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Variable-speed heat pump model for a wide range of cooling conditions and loads

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A modular variable-speed heat pump model is developed from first principles. The system model consists of steady-state evaporator, compressor, and condenser component sub-models. The compressor model currently implemented accounts for re-expansion, valve pressure drop, and back leakage using empirical coefficients obtained from tests covering a wide range of pressure ratio and shaft speed. Variations in heat transfer coefficients with refrigerant and secondary fluid flow rates are modeled over a wide range of capacity. Pressure drops in piping and heat exchangers are also modeled. The resulting heat pump model is flexible and fast enough for use in finding optimal compressor, fan, and pump speeds and optimal subcooling for any specified capacity fraction and operating condition. To confirm the model's accuracy, simulation results are compared to experimental data with condenser inlet air temperatures ranging from 15°C to 45°C, evaporator inlet air (dry) from 14°C to 34°C, and cooling capacity from 1.1 kW to 3.9 kW. The refrigerant charge balance has not been modeled; instead, it is assumed that a liquid receiver maintains the necessary charge balance. Over this wide range of conditions, the coefficient of performance prediction errors are found to be $\pm 10\%$. An example of configuring the HPM with a different evaporator demonstrates the benefits of a modular approach.

Introduction

In the United States, buildings consume about 40% of total energy and more than 70% of electricity. The second and third largest consumers of electricity in commercial buildings are cooling and ventilation systems, and the cooling share is growing (U.S. Energy Information Administration 2008). The majority of cooling energy is used by compressors.

One path to reducing the energy for compression is to decrease the condensing temperature and increase the evaporating temperature. A cooling system that reduces the pressure rise across the compressor is termed “low-lift cooling.” Although the pressure-rise reduction achieved using low-lift technology decreases the compressor energy consumption, it would normally increase the transport energy consumption and/or heat exchanger size and, hence, needs to be carefully balanced. One

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promising way to achieve a balanced low-lift cooling system design is to integrate four available technology elements: variable-speed drive (VSD) motors for the compressor and auxiliary fans and pumps, hydronic radiant cooling system (RCS), thermal energy storage (TES), and a dedicated outside air system (DOAS). While many different studies have shown the benefits of each separate component of this system, the combined system benefits have only recently been investigated (Jiang et al. 2007; Armstrong et al. 2009; Katipamula et al. 2010). These scoping studies of five different U.S. climates and three building types (low performance, mid performance, and high performance) showed possible cooling system energy savings from 30% to 70%, depending on the climate and the building envelope, when all four elements work together.

Analysis of low-lift systems requires a heat pump model (HPM) that is valid over a wide range of compressor and condenser fan speeds and over a wide range of lift conditions. The steady-state air-to-air heat pump and chiller models found in the literature can be divided between relatively simple, fast models, which are usually developed for larger system simulations, and more accurate, detailed, and computationally expensive models, which require large numbers of input parameters. Such models are most often used by manufacturers as design tools. Among high-detail HPMs are Hiller's (1976) air-to-air HPM with the reported accuracy in coefficient of performance (COP) prediction of 4% to 6%, Ellison and Creswick's (1978) and Ellison et al.'s (1979) air-to-air HPM, and Domanski and Didion's (1984) air-to-air HPM with reported maximum errors 2.2% for the capacity, 3.8% for power, and 5.1% for COP. One of the more recent, detailed HPMs is Iu's (2007) model with a unique heat exchanger circuiting algorithm. Probably the best known and most widely used air-to-air, steady-state HPM today, developed from Ellison et al.'s (1979) model, is the DOE/ORNL model (Rice 2006). It offers a great level of detail and is continually improved. For example, recently (Murphy et al. 2007), a tube-in-tube heat exchanger model was added.

Several simpler HPMs found in the literature are Jeter et al.'s air-to-air HPM (1987), Bertsch and Groll's (2008) air-source HPM, Braun et al.'s (1987) variable-speed control chiller model, Fu et al.'s (1988) chiller model, Jin's (2002) physics-based empirical model, and the basic first-principles model developed by Armstrong et al. (2009). Armstrong's model optimizes the heat pump cooling cycle for

the given cooling rate and zone and ambient temperatures by finding the compressor, fan, and pump speeds at which minimal total power is required. The simpler models mentioned above usually combine a volumetric efficiency model for compressor calculations with an ε -NTU (number of transfer units) method or logarithmic mean temperature method for heat exchanger calculations. Although sufficiently simple for larger simulations, most of them neglect the refrigerant pressure drop, variable heat transfer coefficients, superheating, desuperheating, or subcooling, or some combination of them.

Although steady-state HPMs have been around a long time (Stoecker 1971), there is still need for a model that will accurately simulate and optimize not just commercially available heat pumps, but also refinements and new heat pumps designed to operate efficiently over a much broader range of operating conditions than in current practice. To correctly reflect the relative system COP benefits of a wide range of compressor, fan, and pump speeds and of a wide range of evaporating and condensing temperatures, the model should address the impact of evaporator superheating, condenser subcooling, pressure drop, and variable fluid transport rates.

The modular facilitates further development of each component independent from the other parts, replacement of one component sub-model with a different one (different types of heat exchangers, compressors, fans, inverters), or the addition of new components (reversing valve, liquid receiver, expander). One example presented in this article is the evaporator sub-model, which is replaced by a water-to-refrigerant heat exchanger without changing the other sub-models. Other types of compressor and condenser models can be substituted with equal ease. It is shown in the model validation section that with the heat exchanger, pressure drop, and compressor component models currently implemented, there is good agreement between the measured data and HPM predictions of pressures, temperatures, mass flow rates, and compressor speeds. The compressor power predictions show the largest relative errors $\pm 10\%$, which may be a consequence of measurement error and/or of using a relatively simple compressor model.

Model description

The HPM currently consists of three component sub-models within a main solver loop. The

evaporator, compressor, and condenser components are modeled separately using energy balance equations and semi-empirical correlations. An idealized expansion valve has been assumed, which accurately models the real steady-state throttling process in cases where the valve is positioned at the evaporator inlet. It is assumed that the expansion valve delivers the exact refrigerant mass flow rate (assuming the valve free area is sufficient for design capacity during operation at low pressure difference in addition to ideal control) for any specified superheating to be achieved at the evaporator outlet and that the enthalpy at the condenser outlet is equal to the enthalpy at the evaporator inlet. The refrigerant charge balance has not been modeled; instead, it is assumed that the amount of subcooling or the condenser subcooled area fraction can be controlled and that a liquid receiver maintains the necessary charge balance. The process in the suction and discharge lines is also assumed to be isenthalpic.

Conditions where the refrigerant leaves the condenser or evaporator in a two-phase state cannot be handled.

Variable heat transfer coefficients and pressure drops in the components and in the connecting suction and discharge lines have been modeled using correlations explained in Appendix A. The liquid line pressure drop has not been modeled since it is much smaller than either of the vapor line pressure drops and is part of the idealized expansion valve model. The lubricant influence is taken into account for the evaporator and condenser pressure drop calculations and also in the compressor heat balance calculations. The HPM is written in MATLABTM, and the subroutines of REFPROPTM, Version 8.1 (Lemmon et al. 2007) are used to evaluate refrigerant properties.

A schematic of the model is given in Figure 1. Points marked on the temperature-entropy diagram correspond to the subscripts used in sub-model

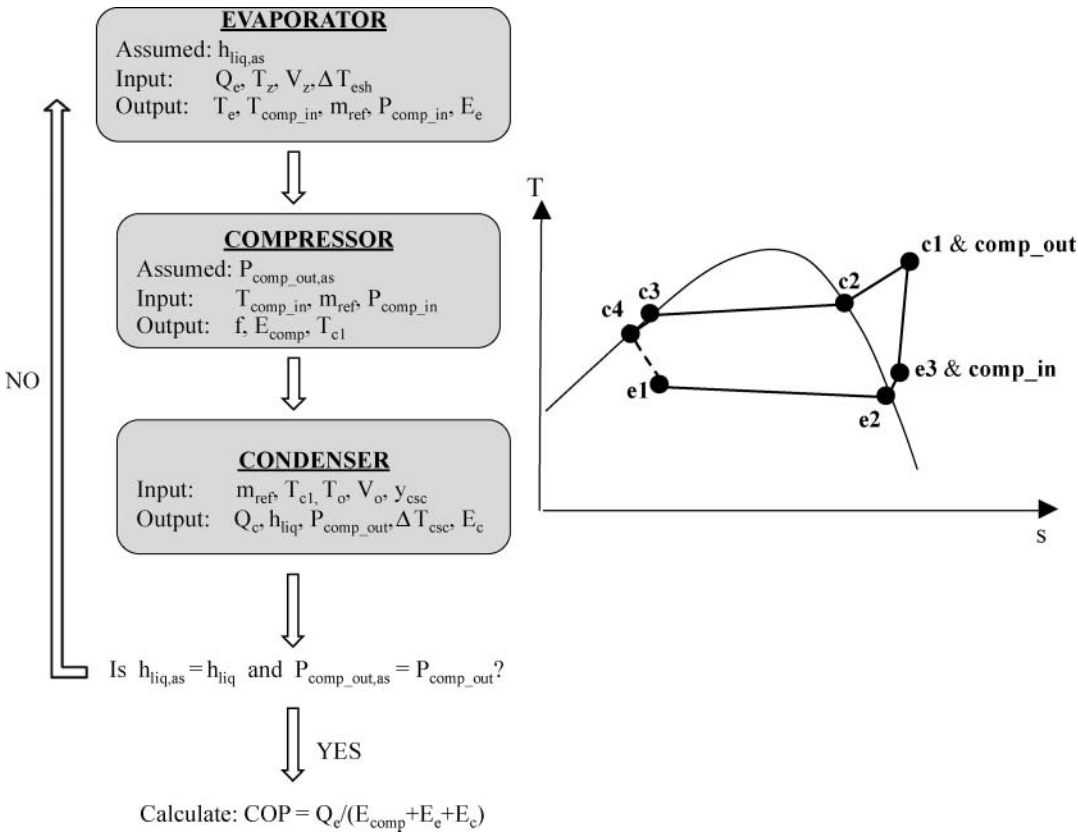


Figure 1. HPM schematic.

equations; e.g., the subscript $e1$ corresponds to evaporator inlet conditions, and $c3$ corresponds to conditions in the condenser where condensation finishes (before sub-cooling). Pressure drops in compressor suction and discharge vapor lines are modeled but not shown in Figure 1.

At the beginning of the simulation, trial values are assigned for condenser outlet enthalpy (equal to the evaporator inlet enthalpy) $h_{liq,as}$, and compressor discharge pressure $P_{comp,out,as}$. The evaporator sub-model calculates the refrigerant mass flow rate, temperatures, and pressure required to satisfy the evaporator energy balance equations for the given cooling load, zone temperature (or water inlet temperature), superheating, evaporator airflow (or water mass flow rate), and trial evaporator inlet enthalpy. For the calculated compressor inlet state, refrigerant mass flow rate, and trial compressor outlet pressure, the compressor sub-model calculates the power, frequency, and discharge temperature. At the end, the condenser temperatures, pressures, and exchanged heat are calculated in the condenser sub-model for the calculated refrigerant mass flow rate (from the evaporator sub-model), condenser inlet temperature (from the compressor sub-model), and given condenser airflow rate, outside temperature, and area ratio devoted to subcooling. Two solver convergence points (Figure 1) are $c1$ for the trial pressure and $c4$ for the trial enthalpy, meaning that the enthalpy assigned at the evaporator inlet needs to be equal to the enthalpy at the condenser outlet calculated from the condenser sub-model and that the pressure assigned at the compressor outlet needs to be equal to the calculated pressure at the condenser inlet plus the pressure drop in the discharge line. The main solver variables are displayed in Table 1.

Heat exchanger sub-model

The air-to-refrigerant evaporator and condenser sub-models are developed for a finned-tube heat exchanger (subscript F), and the water-to-refrigerant sub-model is developed for a brazed-plate heat exchanger (subscript B). The evaporator is divided into the superheating and evaporating region, while the condenser is divided into the desuperheating, condensing, and subcooling region. The finned-tube evaporator model, developed here for sensible cooling only, can be extended to include latent cooling using the enthalpy potential method (Threlkeld 1970).

The steady-state heat exchanger behavior is modeled using the NTU method (ASHRAE 2009) and energy balance equations for the evaporating and condensing region:

$$Q_{ep,F} = m_{ref}(h_{e2} - h_{liq}) = \varepsilon_{ep} C_{ep} (T_z - T_{e1}), \quad (1)$$

$$Q_{ep,B} = m_{ref}(h_{e2} - h_{liq}) = m_w c_w (T_{w,e2} - T_{w,out}) = \varepsilon_{ep} C_{ep} (T_{w,e2} - T_{e1}), \quad (2)$$

$$Q_{cp,F} = m_{ref}(h_{c2} - h_{c3}) = \varepsilon_{cp} C_{cp} (T_{c2} - T_o), \quad (3)$$

where

$$C_{ep,F} = y_{ep} V_z \rho_{air} c_{p,air}, \quad (4)$$

$$C_{ep,B} = m_w c_w, \quad (5)$$

$$C_{cp,F} = y_{cp} V_o \rho_{air} c_{p,air}, \quad (6)$$

Table 1. Main solver variables.

Parameters	Heat exchanger geometry, material properties, constants
Boundary condition	Zone (room) air temperature (T_z) or water inlet temperature ($T_{w,in}$), cooling rate (Q_e), evaporator airflow rate (V_z) or water mass flow rate (m_w), superheating temperature difference at the evaporator outlet (ΔT_{esh}), outside air temperature (T_o), condenser airflow rate (V_o), and condenser area percentage devoted to subcooling (y_{csc}) or subcooling temperature difference (ΔT_{csc})
Unknowns	Enthalpy at the evaporator inlet (h_{liq}) and compressor outlet pressure ($P_{comp,out}$)
Outputs	Refrigerant mass flow rate (m_{ref}), temperatures in the evaporator (T_{e1} , T_{e2} , T_{e3}), temperatures in the condenser (T_{c1} , T_{c2} , T_{c3} , T_{c4}), total heat exchanged on the condenser (Q_c), evaporator and condenser fan power (E_e , E_c), evaporator and condenser fan speeds (w_e , w_c), compressor shaft speed (f), compressor input power (E_{comp}), and COP

$$\varepsilon = 1 - \exp[-UA/C]. \quad (7)$$

For the evaporator superheating, condenser desuperheating and condenser subcooling region:

$$\begin{aligned} Q_{esh,F} &= m_{ref}(h_{e3} - h_{e2}) \\ &= \varepsilon_{esh} C_{esh_min} (T_z - T_{e2}), \end{aligned} \quad (8)$$

$$\begin{aligned} Q_{esh,B} &= m_{ref}(h_{e3} - h_{e2}) = m_w c_w (T_{w,in} - T_{w,e2}) \\ &= \varepsilon_{esh} C_{esh_min} (T_{w,in} - T_{e2}), \end{aligned} \quad (9)$$

$$\begin{aligned} Q_{csh,F} &= m_{ref}(h_{c1} - h_{c2}) \\ &= \varepsilon_{csh} C_{csh_min} (T_{c1} - T_o), \end{aligned} \quad (10)$$

$$Q_{csc,F} = m_{ref}(h_{c3} - h_{c4}) = C_{csc_min}(T_{c3} - T_o). \quad (11)$$

The thermal effectiveness for the superheating, desuperheating, and subcooling regions of the finned-tube heat exchanger is calculated for a cross-flow heat exchanger with both hot and cold stream unmixed, while the thermal effectiveness for the brazed-plate heat exchanger superheating region is calculated using the correlation for a counter-flow heat exchanger (McQuiston et al. 2004):

$$\varepsilon_F = 1 - \exp\left[\frac{\exp(-\beta NTU)}{\beta}\right], \quad (12)$$

$$\varepsilon_B = \frac{1 - \exp[-NTU(1 - \alpha)]}{1 - \alpha \exp[-NTU(1 - \alpha)]}, \quad (13)$$

where

$$\alpha = \frac{C_{min}}{C_{max}}, \quad (14)$$

$$\beta = \frac{C_{min}}{C_{max}} NTU^{-0.22}, \quad (15)$$

$$NTU = \frac{UA}{C_{min}}, \quad (16)$$

$$\begin{aligned} C_{esh_min,F} &= \min\left(y_{esh} V_z \rho_{air} c_{p,air}, y_{esh} V_z \rho_{air} \frac{h_{e3} - h_{e2}}{T_{e3} - T_{e2}}\right), \\ &\quad (17) \end{aligned}$$

$$\begin{aligned} C_{esh_max,F} &= \max\left(y_{esh} V_z \rho_{air} c_{p,air}, y_{esh} V_z \rho_{air} \frac{h_{e3} - h_{e2}}{T_{e3} - T_{e2}}\right), \\ &\quad (18) \end{aligned}$$

$$C_{esh_min,B} = \min\left(m_w, m_w \frac{h_{e3} - h_{e2}}{T_{e3} - T_{e2}}\right), \quad (19)$$

$$C_{esh_max,B} = \max\left(m_w, m_w \frac{h_{e3} - h_{e2}}{T_{e3} - T_{e2}}\right), \quad (20)$$

$$\begin{aligned} C_{csh_min,F} &= \min\left(y_{csh} V_o \rho_{air} c_{p,air}, y_{csh} V_o \rho_{air} \frac{h_{c1} - h_{c2}}{T_{c1} - T_{c2}}\right), \\ &\quad (21) \end{aligned}$$

$$\begin{aligned} C_{csh_max,F} &= \max\left(y_{csh} V_o \rho_{air} c_{p,air}, y_{csh} V_o \rho_{air} \frac{h_{c1} - h_{c2}}{T_{c1} - T_{c2}}\right), \\ &\quad (22) \end{aligned}$$

$$\begin{aligned} C_{csc_min,F} &= \min\left(y_{csc} V_o \rho_{air} c_{p,air}, y_{csc} V_o \rho_{air} \frac{h_{c3} - h_{c4}}{T_{c3} - T_{c4}}\right), \\ &\quad (23) \end{aligned}$$

$$\begin{aligned} C_{csc_max,F} &= \max\left(y_{csc} V_o \rho_{air} c_{p,air}, y_{csc} V_o \rho_{air} \frac{h_{c3} - h_{c4}}{T_{c3} - T_{c4}}\right). \\ &\quad (24) \end{aligned}$$

Correlations and geometry definitions used to calculate the product of the overall heat transfer coefficient and heat exchanger area for each region (UA_{ep} , UA_{esh} , UA_{csh} , UA_{cp} , UA_{csc}) are discussed in Appendix A.

The dependence of the evaporator and the condenser fan speeds on required airflows is modeled using a simple linear relation:

$$w_e = C_1 V_z + C_2, \quad (25)$$

$$w_c = C_3 V_o + C_4, \quad (26)$$

while the fan power versus speed dependence is modeled using power laws (see Appendix C):

$$E_{e-mot} = C_5(w_e)^{C_6}, \quad (27)$$

$$E_{c-mot} = C_7(w_c)^{C_8}. \quad (28)$$

Power versus speed and water flow versus speed relations for a pump are modeled using power laws:

$$w_p = w_{p,0} \left(\frac{V_w}{V_{w,0}} \right)^{C_9}, \quad (29)$$

$$E_{p-mot} = E_{p,0} \left(\frac{w_p}{w_{p,0}} \right)^{C_{10}}. \quad (30)$$

Currently, the condenser and evaporator fan and evaporator pump inverter losses are modeled by the same simple linear relation:

$$E_{inv} = C_{11}E_{mot} + C_{12}. \quad (31)$$

Note that the modular structure allows use of any inverter loss model of the form:

$$E_{inv} = f(E_{mot}, f). \quad (32)$$

For a compressor inverter sensitive to motor power factor, a compressor model that returns power factor can be implemented within the modular framework, and for an inverter cooled by the condenser air stream, its sensitivity to condenser airflow rate and temperature can also be modeled.

The total fan and pump power is then calculated as the sum of the motor and inverter losses, where the constants $C_3, C_4, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, V_{w,0}, w_{p,0}$, and $E_{p,0}$ are generally determined from measurements and semi-empirical models as illustrated in Appendix C.

The evaporator and condenser sub-model variables are shown in Tables 2 and 3, respectively.

Table 2. Air-to-refrigerant and water-to-refrigerant evaporator sub-model variables.

Parameters	Evaporator and suction pipe geometry, material properties
Boundary condition	Zone (room) air temperature (T_z) or water inlet temperature ($T_{w,in}$), cooling rate ($\dot{Q}_e = \dot{Q}_{ep} + \dot{Q}_{esh}$), evaporator airflow rate (V_z) or evaporator water flow rate (m_w), and superheating temperature difference at the evaporator outlet (ΔT_{esh})
Inputs	Enthalpy at the evaporator inlet (h_{liq})
Unknowns	Refrigerant mass flow rate (m_{ref}), temperature at the evaporator inlet (T_{eI}), evaporator area percentage devoted to desuperheating (y_{esh}), pressure drop in the evaporating region (ΔP_{ep}), and pressure drop in the desuperheating region (ΔP_{esh})
Outputs	Refrigerant mass flow rate (m_{ref}), temperatures in the evaporator (T_{e1}, T_{e2}, T_{e3}), compressor inlet temperature ($T_{comp.in}$), compressor inlet pressure ($P_{comp.in}$), evaporator fan or pump power (E_e/E_p), and evaporator fan/pump speed (w_e/w_p)

Table 3. Condenser sub-model variables.

Parameters	Condenser and discharge pipe geometry, material properties
Boundary condition	Outside air temperature (T_o), condenser airflow rate (V_o), and condenser area percentage devoted to subcooling (y_{csc})
Inputs	Refrigerant mass flow rate (m_{ref}), condenser inlet temperature (T_{c1})
Unknowns	Condenser inlet pressure (P_{c1}), condenser area percentage devoted to condensation (y_{cp}), pressure drop in the desuperheating region (ΔP_{csh}), pressure drop in the condensing region (ΔP_{cp}), and pressure drop in the subcooling region (ΔP_{csc}),
Outputs	Temperatures in the condenser ($T_{c1}, T_{c2}, T_{c3}, T_{c4}$), condenser inlet pressure (P_{c1}), subcooling temperature difference at the condenser outlet (ΔT_{csc}), total heat exchanged on the condenser ($\dot{Q}_c = \dot{Q}_{csh} + \dot{Q}_{cp} + \dot{Q}_{csc}$), condenser fan power (E_c), and condenser fan speed (w_c)

Table 4. RMS errors for four different models.

	Model 1	Model 2	Model 3	Model 4
RMS	5.08%	4.94%	1.57%	5.02%

Compressor sub-model

The objective for the compressor sub-model is to calculate the shaft speed, input power, and outlet temperature for the given refrigerant mass flow rate, inlet temperature, inlet pressure, and outlet pressure. The compression process is usually described using the polytropic model for which the volume-pressure relation is modeled in terms of a compression exponent, n :

$$\frac{P_{comp_out}}{P_{comp_in}} = \left(\frac{v_{comp_in}}{v_{comp_out}} \right)^n. \quad (33)$$

For a real gas undergoing isentropic compression, thus no heat transfer and no dissipation during compression, the compression exponent is calculated as

$$n = n_s = \frac{\ln \frac{P_{comp_out}}{P_{comp_in}}}{\ln \frac{\rho_s}{\rho_{comp_in}}}, \quad (34)$$

where ρ_s is the refrigerant density evaluated at inlet entropy and outlet pressure.

The compressor sub-model uses the volumetric efficiency model, for which

$$m_{ref} = Df\rho_{comp_in}\eta_V. \quad (35)$$

Using Gayeski's (2010) experimental data for a rotary-piston type compressor, four volumetric efficiency models have been compared: (1) the model of Jähnig et al. (2000) for small reciprocating compressors, (2) Kim and Bullard's (2002) model for reciprocating and rotary compressors, (3) a simple model developed by the authors that accounts for back leakage losses, and (4) the model developed by Willingham (2009) for a scroll compressor. Al-

though two (Jähnig et al. 2000; Kim and Bullard 2002) of the four models have been developed for constant-speed compressors and would require a different set of constants for each speed, only one set of constants is calculated for each model. The constants are calculated with nonlinear regression of 77 measured points shown in Appendix D. Comparison results are shown in the Table 4.

Model 3, which accounts for re-expansion and back leakage losses, shows the best agreement with the measured data and has been chosen for the current compressor sub-model:

$$m_{ref} = [C_1 f \eta_V - C_3 (P_{comp_out} - P_{comp_in})] \rho_{comp_in}, \quad (36)$$

$$\eta_V = 1 - C_2 \left[\left(\frac{P_{comp_out}}{P_{comp_in}} \right)^{1/n_s} - 1 \right], \quad (37)$$

where the isentropic exponent for a real gas n_s is calculated using Equation 34. Calculated constants for this model are shown in Table 5.

The power model used for the compressor sub-model was developed by Jähnig et al. (2000). It calculates electrical power supplied to the motor ($E_{comp-mot}$) by introducing the combined efficiency (η_{comb}), which represents the ratio of the estimated work required for a polytropic compression process to the total power supplied to the motor:

$$E_{comp-mot} \eta_{comb} = m_{ref} \frac{n_s}{n_s - 1} \frac{P_{comp_in}}{\rho_{comp_in}} \left[\left(\frac{P_{comp_out}}{P_{suction}} \right)^{\frac{n_s-1}{n_s}} - 1 \right], \quad (38)$$

where

$$P_{suction} = P_{comp_in} (1 - C_4), \quad (39)$$

$$\eta_{comb} = C_5 + C_6 \exp(C_7 P_{comp_in}). \quad (40)$$

Table 5. Coefficients for volumetric efficiency model.

	Value	Unit	Physical compressor property
C_1	9.16787×10^{-6} (0.55945)	$\text{m}^3 (\text{in}^3)$	Effective displacement
C_2	3.17178×10^{-2}	—	Clearance volume fraction
C_3	2.42768×10^{-5}	m^3/kPa	Loss in effective displacement per unit back pressure

Table 6. Coefficients and RMS error for power model.

C_4	0
C_5	-0.41535
C_6	1.21272
C_7	-0.04012
RMS	3.88%

Values for the coefficients are listed in Table 6. For the compressor sub-model, $P_{comp.in}$ in Equation 40 has been replaced with pressure ratio $P_{comp.out}/P_{comp.in}$ since it was found that this reduces the root-mean-square (RMS) error from 11% to 3.88%.

The total compressor input power is then calculated as

$$E_{comp} = E_{comp-mot} + E_{comp-inv}, \quad (41)$$

where $E_{comp-inv}$ is from Equation 96 in Appendix C. Inverter parameters are reported in Appendix C.

The temperature at the compressor outlet is determined from the energy balance equation, neglecting the losses from the compressor to environment. These losses are small for the well-insulated compressor used in the test rig. A jacket loss correction can easily be applied for un-insulated compressors by estimating the portions of the jacket exposed internally to suction and discharge conditions.

The compressor energy input is used for the refrigerant enthalpy rise and for heating of the circulating compressor oil:

$$Q_{comp} = Q_{shaft} + Q_{oil}, \quad (42)$$

where

$$Q_{shaft} = m_{ref}(h_{comp.out} - h_{comp.in}), \quad (43)$$

$$Q_{oil} = m_{oil}C_{oil}(T_{comp.out} - T_{comp.in}). \quad (44)$$

The oil mass flow rate has been taken as 4% of the refrigerant mass flow rate, and that fraction remains constant for all calculations where oil is included.

Table 7. Compressor sub-model variables.

Parameters	Compressor constants
Inputs	Refrigerant mass flow rate (m_{ref}), compressor inlet temperature ($T_{comp.in}$), compressor inlet pressure ($P_{comp.in}$), and compressor outlet pressure ($P_{comp.out}$)
Unknowns and outputs	Compressor shaft speed (f), compressor input power (E_{comp}), and compressor outlet temperature ($T_{comp.in}$)

The compressor sub-model variables are shown in Table 7.

Model validation

The heat pump used to validate the accuracy of the HPM is a 2.5-kW mini-split heat pump with refrigerant R410A as the working fluid and a rotary-piston type compressor. Details about experimental measurements have been published in Gayeski et al. (2011), and the data summary can be found in Appendix D.

The comparisons between the HPM outputs and the experimental observations have been made at the component and system level. Certain experimentally measured values have served as the HPM inputs, and the other measured values served as the responses that the model is supposed to accurately predict. The RMS error and relative mean (RM) error are computed after making the errors dimensionless as follows. RMS and RM for temperature are defined as

$$RMS = \sqrt{\frac{\sum \left(\frac{T_m - T_{calc}}{T_{c,avg,m} - T_{e,avg,m}} \right)^2}{N_m}}, \quad (45)$$

$$RM = \frac{\sum \left| \frac{T_m - T_{calc}}{T_{c,avg,m} - T_{e,avg,m}} \right|}{N_m}. \quad (46)$$

RMS and RM for enthalpy are defined as

$$RMS = \sqrt{\frac{\sum \left(\frac{h_m - h_{calc}}{h_{c2,m} - h_{liq,m}} \right)^2}{N_m}}, \quad (47)$$

$$RM = \frac{\sum \left| \frac{h_m - h_{calc}}{h_{c2,m} - h_{liq,m}} \right|}{N_m}, \quad (48)$$

and RMS and RM for all other variables are defined as

$$RMS = \sqrt{\frac{\sum \left(\frac{X_m - X_{calc}}{X_m} \right)^2}{N_m}}, \tag{49}$$

$$RM = \frac{\sum \left| \frac{X_m - X_{calc}}{X_m} \right|}{N_m}, \tag{50}$$

where N_m is the number of points, i.e., number of distinct conditions tested. The range of conditions is summarized in Appendix D.

Component sub-model validation

Air-to-refrigerant heat exchanger validation

The air-to-refrigerant evaporator and condenser sub-models have been validated using the heat exchanger geometry data (summarized in Appendix B) and measured performance data. The experimental data used as evaporator sub-model inputs are cooling rate ($Q_{e,m}$), evaporator airflow rate ($V_{z,m}$), enthalpy at the evaporator inlet ($h_{liq,m}$), entering air temperature ($T_{z,m}$), and superheating temperature difference ($\Delta T_{esh,m}$). The experimental data used as condenser sub-model inputs are refrigerant mass flow rate ($m_{ref,m}$), compressor outlet temperature ($T_{comp\ out,m}$), condenser airflow rate ($V_{o,m}$), outside air temperature ($T_{o,m}$), and subcooling temperature difference ($\Delta T_{csc,m}$).

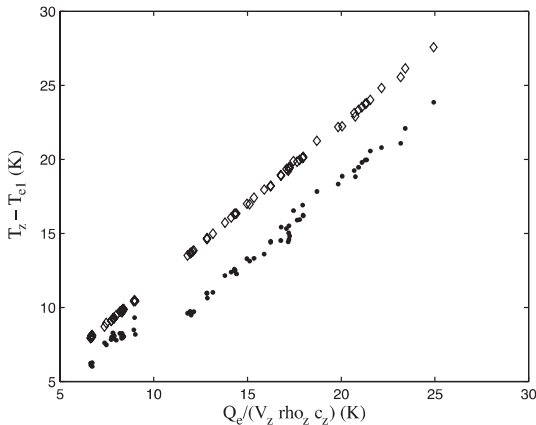


Figure 2. Temperature difference between the refrigerant evaporating temperature and zone temperature as a function of $Q_e/(V_z\rho_z c_{p,z})$; calculated values are marked with diamonds, and measured values are marked with dots.

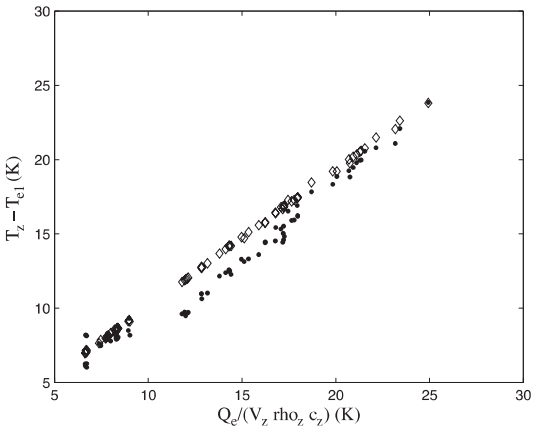


Figure 3. Temperature difference if the evaporator airflow is taken to be 20% larger than inferred value; calculated values are marked with diamonds, and inferred values are marked with dots.

Figure 2 shows the measured and calculated temperature differences between the refrigerant evaporating temperature and entering air temperature; there is a systematic difference between measurement and prediction. The evaporator airflow rate is not directly measured but rather inferred from the measured cooling rate and the air temperature difference. The average air temperature difference is susceptible to nonuniform airflow and temperature distributions over the face area and to errors in the cooling rate measurements. The analysis has shown that an airflow rate 20% larger than that inferred from the measurements would bring the calculated and measured temperature differences into very good agreement (Figure 3). Hence, 20% larger airflow rate than the one calculated from the measurements will be used in subsequent analyses. The RMS errors for evaporator sub-model output variables are shown in Table 8. It is important to notice that an error of a few degrees in the evaporating temperature can cause significant errors in the compressor inlet pressure, due to the strong dependence of pressure drop on density at low evaporating temperatures.

Table 8. Errors for the evaporator sub-model output variables.

	RMS (%)	RM (%)
m_{ref}	0.80	0.72
T_{e1}	3.26	2.75
T_{e3}	3.26	2.75
P_{e1}	2.22	1.82
$P_{comp\ in}$	2.73	2.22

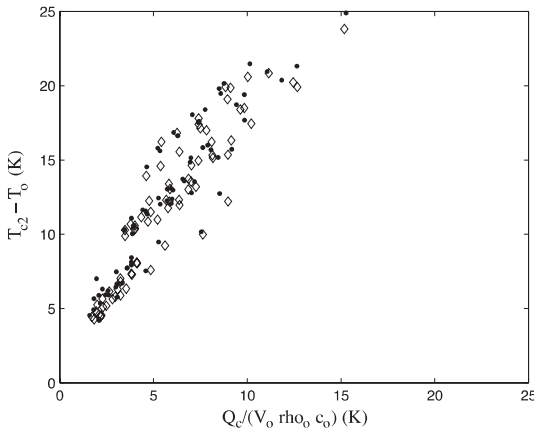


Figure 4. Temperature difference between the refrigerant condensing temperature and ambient temperature as a function of $Q_c/(V_o \rho_o c_o)$; the calculated values are marked with diamonds, and the measured values are marked with dots.

The calculated temperature differences between the refrigerant condensing temperature and outside air temperature show good agreement with measured data (Figure 4). Although condensing temperature errors influence the predicted compressor outlet pressure, this relationship is not as strong as the dependency between the evaporating temperature errors and compressor inlet pressure errors. Two main reasons are high absolute condensing pressures and a significantly lower pressure drop in the condenser than in the evaporator due to larger vapor densities leading to lower velocities. For example, the calculated condenser pressure drop has an order of magnitude 10 kPa to 60 kPa, which is approximately 1.5% of the condensing pressure, whereas the evaporating pressure has an order of magnitude 10 kPa to 200 kPa, approximately 1% to 20% of the evaporating pressure. The RMS errors for the condenser sub-model are shown in Table 9.

Water-to-refrigerant heat exchanger verification

Experimental data used for the chilled-water evaporator sub-model verification have been pub-

Table 9. Errors for the condenser sub-model output variables.

	RMS (%)	RM (%)
Q_c	4.29	3.47
$P_{comp.out}$	1.28	0.98
T_{c2}	1.52	1.20
T_{c4}	1.56	1.30
h_{liq}	0.69	0.55

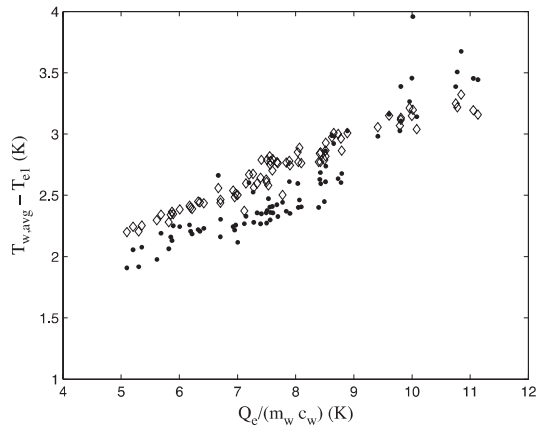


Figure 5. Temperature difference between the refrigerant evaporating temperature and water inlet temperature as a function of $Q_c/(m_w c_w)$; calculated values are marked with diamonds, and measured values are marked with dots.

lished in Gayeski (2010). Although the data were not originally collected for validation purposes and cannot be used for a detailed sub-model validation, they are useful for initial verification of the chilled-water evaporator sub-model. The experimental data used as the evaporator sub-model inputs are cooling rate ($Q_{e,m}$), water flow rate ($m_{w,m}$), enthalpy at the evaporator inlet ($h_{liq,m}$), entering water temperature ($T_{w,in,m}$), and superheating temperature difference ($\Delta T_{esh,m}$).

It can be seen from the evaporating temperature comparison (Figure 5) and the RMS errors (Table 10) that the predicted output variables show good agreement with measured data.

Compressor sub-model verification

The experimental data used as compressor sub-model inputs are the refrigerant mass flow rate ($m_{ref,m}$), compressor inlet temperature ($T_{comp.in,m}$), compressor inlet pressure ($P_{comp.in,m}$), and compressor outlet pressure ($P_{comp.out,m}$).

It can be seen from the compressor output variables in Figures 6 and 7 that for the majority of

Table 10. Errors for the evaporator sub-model output variables.

	RMS (%)	RM (%)
m_{ref}	0.30	0.22
T_{e1}	1.56	1.31
T_{e3}	4.20	4.12
$P_{comp.in}$	4.42	4.34

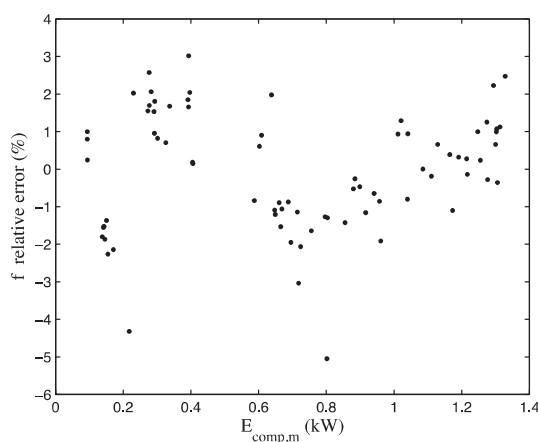


Figure 6. Compressor shaft speed relative error.

points, the relative errors range between $\pm 3\%$ for speed and $\pm 10\%$ for power. Although it is important to predict compressor power as accurately as possible, it is not reasonable to expect lower errors from in situ (as opposed to test stand) measurements and with the simple compressor model used in the HPM. The HPM tends to over-predict the compressor outlet temperatures by as much as 8 K; however, these discrepancies do not significantly influence COP predictions. The outlet temperature over-predictions can be the consequence of neglecting the compressor shell heat losses or assuming too low a circulating oil fraction. Errors in the measured suction temperature and suction and discharge pressures also influence estimates of the calculated inlet and outlet enthalpy and outlet temperature.

The errors for the compressor sub-model are shown in Table 11.

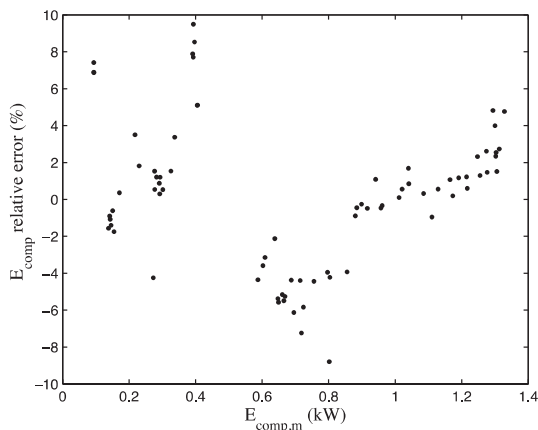


Figure 7. Compressor power relative error.

Table 11. Errors for the compressor sub-model output variables.

	RMS (%)	RM (%)
f	1.57	1.29
E_{comp}	3.88	2.99
T_{comp_out}	4.33	3.54

System validation

The measured values used as the system model inputs are cooling rate ($\dot{Q}_{e,m}$), evaporator airflow rate ($V_{z,m}$), condenser airflow rate ($V_{o,m}$), zone (room) air temperature ($T_{z,m}$), ambient air temperature ($T_{o,m}$), superheating temperature difference at the evaporator outlet ($\Delta T_{esh,m}$), and subcooling temperature difference at the condenser outlet ($\Delta T_{csc,m}$).

It can be seen in Figure 8 that the errors in COP predictions are within $\pm 10\%$ bounds for 68 of 77 points and that the errors are proportional to the compressor power errors shown in Figure 7.

An interesting example is the validation performed with the same inputs as in the validation section but without the pressure drop calculations. This change can result in significant errors for the evaporator sub-model, where pressure drops account for up to 20% of the evaporating pressure. The errors in the evaporator outlet pressure then lead to serious under-predictions of the compressor speed and power consumption. The significant increases in RMS and RM errors for compressor inlet pressure, compressor power, and COP are shown in Table 12b.

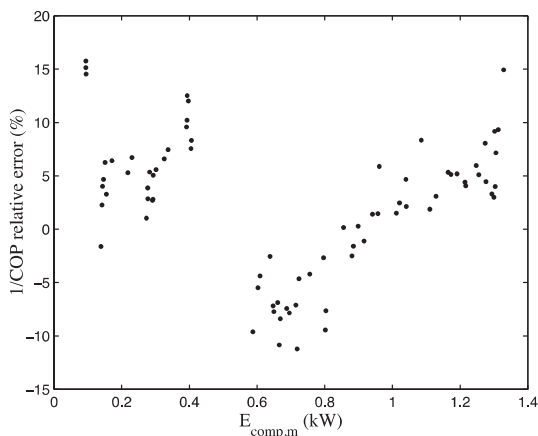


Figure 8. 1/COP relative error.

Table 12. RMS errors for the system output variables.

	(a) With pressure drop calculations		(b) Without pressure drop calculations	
	RMS (%)	RM (%)	RMS (%)	RM (%)
m_{ref}	0.51	0.40	1.63	1.38
T_{e1}	3.26	2.75	3.11	2.57
T_{e3}	3.26	2.75	3.11	2.58
P_{e1}	2.22	1.82	2.13	1.75
$P_{comp.in}$	2.66	2.16	11.83	9.12
$P_{comp.out}$	1.35	1.04	1.56	1.22
P_{rat}	0.0405	0.0046	0.75	0.085
f	3.76	3.10	11.71	9.45
E_{comp}	7.10	5.82	14.03	11.48
$T_{comp.out}$	13.01	10.99	26.48	23.04
Q_c	3.89	3.26	4.59	4.06
T_{c2}	1.62	1.28	1.71	1.38
T_{c4}	1.71	1.41	1.60	1.30
h_{liq}	0.74	0.59	0.71	0.55
COP	6.09	4.97	17.45	13.95

Conclusion

The steady-state HPM is developed from first-principle models of the evaporator and condenser and a semi-empirical compressor model. The required model inputs are the cooling rate, zone (or water inlet) and ambient temperatures, evaporator and condenser airflows (or water flows), desired superheating temperature difference, and subcooling temperature difference (or condenser area percentage devoted to subcooling). For a given set of component parameters and inputs, the model will solve for power consumption, mass flow rate, compressor speed, condenser heat rate, temperatures, and pressures in the evaporator; temperatures and pressures in the condenser; compressor inlet and outlet pressures and pressure drops in the condenser, the evaporator, and compressor suction and discharge lines.

Two evaporator sub-models that describe finned-tube air-to-refrigerant and brazed-plate water-to-refrigerant heat exchangers are presented using the heat balance equations and ϵ -NTU method for the evaporating and superheating region. The finned-tube refrigerant-to-air condenser is modeled in a similar manner, except it consists of the desuperheating, condensing, and subcooling regions. The heat transfer coefficients can be given as constants for each heat exchanger or calculated separately for the air/water stream and two-phase and single-phase

refrigerant flows. The sub-models allow pressure drops to be calculated or ignored. Because refrigerant charge balance has not been modeled, and a liquid receiver is assumed to maintain the necessary charge balance, the user needs to specify the evaporator superheat and condenser subcooling.

The compressor sub-model calculates the compressor speed, compressor power, and discharge temperature for a given mass flow rate, compressor inlet state, and outlet pressure. The shaft speed is calculated using a volumetric efficiency model, and the compressor power is calculated as the power required for isentropic work, corrected by the combined efficiency that takes into account losses in the compressor and motor. The compressor outlet temperature is calculated from the compressor heat balance, through which the lubricating oil is assumed to pass in a constant mass fraction.

The model results for a 2.5-kW mini-split heat pump are compared against 77 measured points collected by Gayeski (2010). Some measured variables have been used as the inputs to the HPM, and other measured values are used for comparison with the model outputs. The measured and predicted values were compared at the component and system levels. The relative error in COP predictions is better than $\pm 10\%$ range for 68 of the 77 test points.

Model accuracy is strongly related to the evaporating temperature and pressure drop predictions. Neglecting the pressure drop, especially pressure

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drop in the evaporator and suction pipe, can have significant consequences for the COP and shaft speed prediction because it influences the compressor suction state. When pressure drop was neglected, with all other inputs unchanged, the RMS error in the COP prediction has increased from approximately 6.1% to 17.5%.

The main goal—to develop an HPM suitable for a wide range of operating conditions and loads and also fast enough to embed in an optimization shell—is achieved using conservation equations and a modified volumetric efficiency compressor model. A modular approach gives the possibility of choosing between different simulation options and makes the model easy to expand and customize. The model can be used for air-to-air, water-to-air, or water-to-water, single compressor or multiple parallel compressor applications. Examples of these capabilities will be included in an article in preparation for several different heat pump variations and three optimization variables (evaporator and condenser fan speeds and condenser subcooling fraction). Currently, the program runs 20 s to 40 s for a single operating point, depending on the complexity (pressure drop calculations, variable heat transfer coefficients, subcooling, etc.) and has a potential to become much faster, because the main limitation for faster execution time is the usage of REFPROP in refrigerant property calculations (30 s out of 40 s, according to a MATLAB profiler). Future work will concentrate on compressor and inverter model improvements and additional component development, as well as the reduction in computational time required for convergence.

Acknowledgments

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Nomenclature

English letter symbols

<i>A</i>	= area, m ²
<i>C</i>	= fluid thermal capacitance rate, W/K
<i>C</i>	= constant, —
<i>c_p</i>	= specific heat at constant pressure, J/(kgK)

<i>c_v</i>	= specific heat at constant volume, J/(kgK)
<i>d</i>	= pipe diameter, m
<i>D</i>	= effective or actual displacement, m ³
<i>e</i>	= pipe roughness, m
<i>E</i>	= power, W
<i>f</i>	= compressor shaft speed, Hz
<i>f</i>	= friction factor, —
<i>g</i>	= acceleration due to gravity, m/s ²
<i>G</i>	= mass velocity, kg/(m ² s)
<i>h</i>	= convective heat transfer coefficient, W/(m ² K)
<i>h</i>	= specific enthalpy, kJ/kg
<i>k</i>	= exponent for ideal gas under isentropic compression, —
<i>k</i>	= thermal conductivity, W/mK
<i>L</i>	= heat exchanger pipe length, m
<i>m</i>	= mass flow rate, kg/s
<i>M</i>	= molar mass, kg/kmol
<i>n</i>	= compression exponent, —
<i>N</i>	= number of measurement points, —
<i>N</i>	= number of heat exchanger pipe rows, —
<i>n_s</i>	= exponent for real gas under isentropic compression, —
NTU	= number of transfer units, —
<i>P</i>	= pressure, Pa
<i>r</i>	= pipe radius, m
RMS	= square root of arithmetic mean of the squared residuals
<i>Q</i>	= heat rate, W
<i>s</i>	= entropy, J/(kgK)
<i>t</i>	= heat exchanger pipe thickness, m
<i>T</i>	= temperature, K
<i>U</i>	= overall heat transfer coefficient, W/(m ² K)
<i>UA</i>	= heat exchanger UA value, W/K
<i>v</i>	= specific volume, m ³ /kg
<i>V</i>	= volumetric air or water flowrate, m ³ /s
<i>w</i>	= fan or pump speed, rpm
<i>w</i>	= mass fraction, kg/kg
<i>w</i>	= mean velocity of fluid, m/s
<i>x</i>	= vapor quality, kg _v /kg
<i>y</i>	= heat exchanger area percentage devoted to desuperheating, subcooling, evaporation, or condensation, —
<i>Z</i>	= resistance factor, —

Greek letter symbols

<i>β</i>	= chevron angle, °
<i>Δ</i>	= difference, —

ε	= cross-flow effectiveness, —
η	= efficiency, —
μ	= dynamic viscosity, Ns/m ²
ρ	= density, kg/m ³
ϕ	= enlargement factor, —
ψ	= mole fraction, kmol/kg

Subscripts

<i>air</i>	= air
<i>avg</i>	= average
<i>c</i>	= whole condenser
<i>c1, c4</i>	= condenser inlet and outlet
<i>c2, c3</i>	= beginning and end of condensing region in condenser
<i>calc</i>	= calculated
<i>comp</i>	= compressor
<i>comp.in</i>	= compressor inlet
<i>comp.out</i>	= compressor outlet
<i>cp, csc, csh</i>	= condensing, subcooling, and desuperheating region of condenser
<i>d</i>	= discharge pipe
<i>discharge</i>	= compressor discharge state
<i>fg</i>	= change from saturated liquid to saturated vapor
<i>fin</i>	= fin
<i>e</i>	= whole evaporator
<i>e1, e3</i>	= evaporator inlet and outlet
<i>e2</i>	= end of evaporating region in evaporator
<i>ep, esh</i>	= evaporating and superheating region of evaporator
<i>ex</i>	= external
<i>h</i>	= hydraulic diameter
<i>in</i>	= internal
<i>inv</i>	= inverter
<i>liq</i>	= evaporator inlet and condenser outlet state
<i>liq</i>	= liquid
<i>m</i>	= measured
<i>max</i>	= maximum
<i>min</i>	= minimum
<i>mix</i>	= refrigerant and oil mixture
<i>o</i>	= outside
<i>o</i>	= oil
<i>out</i>	= outside
<i>ref</i>	= refrigerant
<i>s</i>	= suction pipe
<i>s</i>	= surface
<i>sc</i>	= subcooling region of condenser
<i>sph</i>	= single phase
<i>suction</i>	= compressor suction state
<i>tph</i>	= two phase

<i>tr</i>	= trial value
<i>v</i>	= volumetric
<i>w</i>	= water
<i>w,e2</i>	= end of evaporating region in evaporator, water side
<i>w,in</i>	= water inlet side
<i>w,out</i>	= water outlet side
<i>z</i>	= zone

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Appendix A: Heat transfer and pressure drop correlations

Heat transfer coefficient correlations

Finned-tube heat exchanger

The product of heat conductance and the heat exchanger area UA for a finned-tube heat exchanger is calculated as

$$\frac{1}{UA} = \frac{1}{\eta_s h_{ex} A_{ex}} + \frac{t}{k A_m} + \frac{1}{h_{in} A_{in}}, \quad (51)$$

where the surface efficiency η_s is given by (McQuiston et al. 2004)

$$\eta_s = 1 - \frac{A_{fin}}{A_{fin} + A_{ex}} (1 - \eta_{fin}), \quad (52)$$

and the fin efficiency is calculated using the Schmidt method for continuous-plate fins (McQuiston et al. 2004):

$$\eta_{fin} = \frac{\tanh(m r_{out} \phi)}{m r_{out} \phi}, \quad (53)$$

$$m = \frac{2h_{ex}}{ky}, \quad (54)$$

$$\phi = \left(\frac{R_{equiv}}{r_{out}} - 1 \right) \left[1 + 0.35 \ln \left(\frac{R_{equiv}}{r_{out}} \right) \right], \quad (55)$$

$$\frac{R_{equiv}}{r_{out}} = 1.28 \left(\frac{M}{r_{out}} \right) \left(\frac{L}{M} - 0.2 \right)^{0.5}; \quad (56)$$

M and L are defined in Figure A.1, where L is always selected to be greater than or equal to M .

The external heat transfer coefficient is calculated using Gray and Webb (1986) correlations for a

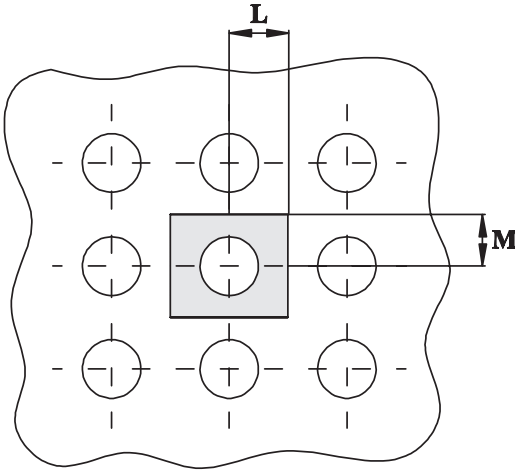


Figure A.1. Rectangular tube array (McQuiston et al. 2004).

plate finned-tube heat exchanger and geometry parameters defined in Figure A.2. The correlation is valid for smooth fins and any number of tube rows:

$$h_{ex} = \frac{j_N}{Pr^{2/3}} c_p \rho_{air} w_{air}, \quad (57)$$

where for four or more tube rows ($N \geq 4$),

$$j_4 = 0.14 Re^{-0.328} \left(\frac{s_v}{s_h} \right)^{-0.502} \left(\frac{s_d}{d_{out}} \right)^{0.0312}, \quad (58)$$

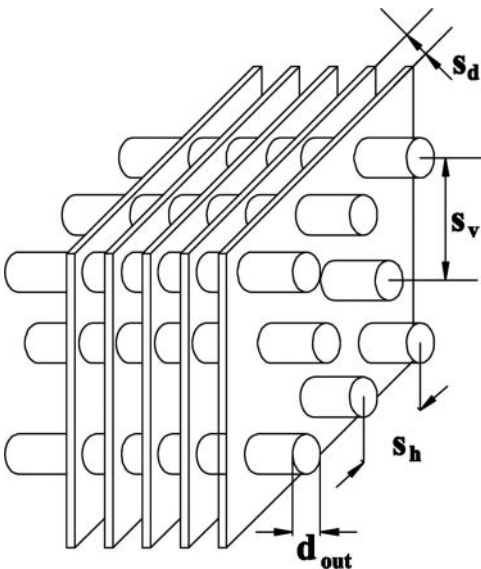


Figure A.2. Heat exchanger geometry.

and for less than four rows,

$$\frac{j_N}{j_4} = 0.991 \left[2.24 Re^{-0.092} \left(\frac{N}{4} \right)^{-0.031} \right]^{0.607(4-N)}. \quad (59)$$

The *internal heat transfer coefficient for a single-phase fluid* is calculated using forced-convection correlations from ASHRAE fundamentals (ASHRAE 2009):

$$\begin{aligned} \text{for laminar flow } (Re < 2300): Nu &= \frac{h_{in} d_{in}}{k} \\ &= 1.86 \left(\frac{Re Pr}{L/d_{in}} \right)^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14}; \end{aligned} \quad (60)$$

$$\text{for turbulent flow: } Nu = \frac{h_{in} d_{in}}{k} = 0.023 Re^{4/5} Pr^x, \quad (61)$$

where $x = 0.4$ if the air temperature is higher than the refrigerant temperature (in the evaporator), and $x = 0.3$ if the air temperature is lower (condenser).

The *internal heat transfer coefficient for evaporation* is calculated using a correlation for refrigerants evaporating in horizontal tubes from ASHRAE Fundamentals (ASHRAE 2001):

$$\begin{aligned} Nu &= \frac{h_{in} d_{in}}{k_{liq}} \\ &= 0.0082 \left[\left(\frac{G_{ref} d_{in}}{\mu_{liq}} \right)^2 \frac{(x_{out} - x_{in}) h_{fg}}{L g} \right]^{0.4}, \end{aligned} \quad (62)$$

where

$$G_{ref} = \frac{m_{ref}}{d_{in}^2 \pi / 4}. \quad (63)$$

The *internal heat transfer coefficient for condensation* is calculated from correlations for film condensation from ASHRAE Fundamentals (ASHRAE 2001):

$$\begin{aligned} Nu_{liq} &= \frac{h_{in} d_{in}}{k_{liq}} \\ &= 13.8 (Pr_{liq})^{1/3} \left(\frac{h_{fg}}{c_{p,liq} (T_{sat} - T_{wall})} \right)^{1/6} \\ &\quad \times \left[\frac{d_{in} G_{ref}}{\mu_{liq}} \left(\frac{\rho_{liq}}{\rho_{vap}} \right)^{1/2} \right]^{0.2}. \end{aligned} \quad (64)$$

Equation 64 is valid for

$$\frac{d_{in} G_{ref}}{\mu_{liq}} < 1,000 \text{ and } \frac{d_{in} G_{ref}}{\mu_{liq}} \left(\frac{\rho_{liq}}{\rho_{vap}} \right)^{1/2} < 20,000.$$

When

$$20,000 < \frac{d_{in} G_{ref}}{\mu_{liq}} \left(\frac{\rho_{liq}}{\rho_{vap}} \right)^{1/2} < 100,000,$$

Equation 65 is used instead of Equation 64:

$$\begin{aligned} Nu_{liq} = \frac{h d_{in}}{k_{liq}} &= 0.1 \left(\frac{c_{p,liq} \mu_{liq}}{k_{liq}} \right)^{1/3} (Pr_{liq})^{1/3} \\ &\times \left(\frac{h_{fg}}{c_{p,liq} (T_{sat} - T_{wall})} \right)^{1/6} \\ &\times \left[\frac{d_{in} G_{ref}}{\mu_l} \left(\frac{\rho_{liq}}{\rho_{vap}} \right)^{1/2} \right]^{2/3} \end{aligned} \quad (65)$$

Since the evaporator is divided into two regions and the condenser into three regions, the areas for each region are expressed as a fraction (y_{ep} , y_{esh} , y_{cp} , y_{csh} , y_{csc}) of the total evaporator or condenser area:

$$A_{e,i} = d_e \pi L_{e,i} = d_e \pi y_{e,i} L_e, \quad (66)$$

$$A_{c,i} = d_c \pi L_{c,i} = d_c \pi y_{c,i} L_c. \quad (67)$$

Brazed-plate heat exchanger

The product of heat conductance and the heat exchanger area UA for the brazed-plate heat exchanger is calculated as

$$\frac{1}{UA} = \frac{1}{h_{ex} A_e} + \frac{t}{k A_e} + \frac{1}{h_{in} A_e}. \quad (68)$$

The *heat transfer coefficient for a single-phase fluid* is calculated for the water side and the evaporator superheating region using the Wanniarachchi et al. equation (ASHRAE 2009) for low Re numbers and Muley and Manglik's (1999) equation for high Re numbers:

$$\begin{aligned} \text{for } Re \leq 10^4: \quad Nu &= \frac{h d_h}{k} = (Nu_l^3 + Nu_t^3)^{1/3} \\ &\times Pr^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.17}, \end{aligned} \quad (69)$$

where

$$Nu_l = 3.65(\beta)^{-0.455} (\phi)^{0.661} Re^{0.339}, \quad (70)$$

$$Nu_t = 12.6(\beta)^{-1.142} (\phi)^{1-m} Re^m, \quad (71)$$

$$m = 0.646 + 0.0011(\beta), \quad (72)$$

for $Re > 10^4$:

$$\begin{aligned} Nu &= \frac{h d_{in}}{k} = [0.2668 - 0.006967(90 - \beta) \\ &+ 7.244 \times 10^{-5}(90 - \beta)^2] \times (20.78 \\ &- 50.94\phi + 41.16\phi^2 - 10.51\phi^3) \\ &\times Re^{[0.728 + 0.0543 \sin[\pi(90 - \beta)/45] + 3.7]} \\ &\times Pr^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}. \end{aligned} \quad (73)$$

The *heat transfer coefficient for evaporation* is calculated using the Cooper's (1984) equation:

$$\begin{aligned} h_{in} &= 55 p^{*(0.12 - 0.2 \log_{10} R_p)} (-\log_{10} p^*)^{-0.55} \\ &\times q^{0.67} M^{-0.5}, \end{aligned} \quad (74)$$

where $p^* = p/p_{cr}$ is reduced pressure, R_p (μm) is roughness, M is molecular weight, and q (W/m^2) is heat flux.

Pressure drop correlations

The pressure drops inside the heat pump have been modeled for the evaporator, condenser, suction pipe, and discharge pipe, separately for single-phase and two-phase flow. The lengths of each evaporator region (L_{ep} and L_{esh}) and condenser region (L_{cp} , L_{csh} , and L_{csc}) are calculated as the fractions of the total evaporator and condenser pipe length, as shown in Equations 66 and 67.

Pressure drop for single-phase flow is calculated from the Darcy-Weisbach equation (ASHRAE 2009):

$$\Delta p_{sph} = \left(f \frac{L}{d_h} + \sum Z \right) \frac{G^2}{2\rho_{avg}} \quad (75)$$

where the resistance factor Z accounts for local pressure drops (U-turns, valves).

The friction factor in the pipes for a finned-tube heat exchanger is calculated from the Colebrook

equation (ASHRAE 2009):

$$\frac{1}{\sqrt{f}} = 1.74 - 2 \log \left(\frac{2e}{d_{in}} + \frac{18.7}{Re\sqrt{f}} \right), \quad (76)$$

while the friction factor for a brazed-plate heat exchanger is calculated using the Wanniarachchi et al. correlation (ASHRAE 2009) for low Re numbers and the Muley and Manglik (1999) equation for high Re numbers:

$$\text{for } Re \leq 10^4: \quad f = (f_l^3 + f_t^3)^{1/3}, \quad (77)$$

where

$$f_l = 1774(\beta)^{-1.026}(\phi)^2 Re^{-1}, \quad (78)$$

$$f_t = 46.6(\beta)^{-1.08}(\phi)^{1+p} Re^{-p}, \quad (79)$$

$$p = 0.00423(\beta) + 0.0000223(\beta)^2; \quad (80)$$

for $Re > 10^4$:

$$\begin{aligned} f = & [2.917 - 0.1277(90 - \beta) + 2.016 \\ & \times 10^{-3}(90 - \beta)^2] \times (5.474 - 19.02\phi \\ & + 18.93\phi^2 - 5.341\phi^3) \\ & \times Re^{-[0.2+0.0577 \sin[\pi(90-\beta)/45]+2.1]} \end{aligned} \quad (81)$$

Pressure drop for two-phase flow in a finned-tube heat exchanger is modeled using pressure drop correlations developed by Choi et al. (1999). The pressure drop correlation is an improved version of the relatively older Pierre's model, which is still often used because of its simplicity and validity. The model includes the oil influence for refrigerant/lubricant mixtures.

The pressure drop due to friction and acceleration in the pipes is calculated as:

$$\Delta p_{ph,f+a} = \left[\frac{f L (v_{out} - v_{in})}{d_{in}} + (v_{out} - v_{in}) \right] G^2, \quad (82)$$

where

$$f = 0.00506 Re_{liq}^{-0.0951} K_f^{0.1554}, \quad (83)$$

$$Re_{liq} = \frac{G d_{in}}{\mu_{mix}}, \quad (84)$$

$$K_f = \frac{\Delta x h_{fg}}{L g}. \quad (85)$$

Modifications that have been made to accommodate the influence of oil on vapor quality, and liquid Reynolds number can be found in Choi et al. (1999). In the HPM, the oil mass flow rate has been taken as 4% of the refrigerant mass flow rate, the same as in the compressor sub-model.

The model developed by Choi et al. (1999) does not investigate the loss due to the flow turn in the 180° return bends. However, in Pierre's original model, that loss is accounted for using the resistance factor Z of magnitude 0.7 to 1.0, where the higher value represents the case when oil is present:

$$\Delta p_{ph,U-turn} = \sum Z \frac{G_{ref}^2}{2 \rho_{ref}} \quad (86)$$

The total pressure drop in the HPM for two-phase flow is then calculated as:

$$\Delta p_{ph,f+a} = \Delta p_{ph,f+a} + \Delta p_{ph,U-turn}. \quad (87)$$

Pressure drops for two-phase flow in a brazed-plate heat exchanger are calculated using Equation 75, where the friction factor f is calculated from Hsieh and Lin's (2002, 2003) correlation for the refrigerant R-410A. The correlation is valid for $2000 < Re < 12,000$ and $0.0002 < Bo < 0.002$:

$$f = 23820 Re_{eq}^{-1.12}, \quad (88)$$

where

$$Re_{eq} = \frac{G_{eq} d_h}{\mu_l}, \quad (89)$$

$$G_{eq} = G \left[(1 - x_{avg}) + x_{avg} \left(\frac{\rho_l}{\rho_g} \right)^{1/2} \right]. \quad (90)$$

The average density for Equation 75 is calculated as

$$\rho_{avg} = \left[\left(\frac{x_{avg}}{\rho_g} \right) + \frac{(1 - x_{avg})}{\rho_l} \right]^{-1}. \quad (91)$$

Appendix B: Geometry data

Geometry data for each component are given in Tables B.1 through Table B.3.

Table B.1. Geometry data for air-to-refrigerant heat exchangers.

	Evaporator	Condenser
Length, m (in.)	0.62 (24.41)	0.86 (33.86)
Height, m (in.)	0.34 (13.39)	0.50 (19.69)
Depth, m (in.)	0.03 (1.18)	0.022 (0.87)
Fin/meter (Fin/in.)	787 (20)	709 (18)
Total number of fins	488	625
Number of tubes	16 + 16 (two branches)	12 + 12 (two branches)
Number of tube rows	2	1
Total tube length, m (ft)	19.84 (65.09)	20.57 (67.49)
Tube inside diameter, m (in.)	0.0052 (0.205)	0.0049 (0.193)
Tube outside diameter, m (in.)	0.0068 (0.268)	0.0065 (0.256)
Inside area, m ² (ft ²)	0.296 (3.187)	0.307 (3.305)
Outside area, m ² (ft ²)	0.421 (4.533)	0.422 (4.541)
Fin area, m ² (ft ²)	7.312 (78.704)	12.6561 (136.229)
Fin thickness, m (in.)	1.02×10^{-4} (4.02×10^{-3})	7.62×10^{-5} (3.00×10^{-3})
M, m (in.)	0.0075 (0.30)	0.010 (0.39)
L, m (in.)	0.009 (0.35)	0.011 (0.43)
Vertical distance between tubes s_v , m (in.)	0.018 (0.71)	0.020 (0.79)
Horizontal distance between tubes s_h , m (in.)	0.015 (0.6)	0
Distance between fins s_d , m (in.)	1.17×10^{-3} (0.046)	1.34×10^{-3} (0.053)
Minimum flow area, m ² (ft ²)	0.03 (0.32)	0.27 (2.91)
Number of U-turns	16 + 16 (two branches)	12 + 12 (two branches)

Table B.2. Geometry data for water-to-refrigerant heat exchanger.

	Evaporator
Length, m (in.)	0.032 (1.26)
Height, m (in.)	0.458 (18.03)
Depth, m (in.)	0.086 (3.39)
Number of plates	10
Number of water channels	5
Number of refrigerant channels	4
Plate thickness, m (in.)	5×10^{-4} (0.0197)
Distance between plates, m (in.)	2.1×10^{-4} (0.0083)
Chevron angle, °	45
Enlargement factor	1.17
Roughness, m (in.)	1.5×10^{-6} (5.9×10^{-5})
Total area, m ² (ft ²)	0.279 (3)

Table B.3. Geometry data for suction and discharge pipe.

	Suction pipe	Discharge pipe
Length, m (ft)	3 (9.8)	1.1 (3.6)
Pipe inside diameter, m (in.)	0.0079 (0.311)	0.0079 (0.311)
Local pressure drop factor	2	2

Appendix C: Flow speed, power speed, and inverter loss models

Above a certain Reynolds number, the momentum imparted by a passing blade to the airstream is sufficient to overcome viscous forces. In the flow regime between the transition Reynolds number and a tip Mach number of about 0.5, the flow rate is

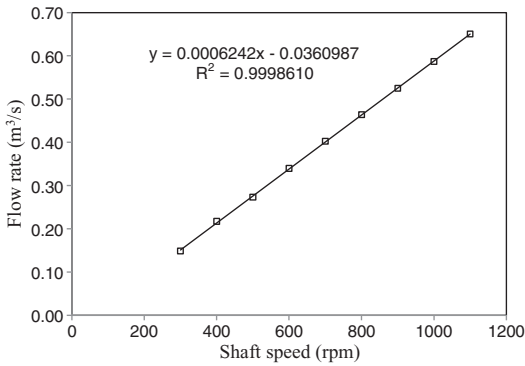


Figure C.1. Condenser fan flow rate versus fan speed.

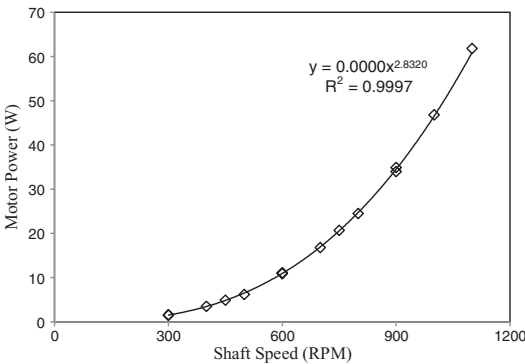


Figure C.2. Condenser motor power versus fan speed.

directly proportional to impeller speed. A typical condenser fan flow-speed (m^3/s -rpm) relation based on 18-point traverses taken at $x = [300:100:1100]$ rpm and shown in Figure C.1 is given by (Gayeski

2010).

$$V_o = 0.000624w_c - 0.0361. \quad (92)$$

For the same condenser fan, a power-speed (W-rpm) relation based on power measurements taken over the same speed range, as shown in Figure C.2, is given (Gayeski 2010) by:

$$E_{c-mot} = 1.48 * 10^{-7}(w_c)^{2.832}. \quad (93)$$

Pump flow speed and power-speed relations are modeled using simple power laws, since detailed experimental measurements for the pump were not performed. Pump flow rate and power are measured at just one operating speed $w_{p,0}$, and the measured values are used for the coefficients $V_{w,0}$ and $E_{w,0}$:

$$V_w = V_{w,0} \left(\frac{w_p}{w_{p,0}} \right), \quad (94)$$

$$E_{p-mot} = 19.5 \left(\frac{V_w}{1.32} \right)^3. \quad (95)$$

The compressor inverter loss is important, especially in low-speed, low-pressure-ratio operation. Details of inverter performance are complex (Bierhoff and Fuchs 2004; Rajapakse 2005). However, semi-empirical models exist to predict inverter loss as a function of carrier frequency, voltage, and fundamental frequency presented to the motor, motor load in terms of direct and reactive RMS currents, and inverter heat sink temperature. Such models can be based on analytical or numerical models using

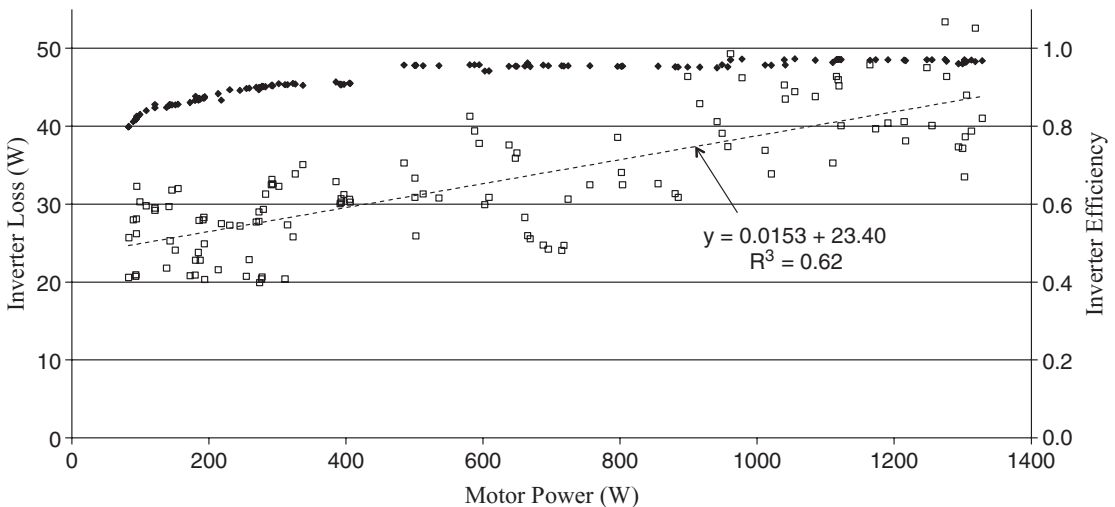


Figure C.3. Inverter efficiency (dots) and loss (squares) as function of compressor motor power.

parameters typically given by manufacturers plus some measured test results (Sheng et al. 2000). Alternatively, one can fit semi-empirical models for special cases, e.g., fixed DC-bus voltage, fixed carrier frequency, and fixed V/f relation, using only measured data.

To satisfy the immediate objectives, the simplest useful model in which loss is a function only of motor load in watts has been implemented. The loss intercept is not zero in part because a portion of the inverter loss is associated with reactive current, and, while true power delivered to the motor covers a wide range (100 W to 1300 W), the reactive current and its associated I^2R and forward voltage losses are relatively constant. The measured compressor inverter loss is plotted against motor load $E_{comp-mot}$ in Figure C.3. The regression line from

these measurements (Gayeski 2010) is given by

$$E_{comp-inv} = 0.015E_{comp-mot} + 23.4. \quad (96)$$

Note that according to these measurements, the inverter is about 97% efficient at a motor load of 1300 W, 95% at 600 W, and 86% at 200 W. Some of the scatter is explained by compressor speed and variations in condenser discharge temperatures to which the inverter heat sink is exposed.

Appendix D: Experimental data conditions

Experimental data conditions are given in Tables D.1 and D.2.

Table D.1. Experimental data range.

Evaporator air inlet temperature, °C (°F)	14 (57), 24 (75), 34 (93)
Condenser air inlet temperature, °C (°F)	15 (59), 22.5 (72.5), 30 (86), 37.5 (99.5), 45 (113)
Compressor speed, Hz	19, 30, 60, 95
Condenser fan speed, rpm	300, 450, 600, 750, 900, 1050, 1200
Evaporator fan speed, rpm	Fixed at the maximum speed
Cooling rate, W (Btu/hr)	From 1074 to 3897 (3665 to 13,300)
Compressor three-phase power, W (Btu/h)	From 94 to 1329 (321 to 4535)
Pressure ratio	From 1.2 to 4.8
Refrigerant mass flow rate, kg/s (lb/min)	From 0.0047 to 0.019 (0.622 to 2.513)

Table D.2. Test conditions actually realized.

Condenser fan speed, rpm	Evaporator air inlet temperature, °C (°F)	Condenser air inlet temperature, °C (°F)	Compressor speed, Hz	<i>N</i>
300	14 (57), 24 (75)	30 (86)	19, 30, 60	6
450	14 (57), 24 (75)	30 (86)	30, 60	4
450	24 (75)	30 (86)	19, 87	2
450	24 (75)	37.5 (99.5)	60	1
450	14 (57)	45 (113)	30, 60, 82	3
600	14 (57)	30 (86), 45 (113)	30, 60, 95	6
600	24 (75)	30 (86)	19, 30, 60, 93	4
750	14 (57), 24 (75), 34 (93)	17 (62.6), 22.5 (72.5), 30 (86)	19, 30, 60, 95	36
750	34 (93)	37.5 (99.5)	86–95	1
750	14 (57), 24 (75), 34 (93)	45 (113)	~80	3
900	14 (57)	30 (86)	30, 60, 95	3
900	14 (57)	37.5 (99.5)	60	1
900	14 (57)	45 (113)	30, 60, 82	3
900	24 (75)	30 (86)	19, 30, 60, 95	4