on the other hand provides a reasonable estimate for the second, third and fourth modes, when $J_s=0{\cdot}23\ WH^2$ and $k_R=2{\cdot}92\times10^{11}$ lb ft/rad, but underestimates the fundamental mode frequency.

55. The value of the foundation restraint obviously exerts a significant influence on the mode shapes. Thus as k_T changes from ∞ to $8{\cdot}6\times10^4$ lb/in., the fundamental mode changes from one where there is considerable relative displacement between floors to one where there is little relative displacement. For the values of $k_T=8{\cdot}61\times10^6$ lb/in. and $k_R=2{\cdot}92\times10^{11}$ lb ft/rad, however, the mode shapes are similar to those shown in Fig. 5. Some of the details of the mode shapes when foundation movement is assumed are shown in Fig. 7.

Discussion of the results

56. Without any analysis, it may be observed that the ratios of the measured frequencies suggest that the building behaves as a shear frame structure.

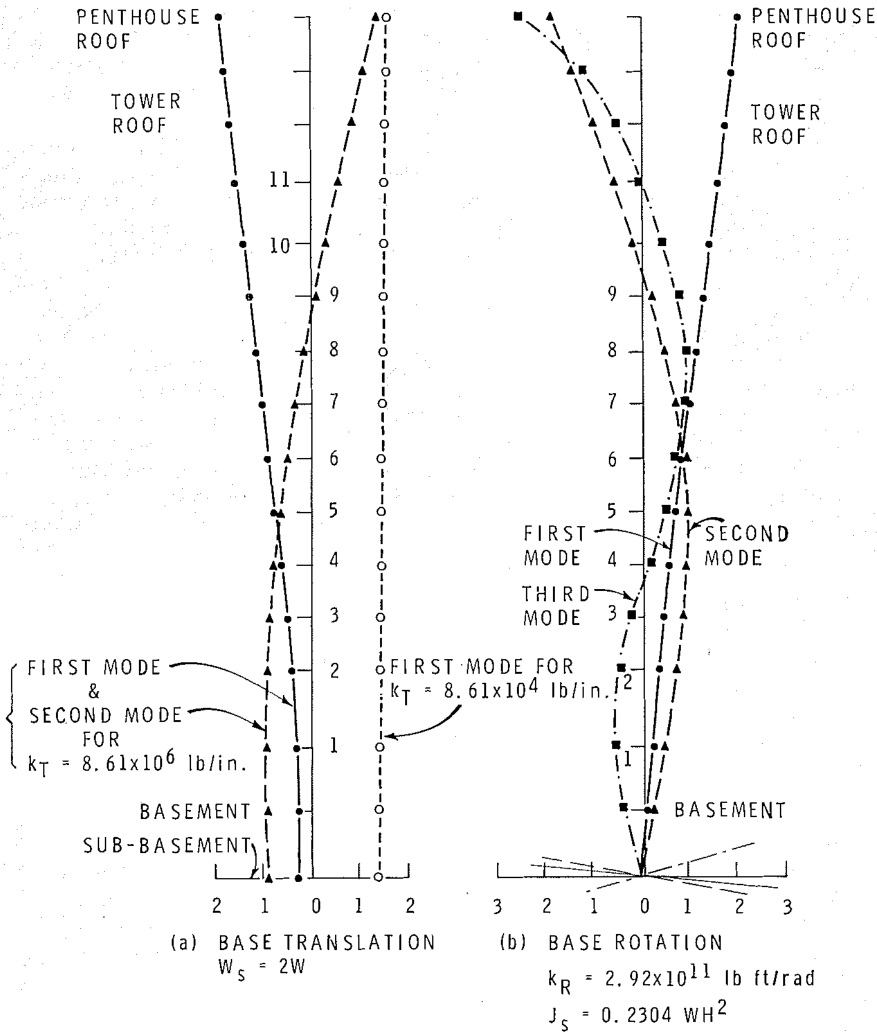


Fig. 7. Some computed mode shapes for foundation-restraint models along short axis

The modal frequencies are very close to the proportion 1:3:5:7, which is characteristic of a simple shear frame system, rather than to the higher proportion of 1:6:17:34 that would be expected of a uniform cantilever structure. This trend is confirmed by a detailed examination of the various theoretical models considered in the calculations. The shear frame model in the direction perpendicular to the long axis appears to give the best agreement with measurements as shown in Table 1. This model cannot, however, be considered a realistic reflexion of the structural behaviour since the substantial stiffness of the shear walls is neglected.

57. The combined shear wall and frame model with the additional assumption of joint rotation gives good agreement for the frequencies of the first mode, but the values for the higher modes are too large. This would be expected, since in the assumed theoretical models the contribution of the shear walls in the cantilever bending mode is significant. If shear deformations in the shear walls were considered there would be very little change in the fundamental frequency, but the second and third mode frequencies might be reduced by about 5% and 10% respectively.

58. It is well to remember at this stage, however, that the results from wind-induced vibrations must be interpreted with some caution. The major disadvantage of the method is that the resultant displacements are small, and there is a possibility that at these low levels of vibration some non-structural elements not included in the analysis can provide a contribution to the stiffness that might disappear at vibration levels which are of engineering significance. Although some recently published work[16] on the forced vibrations of a two-storey building showed that the non-linear behaviour of the structure caused only a 3% shift in the resonance frequency, there is need for more research into this aspect of the dynamic behaviour of buildings to determine whether this order of magnitude is general for most types of structures.

59. The models investigated in this study were comparatively simple, and were mainly concerned with those factors that would have a first-order effect on the vibrations of a structure. Two of the more important second-order factors that were not considered probably tend to cancel each other out; these are the effects of axial deformation and the curtain walls. It can be expected that in buildings up to about 20 storeys the neglect of axial deformations will lead to an overestimation of the fundamental frequency by about 10%; for higher modes this error will decrease.[17] On the other hand, the curtain walls will tend to increase the natural frequencies, and the work reported in Reference 16 indicates that this increase might be of the order of 10%.

60. An investigation of some foundation restraints represented merely by springs showed that this form of restraint can exert an appreciable influence on the first mode of vibration, but this influence decreases for higher modes. A comparison of the results in Tables 1 to 4 shows that the consideration of foundation compliance did not improve the agreement between measured frequencies and those computed from the assumed models. On the contrary, the ratios of frequencies of ascending modes of the various models become larger than the frequency ratios of the corresponding models without foundation movement, and consequently are even further removed from the ratios of the measured frequencies.

61. The foundation model was necessarily simple because there is little

information available on the topic. There is a need for more knowledge about the reactions of actual foundation systems to dynamic loads.

Conclusions

62. In order to improve the methods of earthquake-resistant design of multi-storey structures it is necessary to establish procedures by means of which calculation of reasonable estimates for the modes and frequencies of vibration of these buildings is feasible. The simple models discussed in this Paper indicate to some extent that it is possible to achieve this aim. The results for the structure under consideration have shown that the frequencies of an idealized shear frame model give good agreement with the measured frequencies in the long direction of the building. In the short direction the agreement between measured and computed results is less satisfactory. Further research into the dynamic behaviour of frame structures with shear walls is indicated.

63. Perhaps the most important type of information that is required consists of comparisons between the measured and predicted dynamic characteristics of numerous buildings. The analysis of wind-induced vibrations can readily provide the measured characteristics, but the possible non-linear behaviour of structures at low amplitudes of vibration may be a source of error, particularly in the higher modes of vibration.

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7173 DISCUSSION

Dynamic characteristics of a multi-storey concrete building

H. S. WARD

Mr D. A. Howells and Dr B. Nath

Dr Ward's Paper contains a valuable discussion of the way in which foundation flexibility may affect the frequencies and mode shapes of lateral vibration of a multistorey building. It would be interesting to carry this further, and to consider values of foundation flexibility which may be met in reality and how these may be determined more accurately than hitherto possible by using numerical methods, assuming of course that elastic constants of the soil are known sufficiently accurately.

66. In the commonly accepted simplification of considering the structure as a two dimensional problem in relation to each of two axes and suppressing the motion along the third dimension, there are two degrees of foundation freedom for each plan axis, namely translation and rocking. These cause two very different stress patterns in the soil and in extreme cases of, for example, stratification it may be desirable to use different elastic constants of the soil for each type of motion. In many cases, however, other sources of inaccuracy may mask lack of accuracy in the choice of elastic constants, so that overall values can be used for both types of motion. In most cases it would be reasonable to assume a value of Poisson's ratio based on an inspection of the cores taken during the site investigation. A wave velocity determination will then enable the Young's modulus to be calculated, though at the high values of Poisson's ratio usually encountered in soil dynamics it may eventually be found more convenient to express formulae in terms of one of the other elastic constants.

67. The evaluation of the spring constants and 'equivalent mass' of soil for a material of known elastic constants is the next step. Analytical solutions to the equations of motion have been found for certain boundary conditions, but more realistic examples call for numerical methods of analysis to account for the embedment of basement storeys and, where required, plan shape of the building. For motion along the short axis of a long building a two dimensional analysis using finite differences leads to a solution for the dynamic behaviour of the soil.[18] Briefly, in the case of an embedded rigid foundation block, for example, Lamé's equations of motion can be approximated by their finite difference counterparts, in which the two components of linear displacement are the unknowns to be determined. When relevant boundary conditions are introduced in these equations, including for example that during vertical translation, all points on the block have the same displacement at a given time and the whole problem reduces to finding these displacement components at the pivotal points, for a given frequency of motion. The problem is now easily solved for steady-state conditions of motion, since the original equations reduce to a set of simultaneous equations with components of displacement as unknowns. Once these components at the pivots have been determined, contact pressures under the foundation at a given frequency and subsequently the spring constant can be determined. The computer program can as readily tackle the case of rocking motion.

68. The spring constants, both for translation and rocking, are found to be functions of the exciting frequencies and the method of finite differences outlined above makes it possible to determine this relationship. A more realistic and logical assessment of problems such as that in Dr Ward's Paper would result from the evaluation of frequency dependent spring constants obtained by means of numerical procedures.

69. For multistorey structures foundation flexibility should not be allowed to have

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a significant effect on the dynamic properties of the structure. If foundation flexibility lowers the fundamental frequency by more than 10% from that calculated for a rigid base it would be wise to reconsider the whole project. This sort of analysis will give valuable guidance to the structural designer even when the elastic constants of the foundation material are not known with great precision.

Dr H. S. Ward

The Author is essentially in agreement with everything **Mr Howells** and **Dr Nath** say. As a consequence of trying over a period of five years to correlate experimentally determined dynamic characteristics of tall buildings with theoretical predictions, it became apparent that the factor about which there was least information was the effect of foundation flexibility or more precisely foundation impedance. Nevertheless the simple models used in the Paper to represent foundation impedance were not intended to represent the most realistic situation possible, but merely to see if the inclusion of the extra degree of freedom might account for the relatively poor agreement between the ratios of the measured frequencies and the theoretical values obtained when a rigid base was assumed.

71. It is important of course to develop theoretical techniques that account for ground structure interaction, such as that outlined by Mr Howells and Dr Nath. If these theoretical methods are to reflect truly the performance of actual foundation systems, however, they must incorporate information obtained from experimental observations of such systems. This input of experimental evidence is essential since with our present knowledge of soil mechanics it is unrealistic to predict the dynamic properties of soils from theoretical considerations.

72. Although experimental work in this area dates back to the Degebo[19] study in 1933, there is still a need for careful experimental work because there are many pitfalls for the unwary. With this view in mind the Division of Building Research of NRC are embarking on an experimental programme which will investigate the dynamic impedances of small sized foundation elements (plan dimensions of the order of 10 ft). These measurements will be correlated with such factors as soil profiles and soil strength properties obtained from core samples. The information will also be used to see if it is possible to extrapolate to the dynamic behaviour of actual building foundation systems.

73. It is difficult to be explicit about how much one should allow the foundation flexibility to lower the fundamental frequency of multistorey structures. It might be argued that a 10% reduction from that calculated for a rigid base could be an indication of poor soil strength. On the other hand there are instances in North America where economic factors have encouraged the use of raft foundations rather than piles as the means of supporting multistorey structures on sensitive leda clays. It is conceivable that this solution might lead to more than a 10% reduction in fundamental frequency. In view of our limited knowledge in the area of the dynamic behaviour of foundation systems, however, the figure of a 10% reduction might serve as a limit beyond which more caution should be exercised.

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CARACTERISTIQUES DYNAMIQUES D'UN IMMEUBLE EN BETON A NOMBREUX ETAGES

SOMMAIRE

Les quatre premiers modes et fréquences de vibration latérale d'un immeuble de 15 étages en béton armé ont été déterminés par l'analyse des vibrations créées par le vent. Le bâtiment est constitué de dalles plates supportées par des piliers et des murs de refend. L'auteur a calculé les valeurs théoriques des modes vibratoires à l'aide de différents modèles structuraux en prenant en considération les limites maximales et minimales du poids des planchers et des propriétés des matériaux. Il a étudié théoriquement, à l'aide de quelques modèles choisis, l'influence du rebondissement de l'assise pour les modes de balancement et de glissement horizontal. Les résultats indiquent que les changements de fréquence modale peuvent atteindre des valeurs importantes en fonction des variations des propriétés des matériaux rencontrées en pratique. Pour un des modèles mathématiques, les limites des variations de fréquence mesurées sur place se tenaient en deça des limites des variations calculées. L'auteur montre également que les fréquences des modes de vibrations les plus bas peuvent être abaissées de façon notable par le balancement de l'assise.