3580 (RP-491)

# COMMERCIAL BUILDING TEMPERATURE RECOVERY—PART I: DESIGN PROCEDURE BASED ON A STEP RESPONSE MODEL

**P.R. Armstrong, P.E.**
*Member ASHRAE*

**C.E. Hancock III**

**J.E. Seem, Ph.D.**
*Member ASHRAE*

## ABSTRACT

*A mathematical. model for estimating recovery response is developed based on the dimensionless step response of room temperature to zone heat input. Separate transfer functions are used to represent the walls, floor, and ceiling of a prototypical zone. Additional transfer functions are defined for each of several layered constructions made to represent the contents of a typical office building.*

*A grid of three construction weights (light, medium, and heavy) and five one-node model time constants* (5, *10, 20, 40, and 80 hours) is defined to represent a range of zone thermal characteristics. Dimensionless step responses of the resulting* 15 *prototype zones are generated by simulation.*

*A design method, based on the generalized response curves, is presented as a graphical. hand calculation procedure. A microcomputer implementation is developed based on a reduced order z-transfer function model and a corresponding analytical expression for each of the* 15 *dimensionless simulated step responses. The recovery response model is experimentally verified in part II (the companion paper).*

*The results of research project 491-RP pertain to heating and cooling load calculation (chapters* 25 *and* 26 *of the ASHRAE* Handbook of Fundamentals *[ASHRAE 1989]).*

## INTRODUCTION

To conserve energy, many heating and cooling plants are operated intermittently. This is usually accomplished in commercial buildings by using a lower heating setpoint and a higher cooling setpoint during unoccupied periods. The setback is commonly scheduled to occur at night, hence the terms "night setback" and "morning pickup." A weekend setback schedule is also frequently employed.

### Problem

Operation of the heating or cooling plant must commence before occupants return to allow time for the temperature of the occupied zone to be brought back to the daytime setpoint. The response of the zone to this initial plant operation is known as "temperature recovery."

The plant capacity, the thermal character of the heated or cooled zone, and the conditions (initial zone temperature, outside temperature, internal gains, and daytime setpoint) determine how long it will take for the zone to recover. It is the designer's job to specify a plant capacity that is sufficient for temperature recovery to occur in a reasonable time.

### Background

Recovery time has not always been considered an important factor in the sizing of heating or cooling plants. In the past it was found that the amount of capacity needed to provide a reasonable (<3 hours) recovery time under design conditions was not much (<50%) greater than the capacity needed to satisfy the steady-state design load. In most cases, a designer would apply a safety factor to allow for errors in the building envelope conductance, infiltration, and internal gain estimates. A plant capacity of 1.5 times the design load could be counted on to provide adequately fast recovery from a typical night setpoint under design conditions.

This rule-of-thumb approach has become unreliable because of three trends in the building industry:

1. Buildings are being designed with lower envelope conductance and better resistance to envelope infiltration than in the past.
2. Current design practice tends to reduce fan and pump energy and part-load performance penalties associated, in many cases, with oversized plants by minimizing excess plant capacity; this is accomplished by calculating loads more precisely so that smaller safety factors can be used.
3. Many building operators currently try to use the most extreme night setback possible with a given recovery time in order to increase the energy savings achievable by intermittent heating or cooling.

**Peter R. Armstrong** is a research engineer at Battelle Pacific Northwest National Laboratory, Richland, WA; **Charles E. (Ed) Hancock III** is a research engineer at Colorado Energy Associates, Fort Collins; **John E. Seem** is a research engineer at Johnson Controls, Inc., Milwaukee, WI.

Because of these trends, recovery time has become as important as peak steady-state load and part-load equipment performance in determining the "correct" heating capacity for an energy-efficient building.

## Project Objectives

The ASHRAE *Handbook of Fundamentals* (ASHRAE 1989) gives little guidance on how to estimate recovery time. Technical Committee 9.6 therefore initiated a research project, 491-RP, on the subject of temperature recovery in commercial buildings. The objectives of this project were to measure recovery responses of three commercial buildings under a variety of conditions and to develop an experimentally validated design procedure for estimating recovery response.

Note that a consequence of the trends cited above is that designers are being asked to solve a complex optimization problem rather than to simply determine a steady-state load. To design a system that provides heating and cooling at the lowest life-cycle cost, the designer must consider energy (primary heating and cooling energy plus fan and pump energy) costs as well as capacity-related (primary and distribution equipment) costs. To solve the optimization problem, energy costs and capacity-related costs must both be expressed as functions of night setback parameters. The scope of 491-RP did not include all aspects of the optimization problem, only the question of how capacity and recovery time are related.

## Technical Approach

The technical approach taken in this project had four basic steps. First, the response models that have been used by others were assessed in order to select a theoretical model and set of assumptions appropriate to the problem. Second, the recovery responses of three buildings were monitored under a variety of conditions. Third, the monitored response data were used to validate the theoretical model. Finally, a simple design procedure, based on the general theoretical model, was developed to estimate recovery time given excess capacity and vice versa.

This paper covers the first and fourth steps of the technical approach outlined above. The results of these activities are summarized below. Steps two and three are described in the companion paper.

To describe the thermal characteristics of zones representing a wide range of building construction types, a scheme using the existing ASHRAE categorization by building weight is developed.. This categorization is augmented by the introduction of a second parameter, the one-node model time constant, to account for different levels of insulation in building envelopes. A grid of three construction weights (light, medium, and heavy) and five one-node model time constants (5, 10, 20, 40, and 80 hours) is defined, and dimensionless recovery trajectories for the resulting 15 points on the zone thermal characteristics grid are generated. .

A design method, based on the generalized response curves, is presented as a graphical hand calculation procedure. Two example problems are worked. A computer program that implements the procedure is then described.

## THEORETICAL MODEL

In this section, a model for estimating recovery response is developed from basic principles of heat transfer. A design procedure based on the model is developed in the next section.

The advantage of a good theoretical model is that it can be applied to a wide range of building types. Data from a small number of monitored buildings cannot be expected to yield a general empirical model but can be useful in validating a general theoretical model. Thermal responses of the three buildings monitored in this project and use of the monitored response data to validate the theoretical model are described in the companion paper.

## Model Assumptions

Maximum plant capacity is required when recovery occurs under design conditions. The following four simplifying assumptions can be made regarding design conditions. (1) Under design conditions, the zone temperature has been maintained at the night setback temperature long enough for the zone envelope and its contents to be in a state. of thermal equilibrium. (2) Since buildings are generally unoccupied prior to and during recovery, a constant rate of internal gain is assumed during these periods. (3) When recovery commences, the plant output increases from the rate needed to maintain the night setpoint to its full capacity and remains at full capacity until the day setpoint is satisfied. (4) The zone in question behaves as a linear system under normal recovery conditions.

These assumptions do not exactly represent typical or extreme (design condition) recovery conditions. They are close, however, and are particularly useful because, when used with the standard design conditions,' they correspond to a condition that is slightly conservative. One aspect of the problem that cannot be simplified is the dynamic nature of the response. It is important that this be stated because the first area that is approximated in most simplified energy analyses is the dynamic behavior. of a building.

The foregoing assumptions exactly define the conditions under which a system is said to exhibit its step response. The step response of a linear system has the same shape regardless of the size of the step excitation (excess capacity), the values of steady excitations (outside temperature and internal gains), or the initial value of the output of interest (room temperature). These same assumptions, plus one further simplifying assumption, were used in the general model (Smith 1942) cited in chapter 29 of ASHRAE (1989).

## Evaluation of Step Response

The discrete-time transfer function (also known as the z-transfer function, conduction transfer function, or CTF) is the numerical model that is most widely used to simulate transient thermal responses of buildings. A short time step, one-zone transfer function simulation was developed for this project to examine room temperature step responses for a variety of different construction materials and zone contents. The mathematical formulas and algorithm used in the simulation are presented in Appendix A.

The essential features of the simulation model are as follows. Separate transfer functions are used to represent the walls, floor, and ceiling of a zone. Additional transfer functions are defined for each of several layered constructions made to represent the contents of the zone. The interior surfaces of the enclosure and its contents are coupled to the room air node, and the heating or cooling effect of the HV AC plant is delivered to this same room air node. The air node temperature is evaluated directly (i.e., without iteration or simultaneous solution of surface temperatures or fluxes) at each time step. The surface temperatures and fluxes are then evaluated directly. Note that by determining step responses of entire rooms or zones-rather than individual walls-we eliminate Smith's simplifying assumption of constant heat flux at each interior surface.

The simulation was written to compute the dimensionless recovery temperature as well as the actual room temperature. The dimensionless recovery trajectories generated by the simulation are independent of zone and outside temperature but change when the thermal properties (e.g., capacitance) of the simulated zone are changed. The simulated dimensionless response trajectories of the same building starting at two different temperatures are identical. This and other properties of dimensionless temperature are developed in the next section.

## Properties of Dimensionless Response

The dimensionless response of room temperature to a step change in zone heat input rate $Q$ is defined as

$$\theta(t) = \frac{T_r(t) - T_r(0)}{T_r(\infty) - T_r(0)} \tag{1}$$

*where*

$T_r(t)$ = the room temperature trajectory,
$T_r(0)$ = the initial room temperature, and
$T_r(\infty)$ = the limit to which the room temperature converges.

The limiting temperature for a building zone that has losses only to outside ambient conditions at temperature $T_a$ is given by

$$T(\infty) = T_a + Q/UA \tag{2}$$

*where*

$Q$ = rate of heat input to the zone (negative for cooling), including internal gains, and
$UA$ = the sum of conductance area products for all heat transfer paths, including infiltration, between room air and the ambient.

The limiting temperature for a zone that also has losses to the ground or to adjacent zones that act approximately as constant-temperature heat sinks is

$$T(\infty) = \frac{\sum (UA)_j T_j + Q}{UA} \tag{3}$$

*where*

$T_j$ = $j$th sink temperature,
$(UA)j$ = conductance area product for the heat transfer from room air to the jth sink, and
$UA$ = $\Sigma(UA)_j$ = the sum of conductance area products for all paths to all sinks.

The dimensionless response, $\theta(t)$, depends only on the thermal properties of the zone envelope and its contents and is in no way affected by the number of sinks or the disparity among sink temperatures or sink path conductances. For the purpose of generating dimensionless response curves we therefore used a single sink temperature, $Ta'$ and evaluated the limiting room temperature by Equation 2. In applying the dimensionless recovery curves, however, the designer will use Equation 2 for cases with one sink temperature and Equation 3 for cases with multiple sink temperatures.

Equations 1 through 3 suggest a useful definition for excess capacity:

$$Q_x = Q - UA\Delta T \text{ where } \Delta T = T(0) - T_a$$

for a single sink at temperature $T_a$, or, more generally,

$$Q_x = Q - (\Sigma(UA)_j\, T(0) - \Sigma(UA)_j /T_j$$

for multiple sinks, $T_j$.. In words, excess capacity, $Q_x$, is the total plant capacity minus the fraction of plant capacity needed to maintain the initial room temperature under design conditions. Since the initial temperature of interest is the night setpoint, $Q_x$ should be referred to as the excess *night* capacity.

Since $T_r(t)$ - $T_r(0)$ and $T_r(\infty)$ - $T_r$ $(0)$ are both directly proportional to $Q_x$, one can see from Equation 1 that $\theta(t)$ is unaffected by $Q_x$.

## Generalized Step Response Curves

A family of curves can be used to represent the responses of many buildings with different thermal characteristics. To explore the effects of different parameters, a grid of values of the parameters affecting a zone's dimensionless response was developed. The parameters of interest are

- Time constant (one-node model) of zone.
- Distribution of zone *UA* between massive walls and essentially massless conduction paths (e. g., windows and infiltration).
- Thermal character of construction materials.
- Thermal character of zone contents.

The zone contents and partitions were assumed to be the same for all cases. Six thermally distinct types of room contents and furnishings were considered in order to simulate the variety of furnishings present in an occupied building. To obtain the true distributed lag character of the contents' thermal responses, each furnishing or contents type was modeled as a multilayered construct. The thermal properties of the resulting constructs are listed in Appendix B. The surface areas, weights, and thermal capacitances are summarized in Table 1.

**Weights of Construction**. Three weight categories were used for the points on the grid of construction materials. The thermal properties of the floor and wall constructions for the three weight categories are listed in Appendix C.

The corresponding transfer function coefficients (Armstrong *et al* 1991) were computed using the TARP (Walton 1984) program's transfer function subroutines. The subroutines were compiled under MS-Fortran with the math coprocessor option set. Round-off error was found to result in step response errors as large as 10% (discrepancy between U-value implied by CTF coefficients and U-value implied by wall properties) when using a 15-minute time step with wall constructions of low thermal diffusivity. Critical variables were therefore treated as double precision numbers, which carry a 52-bit mantissa under MS-Fortran. This reduced the error to less than 0.1%. A polishing scheme was used to reduce the step response error to 0.001% with the minimum round off error sequence of response calculations.

The weights of contents and materials of construction for the light-, medium-, and heavyweight categories used in this study are listed in Table 2. The weights used by Mitalas (note b, Table 22, Chapter 26, ASHRAE 1989) in developing room air transfer functions for computing peak air-conditioning loads are also listed in Table 2. The corresponding thermal capacitances for the three weight categories are listed on the right-hand side of the table. Note that for the constructions used in this study the participating capacitance, $C_w$ (defined below), is from 2% to 12% less than the corresponding full thermal capacitance.

**Time Constant**. A one-node model was defined whose deviation from true response is less than the deviation of the "conventional" one-node model's response. We will call the former model's time constant the "one-node model time constant." The time constant of the conventional model is the time for the step response of a system with a lumped thermal capacitance equal to the total distributed capacitance of the system to attain 1 – 1/e of final response. The one-node model time constant has a useful physical interpretation for any distributed-mass system. It uses a capacitance equal to the fraction of the total distributed capacitance such that the change in

TABLE 1
Surface Areas and Weights and Thermal Capacitances
Per Unit Surface Area for Contents of the 2000-Ft$^2$ Simulated Perimeter Zone

| | Area ($ft^2$) | Weight (lbm) | Weight ($lbm/ft^2$) | Thermal Capacitance (Btu/°F) | Thermal Capacitance ($Btu/ft^2$ °F) |
|---|---|---|---|---|---|
| Partitions | 4,000 | 8,567 | 2.14 | 1,691 | 0.423 |
| Shelved Books | 360 | 1,665 | 4.62 | 999 | 2.775 |
| Desks/Furniture | 700 | 1,079 | 1. 54 | 648 | 0.926 |
| Metallic parts | 500 | 1,200 | 2.40 | 120 | 0.240 |
| Acoustic Tile | 4,000 | 3,750 | 0.937 | 750 | 0.188 |
| Wood File | 230 | 1,227 | 5.33 | 405 | 1.760 |
| Metal File | 230 | 1.184 | 5.15 | 377 | 1.641 |
| Total | 10,020 | 18,672 | | 4,990 | |
| Per sf Floor | 5.01 | 9.34 | | 2.50 | |

TABLE 2
Weights and Thermal Capacitances of Zone Contents and Materials of Construction
Note: Mitalas numbers are presented in ASHRAE (1989) as approximate.

| | Weight (lbm/ft2 floor area) | | | Total Capacitance (Bth/F-ft2 floor area) | | |
|---|---|---|---|---|---|---|
| Weight category: | light | medium | heavy | light | medium | heavy |
| This study: | 40 | 81 | 117 | 8.90 | 17.00 | 24.20 |
| Cw for this study: | | | | 7.85 | 16.61 | 23.63 |
| Mitalas study: | 30 | 70 | 130 | 9.10 | 13.30 | 25.00 |

internal energy between the initial and final step response states equals the change. in internal energy that would occur in the true system. We call this fraction of total capacitance the "participating capacitance" and denote it by $C_w$ because in evaluating the participating capacitance, the capacitances of the layers of exterior walls are given different weights.

The participating capacitance for the $j$th exterior wall, with area $A_j$ constructed of $n$ layers (including the film resistance layers), is given by

$$C_j = \frac{A_j \sum_i R_{j,i} t_{j,i} c_{j,i}}{R_j}$$

*where*

$c_{j,i}$ = thermal capacitance per unit volume of the i$^{th}$ layer,

$t_{j,i}$ = thickness of the $i^{th}$ layer,

$R_{j,i}$ = resistance to the middle of the i$^{th}$ layer starting with the outside film resistance, and

$R_j$ = resistance through all $n$ layers of the j$^{th}$ wall.

Thus $R_{j,1}$ is half the outside film resistance and $R_{j,n}$ is the resistance to the inside wall surface plus half the inside film resistance. The one-node model time constant is given by

$$RC_w = (C_c + \sum C_j)/UA$$

where $C_c$ is the total thermal capacitance of contents and interior walls and $UA = \sum(A_j/R_j)$.

The step responses were calculated by the program described earlier, which is listed in Appendix C of Armstrong et al. (1991). An input deck containing the transfer function coefficients of floor, wall, and contents constructions was prepared for each run. Five step responses, corresponding to the five one-node model time constants of 5, 10, 20, 40, and 80 hours, were computed in each run by varying the pure conductance component of the zone $UA$. The input deck for the zone of lightweight construction is listed in Appendix D of Armstrong et al. (1991). All other decks were identical except for the first two sets of transfer function coefficients, which describe the ceiling, floor and exterior wall constructions.

The resulting step response curves are plotted in Figures 1 through 3. These curves will be used with the design procedure developed in the next section. The points on the curves are tabulated in Appendix G of Armstrong et al. (1991). The reader may compare responses for two curves that have roughly the same $UA$ (e.g., light, $RC_w$ = 5 h and medium, $RC_w$ = 10 h) to see that the responses are quite similar initially but that the rate of change of temperature falls off faster for the heavier construction.

## DESIGN PROCEDURE

This section describes a design procedure based on the family of 15 dimensionless recovery curves. The procedure is presented as a hand calculation procedure in which the curves are read to evaluate the time needed to recover or the dimensionless recovery temperature at a given time. The procedure for computing required capacity from dimensionless recovery temperature is also presented. A microcomputer implementation of these procedures is described in the next section.

### Hand Calculation/Graphical Procedure

**Recovery Time.** The procedure for determining recovery time follows. The inputs are plant capacity $Q$, internal gain $Q_i$, day and night setpoints $T_d$ and $T_n$, design temperature(s) $T_a$ or $T_j$ for $j = 1, N_{SINK}$, and the thermal parameters of the building envelope and its contents.

1. Determine the capacitance of the contents plus the fraction of the envelope's capacitance that participates in the step response, $C_w$.

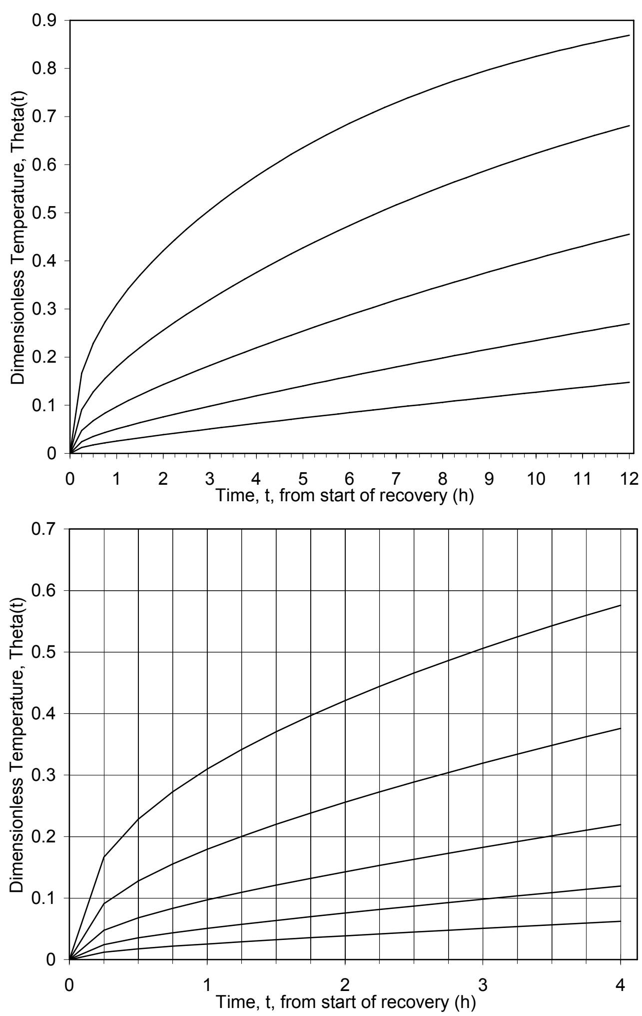


*Figure 1 Dimensionless recovery temperature trajectories for lightweight constructions with one node model time constants, starting from the top, of RCw = 5, 10, 20, 40, and 80 hours.*

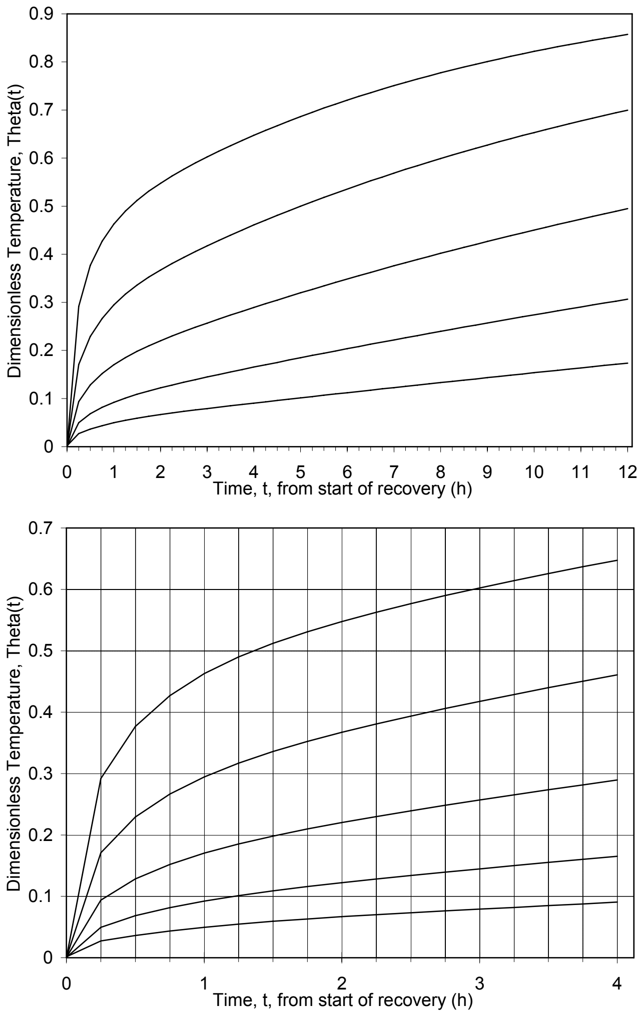


*Figure* 2 *Dimensionless recovery temperature trajectories for medium-weight constructions with one-node model time constants, starting from the top, of* $RC_w$ = 5, *10, 20, 40, and 80 hours.*

2. Determine the conductance, $UA_j$, for each heat loss path that leads to a fixed-temperature sink, $T_j$.
3. Determine the time constant, $RC_w$, of the limiting (one-node model) step response trajectory.
4. Determine the weight category, on the light-medium-heavy scale, for the building envelope and contents in question.
5. Determine the limiting (infinite time) step response temperature for the room air,

   $T(\infty) = \Sigma(UA)_j T_j + Q_i + Q)$, where $1/R = \Sigma(UA)_j$.
6. Determine the dimensionless recovery temperature corresponding to the day setpoint,

   $\theta_d = (T_d - T_n)(T(\infty) - T_n)^{-1}$.
7. Find the recovery times on the four curves in Figures 1 through 3 that bracket the time constant and weight category of the zone in question.

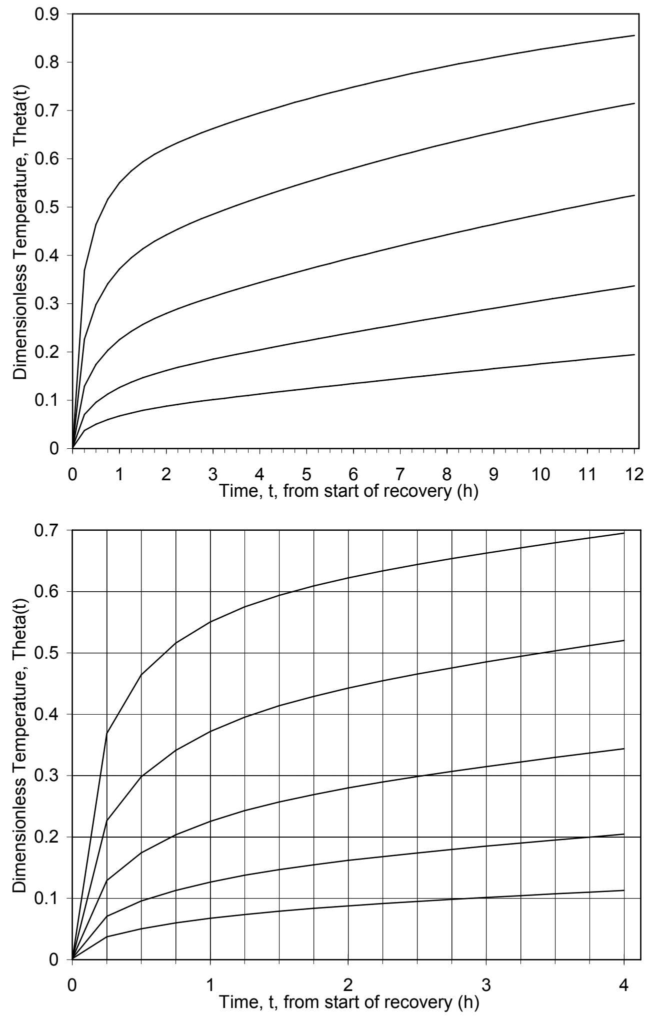


*Figure* 3 *Dimensionless recovery temperature trajectories for heavyweight constructions with one-node model time constants, starting from the top, of* $RC_w$ = 5, *10, 20, 40, and 80 hours.*

8. Linearly interpolate between the four bracketing recovery times to estimate the correct recovery time. Do not extrapolate.

**Plant Capacity.** The same curves of Figures 1, 2, and 3 can be used to determine the plant capacity required for a specified maximum recovery time. After completing steps 1 through 4 above, the designer proceeds as follows.

9. Find the dimensionless recovery temperatures on the four curves in Figures 1 through 3 that bracket the actual time constant and weight category of the zone in question for the chosen recovery time.
10. Linearly interpolate between the four bracketing recovery. temperatures to estimate the exact dimensionless temperature at the end of the recovery period. Do not extrapolate.

11. Determine the limiting temperature, $T(\infty)$, corresponding to dimensionless day setpoint $T_d$:

$$T(\infty) = T_n + (T_d - T_n)/\theta_d.$$

12. Determine the plant capacity corresponding to $T(\infty)$:

$$Q = T(\infty)\sum(UA)_j - \sum T_j(UA)_j - Q_i.$$

**Interpolation** Dimensionless recovery temperature is a function of three variables. One of the variables, time, is represented as a continuum. The other two, $RC_w$ and $C_w/A$, are represented as discrete points on a grid. Three values of $C_w/A$ are represented by the curves in Figures 1 through 3. Five values of $RC_w$ are represented by the five curves on each graph. Having two discretized independent variables means that the linear interpolation performed in steps 8 and 10 involves four grid points rather than the two encountered when only one independent variable is involved.

Interpolation of a discretized function of two variables proceeds as follows. Define the first dimensionless interpolation factor to be

$$X_\tau = \frac{RC_w - RC(i)}{RC(i+1) - RC(i)}$$

where $RC(i)$ and $RC(i+1)$ are the bracketing values of the one-node model time constant, $RC_w$. For example, if $RC_w$= 25h, the bracketing points from the grid values $RC() =$ [5, 10, 20, 40, 80] are $RC(3)$ = 20h and $RC(4)$ = 40h. Thus, $X_\tau$ = .250.

Define the second interpolation factor to be

$$X_W = \frac{C_w/A - W(i)}{W(i+1) - W(i)}$$

where $W(i)$ and $W(i+1)$ are the bracketing values on the weight-of-construction grid. For example, if $C_W/A$ = 15 Btu/ft$^2$°F, the bracketing points from the set of grid values $W()$ = [7.85, 16.61, 23.63] Btu/ft$^2$°F are $W(1)$ = 7.85 and $W(2)$ = 16.61 Btu/ft$^2$°F. Thus, $X_W$ = 0.816.

The interpolated value of dimensionless recovery is given by.

$$\theta(t) = (1 - X_T)((1 - X_W)\theta_{i,j}(t) + X_W\,\theta_{i,j+1}(t))$$
$$+X_T((1 - X_W)\theta_{i,j}(t) + X_W\,\theta_{i,j+1}(t))$$

The foregoing formulas can be used to extrapolate beyond the grid domain by using the two closest grid points. However, extrapolation must be approached with caution. In unusual cases (buildings whose parameters fall outside the grid), the prudent designer will perform a full transient simulation to evaluate plant capacity needed for intermittent heating or cooling.

## Examples

The hand calculation procedures are illustrated in the two example problems presented below.

**Example 1** Find the heating capacity needed to heat a 4,000 ft2, second-floor perimeter zone from a night setpoint of 55°F to a day setpoint of 70°F in two hours with a design outside temperature of -5°F. The heating load during recovery is as follows:

| Load Component | ft$^2$ | Btuh/ft$^2$F | Btuh/F |
|---|---|---|---|
| Outside air (damper leakage) | 4,000 | 0.1 | 400 |
| Exterior wall conductance | 1,950 | 0.08 | 156 |
| Fixed glass conductance | 800 | 0.55 | 440 |
| Total UA | | | 996 |

The nighttime internal gains are assumed to be zero. The nighttime steady-state design heating capacity is therefore (55 - (-5))996 = 59,760 Btuh. The daytime design capacity of about (70 - (-5))1400 = 105,000 Btu/h is considerably larger due to the higher setpoint and larger outside air load. The thermal capacitances of materials of construction and contents are as follows:

| Mass Component | ft$^2$ | lbm/ft$^2$ | Btu/lbm°F | Btu/°F |
|---|---|---|---|---|
| Ferro-concrete frame | 4000 | 15 | 0.2 | 12000 |
| Floor structure | 4000 | 46 | 0.2 | 36800 |
| Partitions | 4800 | 3 | 0.2 | 1920 |
| Ceiling tile | 4000 | 1 | 0.3 | 1200 |
| Ceiling hardware,ducts | 4000 | 1 | 0.1 | 400 |
| $C_w$ of exterior wall | 1950 | | | 1385 |
| Furniture, files | | | 440 | 6000 |
| Total | | | 996 | 59685 |

**Solution** The zone time constant is 59,685/996 = 60h. The thermal capacitance per Unit floor area is 59,685/4000 = 14.9 Btu/ft$^2$°F, which is equivalent to a zone composed of 20% lightweight construction (7.85 Btu/ft$^2$°F) and 80% medium-weight construction (16.61 Btu/ft$^2$°F). The dimensionless temperatures, θ, read the from the four bracketing recovery curves at time = 2h, and the associated interpolation weights are therefore:

| Time | Weight of Construction | Zone Time Constant | Interpolation Weight |
|---|---|---|---|
| 0.078 | light (0.20) | 40 h (0.50) | 0.10 |
| 0.041 | light (0.20) | 80 h (0.50) | 0.10 |
| 0.125 | medium (0.80) | 40 h (0.50) | 0.40 |
| 0.066 | medium (0.80) | 80 h (0.50) | 0.40 |

The interpolated dimensionless recovery temperature is given by

$$0.10(0.078 + 0.041) + 0.40(0.125 + 0.066) = 0.078.$$

The required limiting temperature is therefore

$$T(\infty) = 55 + (70 - 55)/.078 = 247°\text{F},$$

and the required heating capacity is

$$(247 - (-5))°\text{F} \times 996 \text{ Btuh/°F} = 251{,}000 \text{ Btuh}.$$

The night and daytime excess capacities are

100(251,000/60,000−1) = 315% excess night capacity and

100(251,000/105,000−1) = 135% excess daytime capacity.

The large excess daytime capacity may cause the designer to consider a longer recovery time. The designer will select an excess capacity, based on such considerations as air-side temperature rise or equipment availability, and then determine the required recovery time. This approach is considered in the second example.

**Example 2** Find the recovery time needed for the perimeter zone and design conditions described in example 1 given a heating capacity of 180 kBtuh.

**Solution** We first compute the limiting temperature,

$$-5 + 180{,}000/996 = 176°\text{F},$$

and the corresponding dimensionless recovery temperature,

$$(70 - 55)/(176 - 55) = 0.124.$$

The bracketing curves and the associated interpolation weights are the same as those used in example 1 since the same envelope and contents are involved. The recovery times, read from these curves for a dimensionless recovery temperature of 0.124, are

| Time | Weight of Construction | Zone Time-Constant, τ | Interpolation Weight |
|---|---|---|---|
| 4.00 | Light | 40h | 0.10 |
| 9.05 | Light | 80h | 0.10 |
| 1.95 | Medium | 40h | 0.40 |
| 6.75 | Medium | 80h | 0.40 |

and the interpolated recovery time is given by

0.10(4.00 + 9.05) + 0.40(1.95 + 6.75) = 4.8 h.

The foregoing two examples show that recovery time can be quite sensitive to plant capacity. In this case, a 28 % reduction in total capacity (47 % reduction in excess daytime capacity) resulted in a 140 % increase in recovery time under design conditions.

## MICROCOMPUTER IMPLEMENTATION OF THE DESIGN PROCEDURE

The dimensionless recovery curves can be used in a computer-based or a programmable-calculator-based design procedure. The points would be read into a multidimensional array, and linear interpolation would be used to evaluate step response vs. time. However, the number of points involved makes this scheme cumbersome. A better approach is to fit an analytical function to each curve. A computer program based on this approach is presented here. First we show how a physically meaningful, continuous-time expression for step response can be identified from points on the step response curve. We also demonstrate, in the process, that step response expressions for all 15 curves have the same analytical form.

### Transfer Function Model

The parameters of the analytical expression for step response are obtained in two steps. A reduced-order transfer function is first fitted to each curve by linear regression. The important time constants are found from the transfer function parameters by solving for its zeros. The remaining parameters of the corresponding continuous-time (analytical) model are then obtained by another linear regression step. The reduced-order transfer function is identified as follows.

Seem (1987) has shown that the conventional (ASHRAE 1989; Klein 1983; Walton 1984) model in which separate wall transfer functions are connected to a common air node is equivalent to a single transfer function model, known as a comprehensive room transfer function (CRTF) of the form:

$$\Sigma c_k T(t-k) = \Sigma d_k Q(t-k) + \Sigma\Sigma b_{i,k} T_i(t-k)$$

where $T(t)$ is room temperature and $T_i(t)$ is the outside air or sol-air temperature acting on the exterior of the $i$th wall.

The comprehensive room transfer function that is exactly equivalent to the model involving separate transfer functions for each wall will, in general, have the same number of parameters (elements of *b, c,* and *d* vectors) as the set of separate wall transfer functions. However, Seem has shown that there is considerable redundancy when the transfer functions of most common wall constructions are combined and that the high-order CRTF can therefore be closely approximated by a CRTF of a much lower order.

A discrete-time expression for the dimensionless step response of room temperature to heat input that follows directly from the CRTFs the recursive expression:

$$\Sigma c_k \theta(t-k) = \Sigma d_k U(t-k) \frac{\Sigma c_k}{\Sigma d_k}$$

where $U(t)$ is the unit step function. The elements of low order *b, c,* and *d* vectors can be determined from step response data by applying linear least squares to the foregoing expression.

**Continuous-Time Expression for Step Response** Evaluating the discrete-time expression to determine the time required to achieve a given fraction of step response is awkward. There is, however, an explicit continuous time expression that is exactly equivalent (Seem 1987; Seem et at. 1989a,b; Armstrong 1985; Armstrong et at. 1991):

$$\theta(t) = \Sigma w(i)\ \mathrm{EXP}(-t/\tau(i)). \tag{4}$$

By letting $1/\tau(0) = 0$, there will always be a term (the $w(0)\mathrm{EXP}(-t/\tau(0)) = w(0)$ term) that corresponds to the 0-lag part of the transfer function (i.e., the terms involving $c_o$ and $d_o$).

The time constants, $\tau$, are related to the zeros (i.e., roots of the *c* vector) of the CRTF, *R,* by the following expression:

$$\tau(i) = l/ln(R(i)).$$

The *n*th-order discrete-time expression is, in effect, decomposed into an expression involving *n* first-order step responses, $\theta(t) = \Sigma w(i)\ \theta_i(t)$, where $\theta_i(t) = \theta_i(t\text{-}1)/R(i)$.

The coefficients, *w*(), can be determined by simultaneous solution of $n$+1 equations or by linear regression using many or all of the original points on the step response curve. We have taken the latter approach in order to avoid numerical difficulties anticipated with the former.

Goodness-of-Fit The CRTF coefficients (elements of low-order *b, c,* and *d* vectors) were determined by applying linear least squares to the discrete-time expression for step response using each of the 15 simulated dimensionless step responses. The discrete-time response expressions of degree 2 generally provide an adequate (±0.3 %) fit, and expressions of degree 5 provided a very precise fit (±0.001%). The regression errors for the continuous-time expression were higher than for the discrete-time expression because the continuous-time expression is constrained to give a response of 1 at $t = \infty$ and because the roots are not determined exactly. Numerical difficulties were encountered at degree 4 for two of the curves.

Coefficients of the Continuous-Time Expression A program was written to implement the design procedure. The dimensionless response curves are represented by the continuous-time expression of Equation 4.

The parameters of the continuous-time. expressions were determined for the 15 dimensionless response curves by regression. The dominant time constants, *T,* were determined by linear regression, as mentioned above, using the discrete-time expression for step response and points on each step response curve presented in Figures 1, 2, and 3. The weight coefficients, *w*, were then determined by linear regression, using the same points on the step response curve.

The resulting coefficients of the continuous-time expression for each dimensionless step response curve are listed in Table 3. The source code for the design program is listed in Appendix L of Armstrong et at. (1991).

## SUMMARY AND CONCLUSION

A general step-response model has been developed in which the thermal response characteristic of a conditioned perimeter zone is represented by two parameters-the building construction weight category and the one-node model time constant. A third parameter, describing the distribution of exterior insulation between light and heavy wall elements, was found not to be significant (Armstrong et at. 1991). A simplified design procedure, based on the general step-response model, has also been developed. Empirical verification of the model is reported in the companion paper (Armstrong et al. 1992).

## REFERENCES


Armstrong, P.R. 1985. Analysis of building dynamics in the frequency domain. SERI Workshop on Frequency Domain Methods, January, Golden, CO.

Armstrong, P.R., C.E. Hancock, and J.E. Seem. 1991. Commercial building temperature recovery. Final Report, ASHRAE Research Project 491-RP.

Armstrong, P.R., C.E. Hancock, and J.E. Seem. 1992. Commercial building temperature recovery-Part II: Experiments to verify the step response model. *ASHRAE Transactions* 98(1).

*ASHRAE, 1989. ASHRAE handbook—1989fundamentals.* Atlanta: American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc.

Seem, J.E. 1987. Modeling heat transfer in buildings. Ph.D. thesis, University of Wisconsin, Madison.

Seem, J.E., *S.A.* Klein, W.A. Beckman, and J.W. Mitchell. 1987. Transfer functions for efficient calculation of multi-dimensional transient heat transfer. *ASME J. Heat Transfer* 78: 25-33.

Seem, J.E., *S.A.* Klein, W.A. Beckman, and J.W. Mitchell. 1989a. Comprehensive room transfer functions for efficient calculation of transient heat transfer in buildings. *ASME J. Heat Transfer* 111(2): 264-273.

Seem, J.E., *S.A.* Klein, W.A. Beckman, and J.W. Mitchell. 1989b. Model reduction of transfer functions by the dominant root method. *ASME J. Heat Transfer.*

Smith, E.G. 1941a. A simple and rigorous method for the determination of the heat requirements of simple intermittently heated exterior walls. *Journal of Applied Physics* 12 (August).

Smith, E.G. 1941b. The heat requirements of simple intermittently heated interior walls and furniture. *Journal of Applied Physics* 12 (August).

Smith, E.G. 1942. A method of compiling tables for intermittent heating. *Heating, Piping and Air Conditioning* 14(6).

Walton, G. 1984. *TARP-Thermal analysis research program, user manual.* NBSIR 83-2655. Gaithersburg, MD: U.S. Dept. of Commerce.

TABLE 3

Coefficients of Continuous-Time Expressions for the 15 Dimensionless Step Response Curves

(The time constants of each system are given by $\tau$ (i) = $\Delta t/\ln(R(i))$. Note that $\Delta t$ = 0.25 h and that all R(i) > 1 except where model orc;ler = 4 is indicated by w(5) = 0 and R(5) = 1.)

| Weight category | RCw (h) | w(1) R(1) | W(2) R(2) | w(3) R(3) | w(4) R(4) | w(5) R(5) |
|---|---|---|---|---|---|---|
| Light | 5 | -0.72155 | -0.07173 | -0.07863 | -0.03367 | -0.0944 |
| | | 1.03629 | 1.09878 | 1.53281 | 2.28474 | 15.94422 |
| | 10 | -0.85883 | -0.02993 | -0.04533 | -0.01639 | -0.04942 |
| | | 1.02087 | 1.09462 | 1.51246 | 2.26038 | 15.17139 |
| | 20 | -0.92861 | -0.01295 | -0.02245 | -0.00995 | -0.02595 |
| | | 1.01119 | 1.08655 | 1.45851 | 2.07296 | 13.63698 |
| | 40 | -0.96434 | -0.00805 | -0.01300 | -0.01446 | 0.00000 |
| | | 1.00580 | 1.11425 | 1.63813 | 8.66070 | 1.00000 |
| | 80 | -0.98199 | -0.00511 | -0.00512 | -0.00800 | 0.00000 |
| | | 1.00295 | 1.11254 | 1.63264 | 8.61752 | 1.00000 |
| Medium | 5 | -0.49845 | -0.07379 | -0.17298 | -0.06837 | -0.18638 |
| | | 1.02687 | 1.06650 | 1.55944 | 2.23346 | 15.44881 |
| | 10 | -0.68953 | -0.04546 | -0.12417 | -0.03948 | -0.10118 |
| | | 1.01765 | 1.06034 | 1.49816 | 2.13716 | 13.95956 |
| | 20 | -0.82730 | -0.02276 | -0.07904 | -0.01785 | -0.05262 |
| | | 1.01039 | 1.05801 | 1.46900 | 2.15732 | 13.97632 |
| | 40 | -0.90850 | -0.01045 | -0.04474 | -0.00748 | -0.02835 |
| | | 1.00567 | 1.05225 | 1.43458 | 2.01121 | 12.70595 |
| | 80 | -0.95365 | -0.00081 | -0.03565 | 0.01093 | -0.01909 |
| | | 1.00299 | 1.06397 | 1.47241 | 2.85523 | 22.50402 |
| Heavy | 5 | -0.39182 | -0.06501 | -0.18531 | -0.11029 | -0.24750 |
| | | 1.02164 | 1.05756 | 1.53741 | 2.19294 | 16.01633 |
| | 10 | -0.57804 | -0.05541 | -0.16162 | -0.06640 | -0.13808 |
| | | 1.03629 | 1.09878 | 1.53281 | 2.28474 | 15.94422 |
| | 20 | -0.74702 | -0.03031 | -0.11881 | -0.02669 | -0.07567 |
| | | 1.00961 | 1.04558 | 1.42658 | 2.03559 | 13.63823 |
| | 40 | -0.85982 | -0.01364 | -0.07788 | -0.00424 | -0.04235 |
| | | 1.00548 | 1.04059 | 1.39645 | 1.98381 | 13.01106 |
| | 80 | -0.92570 | -0.00674 | -0.04492 | 0.00132 | -0.02224 |
| | | 1.00293 | 1.04145 | 1.39238 | 2.05330 | 13.61381 |

## BIBLIOGRAPHY


ASHRAE. 1975. *Procedure for determining heating and cooling loads for computerized energy calculations: Algorithms for building heat transfer subroutines.* Special publication of the TG on Energy Requirements.

Balcomb, J.D. 1983. Thermal network reduction. Los Alamos National Laboratory Report LA-UR-83-869.

Brisken, W.R., N.J. Bloomfield, and S.G. Reque. 1956. Heat load calculations by thermal response. *ASHRAE Transactions* 62.

Ceylan, H.T., and G.E. Myers. 1985. Application of response-coefficient method to heat-conduction transients. *ASHRAE Transactions* 91(IA).

Davies, M.G. 1973. The thermal admittance of layered walls. *Building Science* 8: 207-220.

Gable, G.K., and K. Koenig. 1977. *ASHRAE Transactions* 83(1).

Harrison, H.L., W.S. Hansen, and R.E. Zelenski. 1981. Development of a room transfer function model for use in the study of short term transient response. *ASHRAE Transactions* 87(1): 461-472.

Horn, J.C. 1982. Investigation of various thermal capacitance models. M.S. thesis, University of Wisconsin, Madison.

Klein, S.A. 1983. TRNSYS: A transient system simulation program. Solar Energy Laboratory, University of Wisconsin-Madison, Engineering Experiment Station Report 38-12.

Koenig, K. 1978. Gas furnace size requirements for residential heating using thermostat night setback. *ASHRAE Transactions* 84(1).

Kusuda, T. 1969. Thermal response factors for multilayer structures of various heat conduction systems. *ASHRAE Transactions* 75: 246.

Madsen, J.M. 1982. Modeling heat transfer in rooms using transfer function methods. M.S. thesis, University of Wisconsin, Madison.

Mitalas, G.P. 1965. An assessment of common assumptions in estimating cooling loads and space temperatures. *ASHRAE Transactions* 71 (II).

Mitalas, G.P. 1968. Calculation of transient heat flow through walls and roofs. *ASHRAE Transactions* 74: 182-188.

Mitalas, G.P. 1972. Transfer function method of calculating cooling loads, heat extraction rate, and space temperature. *ASHRAE Journal* 14(12): 52.

Mitalas, G.P., and *J.G.* Arsenault. 1971. Fortran IV program to calculate z-transfer functions for the calculation of transient heat transfer through walls and roofs. NBS Building Science Series 39, October.

Nelson, L.W. 1965. The analog computer as a product design tool. *ASHRAE Journal,* November.

Nelson, L.W. 1973. Reducing fuel consumption with night setback. *ASHRAE Journal,* August.

Nelson, L.W., and J.W.MacArthur. 15)78. Energy savings through thermostat setback. *ASHRAE Transactions* 84(1).

Pawelski, *M.J., J.W.* Mitchell, and W.A. Beekman. 1979. Transfer functions for combined walls and pitched roofs. *ASHRAE Transactions* 85(2).

Peavy, B.A. 1978. A note on response factors and conduction transfer functions. *ASHRAETransactions* 84: 688-690.

Pipes, L.A. 1957. Matrix analysis of heat transfer problems. *Franklin Institute Journal* 263(3).

Quentzel, D. 1976. Night-time thermostat setback: Fuel savings. in residential heating. *ASHRAE Journal* 18(3): 39-43.

Seem, J.E., P.R. Armstrong, and C.E. Hancock. 1989. Comparison of seven optimal start algorithms. *ASHRAE Transactions* 95(2).

Stephenson, D.G., and G.P. Mitalas. 1971. Calculation of heat conduction transfer functions for multi-layer slabs. *ASHRAE Transactions* 77(2).

Zabinski, M.P., and L. Loverme. 1974. Fuel consumption in residential heating at various thermostat settings. *ASHRAE Journal* 16(12): 67.

---

## APPENDIX A: RECOVERY RESPONSE TRANSFER FUNCTION MODEL

The results presented here are derived in Appendix B of Armstrong et al. (1991). The model uses a star network to approximate radiant and convective heat exchange within an enclosure. A simplified method for estimating star resistances is presented along with the formulas for adjusting transfer function coefficients for a change in inside film resistance.

### Star Network Transfer Function Model

The star network model differs from the combined convection/radiation resistance model (ASHRAE 1989) by interposing a star temperature node between the room air temperature node and the wall surfaces (Seem 1987), as in Figure 4. A single resistance, $R_{rs}$, connects the room and star nodes. The star topology is not as exact as a full radiant network model but it eliminates most of the error inherent in the combined resistance model without adding to the computational effort at each time step.

**Heat Balance Equations**. The conduction transfer function model gives heat flux at the inside surface of the *j*th wall in terms of star temperature, $T_s$, as

$$q_j(t) = \sum_{k=1}^{N_j} d_{jk} q_j(t-k) + \sum_{k=0}^{N_j} b_{jk} T_a(t-k) - \sum_{k=0}^{N_j} c_{jk} T_s(t-k) \qquad \text{(A1)}$$

*where*

$t$ = current time index,

$t$-$k$ = time $k$ time steps in the past,

$T_a$ = outside temperature,

$b_{jk}$, $c_{jk}$, $d_{jk}$ = CTF coefficients of the $j^{th}$ wall,

$N_j$ = order of transfer function representing *j*th wall.

The relation between the true star temperature, $T_s$, and the temperature, $T_{sj}$ that would give the identical conduction heat

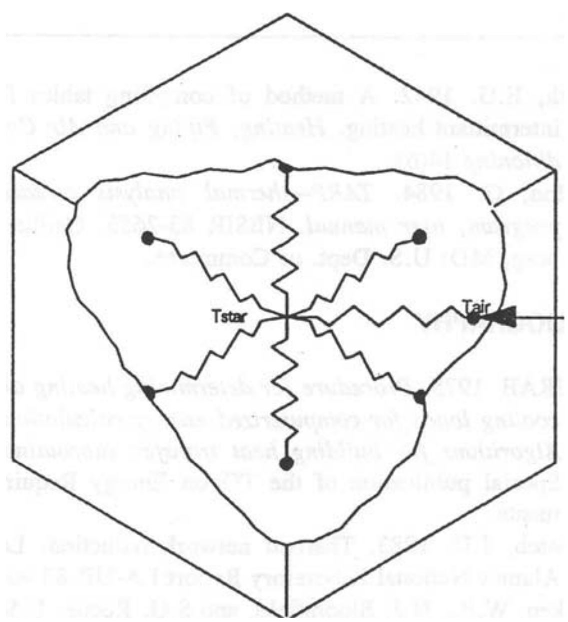


***Figure 4**. Schematic of the star network model for a simple room; heating of room air indicated by the arrow.*

flux at the inside surface of the jth wall in the absence of exogenous radiation is

$$T_{sj} = T_s + q_{rj}/h_{sj} \qquad \text{(A2)}$$

*Where*

$h_{sj}$ = surface-star film coefficient of *j*th wall and
$q_{rj}$ = exogenous radiation from all sources incident on wall *j* averaged over the time interval ending at time *t*.

**Closed Form Solution** Equations Al and A2 are combined, then decomposed into a part that is known and a part that is unknown at time *t*. The known part for exterior walls is

$$p_j(t) = \sum_{k=1}^{N_j} d_{jk} q_j(t-k) + \sum_{k=0}^{N_j} b_{jk} T_a(t-k) - \sum_{k=1}^{N_j} c_{jk} T_{sj}(t-k) - c_{j0}\frac{q_{rj}}{h_{sj}} \qquad (A3a)$$

The known part for symmetric interior walls is

$$p_j(t) = \sum_{k=1}^{N_j} d_{jk} q_j(t-k) + \sum_{k=1}^{N_j} (b_{jk} - c_{jk}) T_{sj}(t-k) + (b_{j0} - c_{j0})\frac{q_{rj}}{h_{sj}} \qquad (A3b)$$

Taking the sum over all walls yields the net rate of heat conduction into an envelope defined by the W inside wall surfaces,

$$Q_E(t) = \sum_{j=1}^{W} A_j q_j(t) = \sum_{j=1}^{W} A_j p_j(t) - T_s \sum_{j=1}^{W} A_j (b_{xj0} - c_{j0}) \qquad \text{(A4)}$$

*where*

$A_j$ = area of the *j*th wall,
$W$ = number of walls,
$b_{xj0}$ = *0* for exterior walls, but
$b_{xj0}$ = $b_{j0}$ for intrazone walls.

The air node heat balance is

$$Q_c(t) + \sum_{j=1}^{W} A_j q_j(t) + (T_r(t-1) - T_r(t))\frac{C_r}{\Delta t} = 0 \qquad \text{(A5)}$$

where $Q_c$ is the total exogenous (HVAC plus convective part of internal gains) heating of the room air node and $\Delta t$ is the time step size.

Combining Equations A4 and A5 to eliminate the common term that contains the unknown heat fluxes, $q_j(t)$, $j = 1{:}W$, and collecting terms in $T_r(t)$, yields

$$T_r(t) = \frac{T_r(t-1)\frac{C_r}{\Delta t} + Q_c(1 + R_{rs} UA_0 + \sum_{j=1}^{W} A_j p_j(t)}{UA_0 + \frac{C_r}{\Delta t}} \qquad \text{(A6)}$$

where

$$UA_0 = \sum_{j=1}^{W} A_j (b_{xj0} - c_{j0})$$

**Perceived Temperature** Operational temperature (ASHRAE 1985) is a weighted average of air temperature and mean radiant temperature that approximates occupant-perceived , temperature:

$$T_{op} = \alpha \frac{\sum_{j=1}^{W} A_j T_j}{\sum_{j=1}^{W} A_j} + (1-\alpha) T_r \qquad \text{(A7)}$$

## Step Response Application of the Model

**Initial Conditions** Initial conditions. must be evaluated before the step response can be evaluated. We solve for the steady-state star temperature that results in occupant-perceived temperature equal to the nighttime setpoint, then compute the corresponding heat flux for each wall.

$$T_s = \frac{T_n - \alpha(T_a \Sigma X_w + \Sigma\Delta T_r) + R_{rs}(T_a \Sigma UA_w + \Sigma Q_r)}{(1-\alpha)(1 + R_{rs}\Sigma UA_w) + \alpha \Sigma X_s} \qquad \text{(A8)}$$

where

$$\Sigma X_w = \frac{\sum_{j=1}^{W} A_j \frac{u_j}{u_j + h_{sj}}}{\sum_{j=1}^{W} A_j},$$

$$\Sigma X_s = \frac{\sum_{j=1}^{W} A_j \frac{h_{sj}}{u_j + h_{sj}}}{\sum_{j=1}^{W} A_j},$$

$$\Sigma\Delta T_r = \frac{\sum_{j=1}^{W} A_j \frac{q_{rj}}{u_j + h_{sj}}}{\sum_{j=1}^{W} A_j},$$

$$\Sigma UA_w = \sum_{j=1}^{W} A_j \frac{u_j h_{sj}}{u_j + h_{sj}},$$

$$\Sigma Q_r = \sum_{j=1}^{W} A_j q_{rj} \frac{h_{sj}}{u_j + h_{sj}}, \text{ and}$$

$u_j$ = U-value of $j$th wall including outside, but not inside, film resistance.

Given the initial star temperature, $T_s$, the initial inside wall temperatures can be evaluated by Equation A2, and the heat fluxes can then be evaluated by

$$q_j(t_{initial}) = \frac{T_s - T_{sj}}{\frac{1}{u_j} + \frac{1}{h_{sj}}} \quad for\ j = 1:W. \tag{A9}$$

*Algorithm*. The trajectory of the temperature that would be sensed by an occupant can now be evaluated by the following algorithm:

1. Evaluate $T_s$ by Equation A8 and initialize $T_s(t - k)$ for $k = 1:\max(N_j)$.
2. Initialize $T_{sj}(t - k)$ and $q_j(t - k)$ for $k = 1:N_j$, $j = 1:W$ by Equations A2 and A9.
3. Evaluate $p_j(t)$ for j = $1:W$ by Equation A3.
4. Evaluate $T_r(t)$ by Equation A6.
5. Evaluate $q_j(t)$ for $j = 1:W$ by Equation A1.
6. Evaluate $T_{op}$ by Equation A7.
7. If $T_{op} < T_{desired}$; set $t \leftarrow t + 1$ and go to step 3.

## Star Network Parameters

**Simplified Evaluation of Star Resistances.** Seem (1987) has developed a method for calculating surface.-star resistances and air-star resistance that minimizes heat balance errors. A simplified method based on the enclosure geometry of a cube is derived in the final report. The resulting formulas are presented below. Typical values of the convective and radiative film coefficients for a room are $h_c$ = 0.5 (2.8) and $h_r$ = 1.0 Btuh/ft$^2$°F (5.7 W/m$^2$K).

| Network Element | Surface | Area | Equivalent Film Coefficient | Network Resistance |
|---|---|---|---|---|
| Star Surface | $j$th | $A_j$ | $h_s = 1.2h_r + h_c$ | $R_s = \frac{1}{h_s A_j}$ |
| Air-Star | all | $\Sigma A_j$ | $h_A = h_c\left(1 - .833\frac{h_c}{h_r}\right)$ | $R_A = \frac{1}{h_a \Sigma A_j}$ |

**Adjusting CTF Coefficients for the Star Network** Tabulated values of transfer function coefficients cannot generally be used in the star network model without first adjusting the coefficients for the new inside surface film resistance. The formulas for adjusting transfer function coefficients are

$$d'_k = \frac{d_k + rc_k}{1 + rc_0}, \quad b'_k = \frac{b_k}{1 + rc_0}, \quad and\ c'_k = \frac{c_k}{1 + rc_0}$$

where $1/h_c$ is the inside resistance used to compute the original $b$, $c$, and $d$ coefficients; $1Ih_c'$ is the new inside surface.-star resistance; and $r = 1Ih_c' - 1Ih_c$.

If the original inside film resistance is 0, the formulas reduce to

$$d'_k = \frac{h'_c d_k + c_k}{h'_c + c_0}, \quad b'_k = \frac{h'_c b_k}{h'_c + c_0}, \quad and\ c'_k = \frac{h'_c c_k}{h'_c + c_0}$$

# APPENDIX B
# ZONE CONTENTS MODEL

Physical Properties of Multilayered Wall Constructions
Used to Simulate the Contents of All Fifteen Prototypical Buildings

| NL | CODE | MATERIAL | R-CUM | CAP | THICK | K | DENSITY | S.H. | RES |
|---|---|---|---|---|---|---|---|---|---|
| WALL: Shelved Books, low hc | | | | | | | | | |
| 1 | US | AIR SURFACE RES-INT | 0.75 | 0.00 | | | | | 1.50 |
| 2 | US | wood | 3.64 | 6.66 | 0.300 | 0.070 | 37.0 | 0.60 | |
| 3 | US | AIR SURFACE RES-INT | 6.54 | 0.00 | | | | | 1.50 |
| WALL: Shelved Books, high hc | | | | 0.00 | | | | | |
| 1 | US | AIR SURFACE RES-INT | 0.25 | 0.00 | | | | | 0.50 |
| 2 | US | wood | 2.64 | 6.66 | 0.300 | 0.070 | 37.0 | 0.60 | |
| 3 | US | AIR SURFACE RES-INT | 5.04 | 0.00 | | | | | 0.50 |
| WALL: Shelved Books, 3" thick | | | | | | | | | |
| 1 | E0 | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | US | wood | 2.47 | 5.55 | 0.250 | 0.070 | 37.0 | 0.60 | |
| 3 | E0 | AIR SURFACE RES-INT | 4.60 | 0.00 | | | | | 0.68 |
| WALL: Wood File | | | | | | | | | |
| 1 | E0 | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | D25 | Softwood | 0.99 | 0.44 | 0.042 | 0.067 | 32 | 0.33 | |
| 3 | B1 | AIR SPACE | 1.76 | 0.00 | | | | | 0.91 |
| 4 | D25 | Softwood | 4.08 | 2.64 | 0.250 | 0.067 | 32 | 0.33 | |
| 5 | B1 | AIR SPACE | 6.40 | 0.00 | | | | | 0.91 |
| 6 | D25 | Softwood | 7.17 | 0.44 | 0.042 | 0.067 | 32 | 0.33 | |
| 7 | E0 | AIR SURFACE RES-INT | 7.82 | 0.00 | | | | | 0.68 |
| WALL: Partition | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | GypBoard | 0.73 | 0.42 | 0.042 | 0.400 | 50.0 | 0.20 | |
| 3 | BAT | Air-MtlStud Cokmposite | 1.20 | 0.01 | 0.292 | 0.350 | 0.4 | 0.10 | |
| 4 | BAT | GypBoard | 1.67 | 0.42 | 0.042 | 0.400 | 50.0 | 0.20 | |
| 5 | BAT | AIR SURFACE RES-INT | 2.07 | 0.00 | | | | | 0.68 |
| WALL: Shelved Books, 3.6" thick | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | wood | 2.82 | 6.66 | 0.300 | 0.070 | 37.0 | 0.60 | |
| 3 | BAT | AIR SURFACE RES-INT | 5.31 | 0.00 | | | | | 0.68 |
| WALL: Shelved Books, 4" thick | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | wood | 3.06 | 7.40 | 0.333 | 0.070 | 37.0 | 0.60 | |
| 3 | BAT | AIR SURFACE RES-INT | 5.79 | 0.00 | | | | | 0.68 |
| WALL: Furniture | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | wood | 0.98 | 0.93 | 0.042 | 0.070 | 37.0 | 0.60 | |
| 3 | BAT | AIR SURFACE RES-INT | 1.74 | 0.00 | | | | | 0.91 |
| 4 | BAT | wood | 2.49 | 0.93 | 0.042 | 0.070 | 37.0 | 0.60 | |
| 5 | BAT | AIR SURFACE RES-INT | 3.13 | 0.00 | | | | | 0.68 |
| WALL: Wood | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | Steel | 0.68 | 0.10 | 0.002 | 26.000 | 480.0 | 0.10 | |
| 3 | BAT | AIR SURFACE RES-INT | 1.02 | 0.00 | | | | | 0.68 |
| WALL: Metal | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | Steel | 0.68 | 0.10 | 0.002 | 26.000 | 480.0 | 0.10 | |
| 3 | BAT | AIR SURFACE RES-INT | 1.02 | 0.00 | | | | | 0.68 |
| WALL: SAT w/R.A. plenum above | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | ACOUSTIC TILE | 1.58 | 0.38 | 0.063 | 0.035 | 30.0 | 0.20 | |
| 3 | BAT | AIR SURFACE RES-INT | 2.81 | 0.00 | | | | | 0.68 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| WALL: Wood File | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | Softwood | 1.07 | 0.55 | 0.052 | 0.067 | 32 | 0.33 | |
| 3 | BAT | AIR SPACE | 1.91 | 0.00 | | | | | 0.91 |
| 4 | BAT | Softwood | 4.55 | 3.09 | 0.293 | 0.067 | 32 | 0.33 | |
| 5 | BAT | AIR SPACE | 7.20 | 0.00 | | | | | 0.91 |
| 6 | BAT | Softwood | 8.04 | 0.55 | 0.052 | 0.067 | 32 | 0.33 | |
| 7 | BAT | AIR SURFACE RES-INT | 8.77 | 0.00 | | | | | 0.68 |
| WALL: Metal File | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | Steel | 0.68 | 0.10 | 0.002 | 26.000 | 480.0 | 0.10 | |
| 3 | BAT | AIR SPACE | 1.14 | 0.00 | | | | | 0.91 |
| 4 | BAT | Softwood | 3.78 | 3.09 | 0.293 | 0.067 | 32.0 | 0.33 | |
| 5 | BAT | AIR SPACE | 6.42 | 0.00 | | | | | 0.91 |
| 6 | BAT | Steel | 6.87 | 0.10 | 0.002 | 26.000 | 480.0 | 0.10 | |
| 7 | BAT | AIR SURFACE RES-INT | 7.21 | 0.00 | | | | | 0.68 |

# APPENDIX C
# ENVELOPE MODEL

Physical Properties of Multilayered Walls Used to Simulate
the Prototypical Buildings of Light, Medium, and Heavy Construction

| NL | CODE | MATERIAL | R-CUM | CAP | THICK | K | DENSITY | S.H. | RES |
|---|---|---|---|---|---|---|---|---|---|
| WALL: Light floor | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | H.W. CONCRETE | 0.77 | 4.67 | 0.167 | 1.000 | 140.0 | 0.20 | |
| 3 | BAT | AIR SURFACE RES-INT | 1.19 | 0.00 | | | | | 0.68 |
| WALL: Light R-11 wall | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-EXT | 0.17 | 0.00 | | | | | 0.33 |
| 2 | BAT | STUCCO | 0.44 | 1.93 | 0.083 | 0.400 | 116.0 | 0.20 | |
| 3 | BAT | AIR SPACE | 6.04 | 0.00 | | | | | 11.00 |
| 4 | BAT | PLASTER | 11.62 | 1.25 | 0.063 | 0.420 | 100.0 | 0.20 | |
| 5 | BAT | AIR SURFACE RES-INT | 12.03 | 0.00 | | | | | 0.68 |
| WALL: Medium floor | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | H.W. CONCRETE | 0.85 | 9.32 | 0.333 | 1.000 | 140.0 | 0.20 | |
| 3 | BAT | AIR SURFACE RES-INT | 1.36 | 0.00 | | | | | 0.68 |
| WALL: Medium R-11 wall | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-EXT | 0.17 | 0.00 | | | | | 0.33 |
| 2 | BAT | STUCCO | 0.44 | 1.93 | 0.083 | 0.400 | 116.0 | 0.20 | |
| 3 | BAT | AIR SPACE | 6.04 | 0.00 | | | | | 11.00 |
| 4 | BAT | H.W. CONCRETE | 11.71 | 9.33 | 0.333 | 1.000 | 140.0 | 0.20 | |
| 5 | BAT | AIR SURFACE RES-INT | 12.22 | 0.00 | | | | | 0.68 |
| WALL: Heavy floor | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-INT | 0.34 | 0.00 | | | | | 0.68 |
| 2 | BAT | H.W. CONCRETE | 0.93 | 14.00 | 0.5 | 1.000 | 140 | 0.2 | |
| 3 | BAT | AIR SURFACE RES-INT | 1.52 | 0.00 | | | | | 0.68 |
| WALL: Heavy R-11 wall | | | | | | | | | |
| 1 | BAT | AIR SURFACE RES-EXT | 0.17 | 0.00 | | | | | 0.33 |
| 2 | BAT | STUCCO | 0.44 | 1.93 | 0.083 | 0.400 | 116.0 | 0.20 | |
| 3 | BAT | AIR SPACE | 6.04 | 0.00 | | | | | 11.00 |
| 4 | BAT | H.W. CONCRETE | 11.79 | 14.00 | 0.5 | 1.000 | 140 | 0.2 | |
| 5 | BAT | AIR SURFACE RES-INT | 12.38 | 0.00 | | | | | 0.68 |