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TECHNICAL NOTE

RJR

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SUBJECT INSULATION TO PREVENT GROUND FREEZING

The ground can be kept unfrozen all winter if it is covered at the end of the summer with a thick layer of straw (or other insulation). This note gives a method of determining how much insulation is required. It assumes that the straw-covered area is large enough that edge effects can be neglected. This means that the cover must extend about 10 ft beyond the perimeter of the area that must be protected.

The other assumptions are:

- 1) At the time when the straw is applied the temperature of the ground is T_0 at all depths and that the temperature of the air is also T_0 at that time.
- 2) The ground is homogeneous with a thermal conductivity K_{soil} and diffusivity α_{soil} .
- 3) The straw cover has a known conductivity, K_{straw} , and a negligible heat storage capacity.
- 4) The straw remains dry so there is no latent heat released as the 32°F isotherm penetrates into the straw.

The variation in air temperature with time can be represented as the sum of a series of ramp functions with different slopes and different starting times and with the same temperature datum. For example, a ramp with a slope of -20°F/month plus one starting 3 months later with a slope

of +20°F/month is equivalent to a temperature curve that decreases linearly for 3 months and then remains constant thereafter. Since the heat conduction equation is linear, the variation of the soil surface temperature is just the sum of the effects caused by each component of the air temperature acting independently.

The temperature, T_s , at the surface of a semi-infinite solid with a finite surface heat conductance h is given by

$$T_s = T_o + \frac{C}{\gamma} \{F(\gamma^2 t)\}$$

when $T_A = T_o + Ct$, where T_A is the air temperature.

Temperature at all depths in ground = T_o at $t = 0$

$$\gamma = \frac{h \sqrt{\alpha_{\text{soil}}}}{K_{\text{soil}}}$$

$$F(\gamma^2 t) = 1 - e^{-\gamma^2 t} \operatorname{erfc} \gamma \sqrt{t} + \gamma^2 t - \frac{2 \gamma \sqrt{t}}{\sqrt{\pi}}$$

For values of $\gamma^2 t$ less than 0.2 the function $F(\gamma^2 t)$ is very well represented by the simpler expression

$$F(\gamma^2 t) \div 0.53 (\gamma^2 t)^{1.43}$$

Example:

$$T_o = 40^\circ\text{F}$$

T_A drops 20°/month for 3 months, remains constant at -20°F for 1 month, then rises at 20°/month.

Find the depth of straw needed to keep the soil surface temperature from dropping below 32°F.

$$K_{\text{soil}} = 8 \text{ Btu/hr ft}^2 (\text{°F/in.})$$

$$K_{\text{straw}} = 0.5 \text{ Btu/hr ft}^2 (\text{°F/in.})$$

$$\alpha_{\text{soil}} = 0.025 \text{ ft}^2/\text{hr}$$

(a) For 2-ft layer of straw

$$h = \frac{K_{\text{straw}}}{l_{\text{straw}}}, \text{ where } l \text{ is the thickness of straw.}$$

$$\begin{aligned} \gamma^2 &= \left(\frac{K_{\text{straw}}}{K_{\text{soil}}} \right)^2 \frac{\alpha_{\text{soil}}}{(l_{\text{straw}})^2} \\ &= \left(\frac{0.5}{8.0} \right)^2 \times \frac{0.025}{(2.0)^2} = 24.4 \times 10^{-6} \text{ hr}^{-1} \\ &= 0.0176 \text{ months}^{-1} \end{aligned}$$

Time Months →	0	1	2	3	4	5	6	7
$T_o = \text{Constant}$	40	40	40	40	40	40	40	40
Ramp 1	0	-20	-40	-60	-80	-100	-120	-140
Ramp 2	0	0	0	0	20	40	60	80
Ramp 3	0	0	0	0	0	20	40	60
Sum = T_A	40	20	0	-20	-20	0	20	40

The constant plus the three ramp functions produce the required time variation of the ambient temperature T_A . The resulting surface temperature is just the sum of the values that would occur if each component of the ambient temperature acted independently.

The surface temperature resulting from a ramp with slope $20^\circ/\text{month}$ is given in the following table.

For $\gamma^2 = 0.0176 \text{ months}^{-1}$

t months	$\gamma^2 t$	$F(\gamma^2 t)$	$\frac{20}{\gamma} \left[F(\gamma^2 t) \right]$
0	.0000	.0000	0.00
1	.0176	.00164	1.86
2	.0352	.0044	5.00
3	.0527	.0078	8.87
4	.0703	.0118	13.42
5	.0879	.0163	18.54
6	.1055	.0197	22.41
7	.1230	.0261	29.69

Soil Surface Temperature

Time Months →	0	1	2	3	4	5	6
$T_o = \text{Constant}$	40	40	40	40	40	40	40
Ramp 1	0	-1.86	-5.00	-8.87	-13.42	-18.54	-22.41
Ramp 2	0	0	0	0	+1.86	+5.00	+8.87
Ramp 3	0	0	0	0	0	+1.86	+5.00
Sum = T_s	40°	38.14°	35.00°	31.13°	28.44°	28.32°	31.46°

(b) For 3 ft straw $\gamma^2 = 0.0078 \text{ months}^{-1}$

Time Months	$\gamma^2 t$	$F(\gamma^2 t)$	$\frac{20}{\gamma} F(\gamma^2 t)$
0	0.0000	0.0000	0.00
1	0.0078	0.00050	1.28
2	0.0156	0.00137	3.51
3	0.0234	0.00245	6.28
4	0.0313	0.0037	9.49
5	0.0391	0.0051	13.08
6	0.0469	0.0066	16.92
7	0.0547	0.0083	21.28

Soil Surface Temperature

Time Months	→	0	1	2	3	4	5	6
$T_o = \text{Constant}$		40	40	40	40	40	40	40
Ramp 1		0.00	-1.28	-3.51	-6.28	-9.49	-13.08	-16.92
Ramp 2		0	0	0	0	+1.28	+3.51	+6.28
Ramp 3		0	0	0	0	0	+1.28	+3.51
Sum = T_s		40°	38.72°	36.49°	33.72°	31.79°	31.71°	32.87°

These results indicate that just over 3 ft of straw would be enough to keep the ground surface from freezing if the minimum mean monthly temperature is -20°F and the ground is at $+40^\circ\text{F}$ when the straw is spread. The thickness of the insulation needed depends on the thermal properties of the soil as well as on the conductivity of the insulation. The more general conclusion to be drawn from the example is that the parameter, γ^2 should be not more than $0.0078 \text{ months}^{-1}$, (i. e., $1.1 \times 10^{-5} \text{ hr}^{-1}$).

$$\text{Thus } l_{\text{insul}}^2 > \left(\frac{K_{\text{insul}}}{K_{\text{soil}}} \right)^2 \frac{\alpha_{\text{soil}}}{1.1} \times 10^5$$

Another generalization can be made. The insulation should be applied when the heat content of the ground is at its annual maximum. This is usually the latter part of August. If the insulation is applied this early the T_o will usually be greater than the 40°F assumed in the example. In fact, 2 ft of straw applied at the end of August would probably be adequate to keep the ground unfrozen in those parts of Canada where the minimum mean monthly temperature is the order of -20°F .