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Dikes on permafrost: predicting thaw and settlement\textsuperscript{1,2}

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Prediction of permafrost thaw and settlement of dikes constructed on perennially frozen ground is important for maintenance and foundation stability. A method is developed for estimating the rates of thaw and settlement, based on simple heat conduction theory. A comparison of calculated and observed rates of thaw for dikes on ice-rich foundation soils at the Kelsey Generating Station in northern Manitoba shows good agreement and indicates that thaw and settlement rates can be predicted with reasonable accuracy.

Introduction

Detailed performance observations have been reported of two pervious dikes located on permafrost at Manitoba Hydro's Kelsey Generating Station in northern Manitoba (Johnston 1969). It was the object of the study to determine rates of permafrost thaw and settlement of the dikes after flooding of the forebay.

Prediction of thaw and settlement is important for two main reasons. (1) The amount and rate of settlement, which depend on the depth and rate of thaw, are of interest in assessing the need for and the scheduling of future maintenance. (2) The stability of the foundation, which is dependent upon the rate at which thaw water is redistributed or escapes, is also related to rate of thaw.

It was recognized that thaw and subsequent settlement would occur as a result of heat transfer between the water of the forebay and the frozen ground under and adjacent to the pervious dikes. Rate of thaw, however, depends on the temperature and quantity of water flowing through a dike (the degree of permeability) as well as on the thermal properties of the ground. As none of these quantities was known, it was only possible to make very rough estimates of thaw and settlement rates.

\textsuperscript{1}NRCC 11540.

\textsuperscript{2}This paper is published with the approval of the Director of the Division.

During the 7-year period of observation from 1960 to 1967, large, irregular settlements of about 5 ft (1.5 m) occurred while thaw progressed to depths of about 17 ft (5 m); to date no serious problems of foundation instability have been encountered. The record of thaw and settlement can now be analyzed with the help of simple heat transfer theory to indicate how they may be interpreted for other situations.

Heat Transfer Considerations for Dikes on Thawing Ground

Figure 1(a) is a schematic representation of a dike and forebay. The dike may be pervious or impervious to water flow. For the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Dikes on permafrost: thermal considerations.}
\end{figure}
pervious case (Fig. 1(b)), water may flow through the dike at rates sufficiently high for the temperature on the original ground (foundation) surface to be essentially the same as that in the forebay. Thawing under the dike will proceed at approximately the same rate as that under the forebay. For an impervious dike (Fig. 1(c)), the temperature on the upstream face is that of the water, and the temperature on the remaining dike surface is determined by local climate. The pattern of thawing in the two cases differs greatly. With a pervious dike, thawing under the entire dike begins when the forebay is filled, whereas with an impervious dike thawing commences only from the upstream face and is delayed under the main body of the dike.

The initial stages of thawing, for a period of about 1 to 3 years, are also dependent on the time of year when construction takes place and when water first enters the forebay. It would be impracticable to take account of all the above factors in a calculation of thaw rates. The following elementary theory should be adequate to describe the progression of thaw under the forebay and under a pervious dike. (An impervious dike will be considered later.)

Figure 2 illustrates the assumed condition in the ground at time \( t \) after introducing water at temperature \( T_w \) to the original foundation surface (i.e., dike-ground interface). It is important to note that at the time under consideration the thaw plane is at a depth \( x \) below the original foundation surface and settlement of \( s \) ft has taken place. Neglecting soil heat capacity, the heat conducted from the foundation surface to the thaw plane per unit area in time interval \( dt \) is:

\[
Q = \frac{k(T_w - 32) \, dt}{(x - s)^2}
\]

This is the basic equation of one-dimensional heat conduction

where \( Q = \) heat conducted, Btu/sq ft
\( k = \) thermal conductivity of the unfrozen soil, Btu/h, °F, ft
\( T_w = \) water temperature at the ground surface, °F
\( x = \) depth below original foundation surface
\( s = \) amount of settlement, ft
\( t = \) time, h.

Assuming that settlement results only from the removal of water (ice) in excess of the saturation content of the soil, then the volume fraction of this excess water is

\[
s/x = a
\]

Thus, eq. [1] becomes

\[
Q = \frac{k(T_w - 32) \, dt}{x(1 - a)}
\]

Soil heat capacity should not be entirely neglected. It will be recognized from the geometry of Fig. 2 that as thawing progresses by a distance \( dx \), additional heat of amount

\[
Q_t = \frac{C_u(T_w - 32) \, dx}{2}
\]

is required to raise the temperature of the thawed soil.

Here \( C_u = \) volumetric heat capacity of the unfrozen soil

\[
= \gamma_a \left( c + \frac{w}{100} \right) \text{Btu/cu ft, °F}
\]

where \( \gamma_a = \) dry unit weight of soil, lb/cu ft
\( w = \) water content in \% of dry weight of soil
The specific heat of most dry soils near the freezing point may be assumed constant at a value of 0.17 Btu/lb (0.40 kJ/kg), °F. Therefore

\[ C_u = \gamma_d \left( 0.17 + \frac{w}{100} \right) \]

In addition, the excess water percolating through the soil carries away with it a quantity of heat equal to

\[ Q_2 = 62.4a(T_w - 32) \, dx \]

Here the volumetric heat capacity of water is 62.4 Btu/cu ft (2.33 J/cm³), °F.

From eqs. [3], [4], and [6] the net heat available by conduction at the thaw plane is

\[ Q_p = Q - (Q_1 + Q_2) \]

where

\[ Q_p = \frac{k(T_w - 32) \, dt}{x(1 - a)} \left( \frac{C_u}{2} + 62.4a \right) \times (T_w - 32) \, dx \]

This quantity of heat thaws the soil an additional depth \( dx \), the quantity of heat removed per unit area (sq ft) being

\[ Q_L = L \, dx \]

with

\[ Q = \frac{w_i}{100} L \]

and \( w_i \) = ice content in % of dry weight of soil

\[ L_r = \text{latent heat of fusion} \ 144 \ \text{Btu/lb (335 kJ/kg)} \]

As \( Q_L = Q_p \), then equating [7] and [8] gives

\[ x \, dx = \frac{k(T_w - 32) \, dt}{(1 - a)L \left[ 1 + \left( \frac{C_u}{2} + 62.4a \right) \frac{(T_w - 32)}{L} \right]} \]

whence, on integration,

\[ x = \sqrt{ \frac{2k}{(1 - a)L} \int_0^t \left[ 1 + \left( \frac{C_u}{2} + 62.4a \right) \frac{(T_w - 32)}{L} \right] \, (T_w - 32) \, dt } \]

Equation [10] permits evaluation of thaw depth at any time after water is first introduced. Following an initial period of one or two years, however, it is sufficient to insert into eq. [9] a constant value \( T_w \) equal to the mean annual bottom water temperature. For this condition eq. [10] becomes:

\[ x = \sqrt{ \frac{2k(T_w - 32)t}{(1 - a)L \left[ 1 + \left( \frac{C_u}{2} + 62.4a \right) \frac{(T_w - 32)}{L} \right] } } \]

**Calculation of Thaw at Kelsey**

It is now possible to insert values into eq. [11] for comparison with the measurements made at Kelsey. In the previous paper (Johnston 1969) the water temperature \( T_w \) at the bottom of the forebay was found to have a mean annual value of \( T_w = 42 \) °F (5.6 °C). The ice content of the frozen soil was determined by visual inspection of bore hole samples (Johnston 1969, Fig. 3) and was found to be about \( \frac{1}{3} \) by volume; fraction \( a \) (eq. [2]) is thus \( 0.9 \times \frac{1}{3} = 0.30 \) (the factor 0.9 accounts for the difference in specific volume of ice and water). This figure is in good agreement with the ratio of observed settlement to depth of thaw below original ground surface.
No moisture content determinations were made on the unfrozen soil, but silt-clay soils of the type found at Kelsey usually have saturation water contents about 30% of dry weight, corresponding to soil dry densities of about 95 lb/cu ft (1520 kg/m³), and have a thermal conductivity of about 0.8 Btu/h ft (0.234 w/ºF ft) (Kersten 1963). The total water/ice content of the frozen soil/ice would thus be expected to be about $\frac{1}{2} + 0.30 = 0.63$ volume fraction.

It has been shown (Williams 1968), however, that many soils will have a significant unfrozen water content, particularly at temperatures just below freezing. As indicated by the typical example given in Fig. 3, it is possible that only about half the soil water would be in the frozen state. Thus the total amount of moisture present in the form of ice at Kelsey would probably be about $\frac{1}{2} + 0.15 = 0.48$ volume fraction. As the unit weight of ice is 57 lb/cu ft (912 kg/m³), and it has a latent heat of 144 Btu/lb (335 kJ/kg), the heat ($L$) required to thaw the ice is $57 \times 0.48 \times 144 = 3940$ Btu/cu ft (147 J/cm³) of soil.

Inserting values obtained above for $a$, $k$, and $L$ and with $C_u = 45$ Btu/cu ft (1.68 J/cm³) (from eq. [5]), eq. [11] gives for the depth of thaw at Kelsey

$$x = \sqrt{\frac{2 \times 0.8 \times 10 \times 8766t}{0.7 \times 3940 \left[ 1 + \left( \frac{22.5 + (0.30 \times 62.4)}{3940} \right)^{10} \right]}}$$

and

$$x = 6.8\sqrt{t}$$

Here, time $t$ is in years (1 year is 8766 h).

**Pervious Dikes: Observed and Calculated Thaw Rates**

The depth of thaw at Kelsey has been calculated using eq. [12] and the result compared with the actual field observations in Fig. 4. Agreement is good, demonstrating that thaw rates can be predicted by elementary heat transfer theory for dikes of this kind. (Agreement may be partly coincidental, because the properties of the unfrozen soil were estimated, not measured. Nevertheless, these properties enter eq. [12] under the square root sign, thereby reducing the effect of their uncertainty.)

That thaw rates under the dike are similar to those under the forebay is an indication that this particular dike is quite pervious. This conclusion is borne out by observations reported in the earlier paper that (1) summer temperatures at the original ground surface under the dike reach about 60 ºF (16 ºC) (the maximum water temperature in the forebay is about 65 ºF (18 ºC)) and, (2) seepage is observed at the downstream toe throughout the winter.

**Settlement of Thawing Soils**

Widely varied, large differential movements of structures such as the Kelsey dikes are to be expected on thawing soils because of the erratic (and relatively unpredictable) occurrence of perennially frozen layers together with
unfrozen patches in the foundation and the highly variable and complex occurrence of ice and water in the frozen foundation soils. Investigations, mainly in the Soviet Union, have shown that most of the total settlement, which is of major concern with respect to dike performance, is the result of thawing of the ice lenses and inclusions and occurs at about the same rate as the frozen ground thaws. The remainder, and much smaller portion of the total settlement, results from consolidation of the thawed soil and takes place at rates dependent on the drainage and compressibility characteristics of the soil.

Russian analytical and experimental studies (Tsytovich 1958; Tsytovich et al. 1959) indicate that “thermal settlement” (caused by the abrupt change in void ratio that occurs during thawing of the ice in the soil and results in rapid subsidence approximately equal to the thickness of ice inclusions) is independent of external load and proportional to depth of thawing, and occurs at a rate proportional to the square root of time. For a considerable period after complete thawing has occurred consolidation settlement proceeds at a reduced rate and is proportional to the logarithm of the time. Studies by Tsytovich et al. (1965), Malyshev (1966), Zaretskii (1968) well illustrate the complexity of thaw-consolidation and the difficulty of predicting total settlement.

No piezometric measurements were made in the field and a quantitative evaluation of soil water drainage, and hence stability, was, therefore, not possible. Serious problems of foundation instability have not been experienced and it may be assumed that thaw water was able to dissipate at a rate that was not critical. The fact that maximum settlements occurred rapidly during the yearly period of maximum thaw, i.e. mid-summer, suggests that thaw water was able to escape fairly readily, presumably laterally to the sand drains since the soil is relatively impermeable in a vertical direction.

**Impervious Dikes: Thaw and Settlement**

It is appropriate to consider also the performance of an impervious dike. The thaw behavior will be quite different from that of a pervious dike. The thermal situation for an impervious dike becomes that shown in Fig. 5, where the approximate position of the thaw plane (32 °F isotherm) is given for different times after flooding. To develop Fig. 5 the conditions at Kelsey were assumed for thaw penetration under the forebay. The thaw penetration under the dike, however, is influenced by both water temperature (on the upstream face) and climate over the remainder of the dike surface (Fig. 1(c)). It can be shown that the combined effects result in thaw penetration vertically under the dike/water interface of about one-half that under the forebay and slightly less in a horizontal direction. It should be kept in mind that this is an idealized “no flow” case. (“No flow” means that the amount of seepage is not sufficient to influence the thermal regime of the foundations.)

Because of the lateral penetration of thaw in the impervious dike, settlement and stability considerations will be more complex for it than for the pervious dike, where thawing progresses downward at essentially the same rate under the whole dike and the adjacent forebay. When
thaw progresses laterally and at unequal rates under the dike and adjacent forebay, significant differential settlements and cracking can be expected, only tolerated with difficulty by a typical, compacted, impermeable core material. In addition, stability of the upstream slope would be questionable. In general, therefore, selection of an impervious (clay core) structure should be considered with great caution where soil and permafrost conditions are similar to those at Kelsey.

Discussion and Conclusion

Comparison of calculated and observed rates of thaw in the ice-rich foundation soils at Kelsey shows good agreement and indicates that thaw rate can be predicted with reasonable accuracy by means of elementary heat conduction theory. The magnitude and rate of settlement that occurred were also as anticipated.

In order to predict thaw and settlement it is evident from eq. [11] that it is necessary to have prior field knowledge of moisture content and water and ground temperature. From Williams’ data (Fig. 3) it is clear that if the mean ground temperature is below about 28 °F then effectively all of the moisture content is initially in the frozen form and it should be possible to predict thaw rates with considerable accuracy. For mean ground temperatures between about 30 and 32 °F (−1 and 0 °C), however, as at Kelsey, there will always remain some uncertainty about the amount of moisture that is initially present in the unfrozen form. The importance of prior knowledge of water temperature should also be noted. For example, if a value of 37 °F (2.8 °C) has been estimated for Kelsey instead of the 42 °F (5.6 °C) actually measured, then by eq. [12] the calculated depth of thaw would be in error by about 30%.

It is essential, therefore, for predicting thaw rates, that adequate field investigations be carried out to provide the required information. The most important quantities to be determined are total moisture content (ice plus water) and mean annual ground and water temperature. Next, but somewhat less important, is a determination of the thermal conductivity of the unfrozen soil. (This can be measured directly in the laboratory or estimated from soil type and unfrozen moisture content.) At the present time there is a need to improve both field and laboratory sampling and measurement techniques.


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