NOT FOR PUBLICATION

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## SUBJECT

PREVENTING EXPOSED WATER PIPES FROM FREEZING

A water pipe that is exposed to an environment at temperatures lower than the freezing point of water will not necessarily freeze even without insulation if there is a continuous flow through it, but when there is no flow it will freeze regardless of insulation. The required minimum flow rate depends on the temperature of the water entering the exposed section of pipe and the resistance to heat transfer from the water to the environment. This note presents an equation relating these parameters. It can be used to solve for any one of the three variables when the other two are known. Information is also presented which facilitates the calculation of the thermal resistance between water in the pipe and the environment, for conditions that could cause freezing.

## Basic Equation for Heat Loss

The rate of heat transfer from fluid flowing through a pipe is:

$$
Q=\frac{L^{\cdot} \cdot \Theta_{\text {mean }}}{R_{\text {Total }}}
$$

where $R_{\text {Total }}$ is the resistance to heat flow per unit length of pipe,
$L$ is the length of the exposed section of pipe,
$\theta$ is the difference between the fluid temperature and the ambient air temperature.

Let $T_{i n}=$ temperature of fluid entering pipe, $T_{\text {out }}=$ temperature of fluid leaving pipe,

$$
\mathrm{T}_{\text {ambient }}=\text { temperature of the environment. }
$$

Then

$$
\begin{aligned}
& \theta_{\text {in }}=T_{\text {in }}-T_{\text {ambient }} \\
& \theta_{\text {out }}=T_{\text {out }}-T_{\text {ambient }} \\
& \theta_{\text {mean }}=\frac{\theta_{\text {in }}-\theta_{\text {out }}}{\ln \left(\theta_{\text {in }} / \theta_{\text {out }}\right)}=\frac{T_{\text {in }}-T_{\text {out }}}{\ln \left(\theta_{\text {in }} / \theta_{\text {out }}\right)}
\end{aligned}
$$

The heat loss can also be related to the fluid flow rate and the temperature drop between inlet and outlet.
$Q=W C\left(T_{\text {in }}-T_{\text {out }}\right)$
where $W$ is mass flow rate through the pipe, $C$ is the specific heat of the fluid ( $=1.0$ for water).

Combining the two independent expressions for $Q$ gives:

$$
\begin{aligned}
& \ln \left(\theta_{\text {in }} / \theta_{\text {out }}\right)=\frac{L}{W C R} \text { Total } \\
& \text { or } \theta_{\text {in }}=\theta_{\text {out }} e^{L / W C R} \text { Total }
\end{aligned}
$$

## Thermal Resistances

The total resistance to heat flow between fluid flowing through a pipe and the outside environment is the sum of the following four components:
(a) $\mathrm{R}_{\text {inside }}$, which depends on the rate of flow and the inside diameter of the pipe. Figure 1 gives the relationship for cold water flowing through a long pipe.
(b)

$$
R_{\text {pipe }}=\frac{\ln (O . D . / \text { I.D. })_{\text {pipe }}}{2 \pi K_{\text {pipe }}}
$$

where O.D. and I.D. are the outside and inside diameters of the pipe respectively and $K_{\text {pipe }}$ is the thermal conductivity of the pipe material.

| Material | $\mathrm{K}\left(\frac{\mathrm{Btu}}{\mathrm{ft} \mathrm{hr}{ }^{\circ} \mathrm{F}}\right)$ |
| :--- | :---: |
| Copper | 220 |
| Aluminum | 120 |
| Steel | 28 |
| Plastic | 0.1 |

(c) $R_{\text {insulation }}=\frac{\ln (O . D . / \text { I.D. })_{\text {insulation }}}{2 \pi K_{\text {insulation }}}$

For most pipe insulations the conductivity has a value of about 0.025 Btu/hr ft ${ }^{\circ} \mathrm{F}$.
(d) $R_{\text {outside }}=\frac{1}{H_{c}+H_{R}}$
where $H_{c}$ and $H_{R}$ are the conductances per lineal foot of pipe due to convection and radiation respectively. $H_{c}$ depends on the outside diameter of the cylinder and the velocity of the air blowing across it. The relationship is shown graphically in Figure 2. $H_{R}$ depends on the diameter of the cylinder and the emissivity of the outer surface. For surfaces with a high value of emissivity, $H_{R}$ is approximately equal to twice the outside diameter expressed in feet. This is usually small compared to $H_{c}$ and a lower emissivity surface makes it smaller still.

## Minimum Water Temperature

If a pipe is to remain completely free of ice, its inside surface temperature must not fall below $32^{\circ} \mathrm{F}$, which means that the water temperature must be above $32^{\circ} \mathrm{F}$. The minimum value of $\mathrm{T}_{\text {out }}$ is given by:

$$
T_{\text {out }}=32+\left(\frac{R_{\text {inside }}}{R_{\text {pipe }}+R_{\text {insulation }}+R_{\text {outside }}}\right)\left(32-T_{\text {ambient }}\right)
$$

or $\quad \theta_{\text {out }}=\left(\frac{R_{\text {Total }}}{R_{\text {pipe }}+R_{\text {insulation }}+R_{\text {outside }}}\right)\left(32-T_{\text {ambient }}\right)$

## Example Problem

Find the required inlet water temperature to prevent freezing in a $500-\mathrm{ft}$ length of $6-\mathrm{in}$. schedule 40 steel pipe covered by a l-in. layer of insulation. The minimum water flow rate will be 3 gallons $/ \mathrm{min}$. ( 1800 $\mathrm{lb} / \mathrm{hr}$ ) and the ambient conditions are $-10^{\circ} \mathrm{F}$ with a 30 -mile-per-hour wind.

Data: 6-in. schedule 40 steel pipe
I. D. pipe $\quad=6.065 \mathrm{in}$.
O.D. pipe $=6.625 \mathrm{in}$.
O. D. insulation $=8.625 \mathrm{in}$.

## Solution:

A. Calculation of Thermal Resistances
$\frac{\mathrm{W}}{\pi \times \text { I. D. }}=\frac{1800}{\pi \times 6.065 / 12}=1134$
From Figure 1, $R_{\text {inside }}=0.25$
$R_{\text {pipe }}=\ln (6.625 / 6.065) /(2 \pi \times 28)=0.0005$ i.e., negligible
$R_{\text {insulation }}=\ln (8.625 / 6.625) /(2 \pi \times 0.025)=1.68$
$R_{\text {outside }}=\frac{1}{H_{c}+H_{R}}$
Wind Speed $\times$ O.D. ${ }_{\text {insulation }}=30 \times \frac{8.625}{12}=21.6$
$\therefore$ From Figure 2, $\mathrm{H}_{\mathrm{c}}=19.8$
$H_{R}=\frac{2 \times 8.625}{12}=1.4$
$R_{\text {outside }}=\frac{1}{19.8+1.4}=0.05$
$R_{\text {Total }}=0.25+1.68+0.05=1.98$
B. Calculation of Water Outlet Temperature

$$
\begin{aligned}
\theta_{\text {out }} & =\left(\frac{R_{\text {Total }}}{R_{\text {pipe }}+R_{\text {insulation }}+R_{\text {outside }}}\right)\left(32-T_{\text {ambient }}\right) \\
& =\frac{1.98}{1.73} \times 42=48 \text { degrees } \\
T_{\text {out }} & =T_{\text {ambient }}+\theta_{\text {out }}=-10+48=38^{\circ} \mathrm{F}
\end{aligned}
$$

C. Calculation of Water Inlet Temperature

$$
\begin{aligned}
& \Theta_{\text {in }}=\theta_{\text {out }} \mathrm{e}^{\mathrm{L} / \mathrm{WCR}} \text { Total } \\
& \mathrm{L} / \mathrm{WCR}_{\text {Total }}=\frac{500}{1800 \times 1 \times 1.98}=0.140 \\
& \Theta_{\text {in }}=48 \mathrm{e}^{0.140}=55 \text { degrees } \\
& \mathrm{T}_{\mathrm{in}}=\mathrm{T}_{\text {ambient }}+\Theta_{\mathrm{in}}=-10+55=45^{\circ} \mathrm{F}
\end{aligned}
$$

The problem might have been to determine the minimum allowable flow rate for a given inlet temperature, say $\mathrm{T}_{\text {in }}=48^{\circ} \mathrm{F}$.

## Solution:

A. as in first example assuming that flow will be laminar and $R_{\text {inside }}=.25$ regardless of flow rate
B. as in first example
C. Calculation of Minimum Flow Rate

$$
\begin{aligned}
& L / W_{\text {Total }}=\ln \left(\Theta_{\text {in }} / \Theta_{\text {out }}\right) \\
& \theta_{\text {in }}=T_{\text {in }}-T_{\text {ambient }}=48+10=58 \\
& \ln \left(\Theta_{\text {in }} / \Theta_{\text {out }}\right)=\ln (58 / 48)=0.190 \\
& \mathrm{~W}=\frac{\mathrm{L}}{\mathrm{CR}_{\text {Total }} \ln \left(\theta_{\mathrm{in}} / \Theta_{\text {out }}\right)}=\frac{500}{1 \times 1.98 \times 0.190}=1330 \mathrm{lb} / \mathrm{hr} .
\end{aligned}
$$

## FIGURE 1

THERMAL RESISTANCE BETWEEN PIPE AND WATER FLOWING THROUGH IT


FIGURE 2
CONVECTIVE HEAT TRANSFER FROM CYLINDER TO AIR IN CROSSFLOW

