Control with Building Mass—
Part I: Thermal Response Model

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ABSTRACT

In this paper we derive a general model and reliable identification procedures that can be applied autonomously for online forecasting of wall, zone, and whole building transient thermal responses. Several important aspects of building HVAC operation—fault detection, curtailment, verification, model-based control, and commissioning—are enabled or facilitated by use of accurate building-specific models. The methods of thermal response model identification developed in this work require little or no user intervention. Test cases for the model and identification procedures include a laboratory test room and a 60-unit apartment building in Ryazan, Russia.

INTRODUCTION

Identification of transient thermal response models is motivated by three broad areas of application: peak shifting (including curtailment and night precooling), model-based control, and fault detection and diagnosis (FDD). Additional applications in which an accurate building-specific model is extremely useful include commissioning,1 planning (e.g., for efficiency retrofits), and verification of energy and demand savings.2

Load-shedding and peak-shifting strategies are constrained by building and HVAC plant characteristics as well as climate, utility rate structure, and building occupancy requirements. The analytical framework for dealing with different climates, rate structures, and occupancies is well established (Braun 1990; Kintner-Meyer and Emery 1995; Morris et al. 1994; Norford et al. 1985; Xing 2004). Reliable one-day-ahead weather forecasts can be downloaded daily or hourly. Internal gains, predominantly light and plug loads, can be disaggregated from the building total (Laughman et al. 2003; Kintner-Meyer 1994), and future loads can be forecast from the accumulated history (Seem and Braun 1991). However, thermal response to weather, internal loads, and HVAC inputs is unique to each building and difficult to characterize empirically (Braun and Chaturvedi 2002; Norford et al. 1985; Pryor and Winn 1982; Rabl and Norford 1991; Taylor and Pratt 1988). The development of a general model and model identification procedure sufficiently robust to be automated is therefore a central and challenging prerequisite to the implementation of useful control strategies such as curtailment or peak-shifting (Stoecker et al. 1981), pre-cooling (Brandemuehl et al. 1990; Braun 1990; Rabl and Norford 1991; Keeney and Braun 1997), and optimal start (Seem et al. 1989).

Model-based control, particularly of zone temperature control loops, can be used to improve comfort (reduce deviations from setpoint) and reduce the frequency and amplitude of control variable excursions. In a room temperature control application, the model provides a rational way to handle some or all of the measurable system excitations (sunshine, outside temperature or sol-air temperature, light and plug loads) that are normally treated as “disturbances” in traditional PID control.

Finally, on-line identification of zone and building thermal response models is essential to detection and diagnosis of HVAC faults that affect comfort and overall HVAC system

1. In future we expect that self-configuration and model identification will be key elements of automated commissioning.
2. In a building with conventional controls—i.e., constant setpoint during occupied periods—demand and energy savings are often rated separately because transient response, while important to demand savings, has little effect on energy savings.

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efficiency (Rossi and Braun 1993; Katipamula and Brambley 2005). Fault detection can be as simple as checking that zone temperatures respond to HVAC control modes (e.g., economizer operation, control of openings for natural cooling, changes in air or water flow rate and supply temperature, etc.) as predicted by a properly identified model. Faults of interest include simultaneous heating and cooling and open windows, doors, or dampers.

Accurate building-specific plant models, common to all of the above applications, will be economically useful only if the process of identification can be reliably automated. Models and identification methods that have been proposed for on-line identification of building thermal response are reviewed in the “Previous Work” section. The model structure found to be most generally useful and the new identification methods developed in this research are presented next. The metrics needed by an autonomous system to ensure that an identified model is of sufficient quality for control purposes are described, and typical results of a model identification exercise are presented in the “Model Application” section. A companion paper (Armstrong et al. 2006) presents formulations of model-based optimal control and estimates the benefits in a case study. Both papers summarize research documented in Armstrong (2004).

PREVIOUS WORK

Four approaches to thermal response model identification are steady-state models, calibrated forward models, finite-difference models, and transfer-function models. Degree-day models have been used to estimate fuel use since the early days of home coal delivery. A simple steady-state model can be used to estimate wall U-factors (Flanders 1985); more elaborate steady-state models have been used to estimate effective solar aperture, UA, and wind loss coefficient at the zone or building level (Quentzel 1976; Bushnell 1978). In these models, indoor temperatures are measured and typically processed, along with other time-series observations, as one-day or longer average values. The regression coefficients of a steady-state model provide no information about thermal capacitance or transient response.

The calibrated simulation approach lies at the other extreme. In this approach, an engineering description of the building is first developed and unknown parameters are identified. A detailed hour-by-hour simulation model that implements the heat balance method (ASHRAE 2005) is then run with the measured excitations, and the simulated and measured responses are compared. The parameters are adjusted and successive simulations are run until the measured and observed response discrepancies are reduced to some satisfactory level.

3 These are usually time domain transfer functions, although frequency domain methods have been used for network reduction by Green and Ulge (1979) and Balcomb (1983).

Topo-specific difference equations can usually be forced to return thermodynamically feasible models in a straightforward manner, e.g., by constraining the heat transfer and capacitance coefficients to be positive. However, the method is limited in practice to first- and second-order models, and the second-order topologies that can be handled require the analyst to associate one capacitance with the envelope and the other with contents. First-order models have been developed mainly for the purpose of long-term energy simulation of single zones or walls (Sonderegger 1978; Dexter 1981; Richardson and Berman 1981; Horn 1982). Second-order models have been developed for control and for more accurate estimation of equivalent thermal parameters (Pryor and Winn 1982; Taylor and Pratt 1988). Higher-order transfer function models have been derived, not from measured responses, but simulated responses of high-order, complex topology, state-space, and aggregate transfer-function models (Mitalas and Stephenson 1967; Pawelski et al. 1979; Ceylan and Meyers 1980; Harrison et al. 1968; Balcomb 1983; Seem 1987); the motivation for this model-order reduction was to run long-term simulations at low computational cost. Hybrid methods that use engineering models to produce initial guesses or basis functions (Subbarao 1985; Braun and Chaturvedi 2002) have also been developed. Topo-specific methods require some level of analytical skill and experience to develop a model kernel appropriate to a given building.

Transfer-function models have been successfully applied to data from laboratory and field tests of building components (Brown 1993; Armstrong et al. 2000). Rabl (1988) provides an extensive review and derivation of model identification procedures, including transfer function methods, for building envelopes. The few cases in which second- and higher-order transfer function models have actually been extracted from data (Seem and Hancock 1985; Subbarao 1985; Barakat 1987; Armstrong et al. 2000) have been accomplished only with considerable manipulation by a skilled analyst. Time-domain transfer function models are the most general linear models that have been used, and their coefficients can be easily estimated by ordinary least squares (OLS). However, the relation between model coefficients and the building’s physical parameters is complex. If these relationships are ignored (as is usual when taking a purely statistical approach, e.g., applying a multi-input ARMAX [Ljung 1999] model), the resulting model is likely to violate conservation of energy, to exhibit resonant behavior, or to be unstable or noncausal.

In this paper, constraints that ensure physically realistic behavior are described and identification methods are developed to impose the constraints automatically. These developments allow a practitioner to take advantage of the generality and model-order flexibility of transfer function models without getting into trouble with such pitfalls as overfitting.

4 These are time-discretized differential equations, usually based on a topology-specific discrete-time state-space model.
Figure 1 A general model should describe the transient interaction of zone temperature \(T\) and the rate of controlled zone heating or cooling \(Q_{HC}\) as well as the responses to uncontrolled conditions such as internal gains, \(Q_{L&P}\), and weather excitations such as solar-air temperature, incident solar fluxes, and wind-driven infiltration.

MODEL SYNTHESIS

Room air temperature, \(T\), responds to weather, internal gains, and thermal energy delivered by the HVAC system as shown schematically in Figure 1. “Weather” includes the effects of sun, wind, and outdoor temperature on all exterior surfaces, as well as direct gains through windows. Internal loads are usually dominated by office equipment and lighting, i.e., “light and plug loads,” \(Q_{L&P}\). The heating or cooling effect, \(Q_{HC}\), may be delivered by forced air, natural ventilation, or various types of hydronic terminal equipment.\(^5\)

The response to a given input is determined by the physical parameters and geometry of the building envelope, internal partitions, furniture, and other contents. Transient response, in particular, is determined by the thermal capacitance or “mass” as well as the thermal transmission properties of the building fabric. The engineering data needed to predict how a given building will respond are complex and subject to substantial uncertainty and, in many cases, to variation over how a given building will respond are complex and subject to substantial uncertainty and, in many cases, to variation over time.

For each application discussed in the “Introduction,” there is an objective function or control law that has a thermal response model embedded in it. It is critical that the thermal response model represent not the intended design but rather the building as built and currently configured, furnished, and used. We address the problem of producing such a model, with little or no analyst intervention, by new robust identification methods. These methods enable one to reliably obtain models based on measured excitations and the corresponding response. It is our experience that failure to obtain a satisfac-

tory model can always be traced to sensor error, to a failure to measure important boundary conditions, or to time-varying parameters.

The derivation of a model that is generally applicable to linear and weakly nonlinear thermal systems begins with Fourier’s diffusion equation (ASHRAE 2005; Carslaw and Jaeger 1959):

\[
\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} (1 - D)
\]

where \(k\) (conductivity, assumed isotropic) and \(c_p\) (thermal capacitance) are the local material properties of interest. Integrating the steady-state version of Equation 1 and applying boundary conditions for a set of walls that define a space gives

\[
Q = \sum_{w=1}^{W} u_w (T_w - T),
\]

where \(T_w\) is a uniform exogenous temperature for a surface,\(^6\) \(T\) is the zone temperature (Seem 1987),\(^7\) \(Q\) is the net heat input (flux integrated over area, \(W\) or Btuh) to the zone, and \(u = \Sigma UA\) (W/K or Btuh/°F) is the parallel conductance of one or more subareas that define the element.

An important constraint implicit in Equation 2 will become apparent as we develop a transient model. Consider the discrete-time, linear, time-invariant dynamic system with one wall. We have used the symbol \(t\) for continuous time (Equation 1) but \(i\) is now, and hereafter, defined as the discrete time index. It can be shown (Pipes 1957; Armstrong et al. 1983; Seem 1987) via a state-space formulation and the state-transition matrix of the resulting diagonalized system that Equation 1 corresponds to the following discrete time expression:

\[
B^{0:n}(\theta) + B^{0:n}(\theta, T_w) - B^{0:n}(\theta, T) = 0
\]

where the coefficient vectors each have \(n + 1\) elements,

\[
\phi = [\phi_0 \ \theta_1 \ \cdots \ \phi_n],
\]

\[
\theta = [\theta_0 \ \theta_1 \ \cdots \ \theta_n],
\]

\[
\theta_w = [\theta_{w,0} \ \theta_{w,1} \ \cdots \ \theta_{w,n}],
\]

the time-varying excitation and response vector terms have corresponding elements,

\[
Q = [Q(t) \ Q(t-1) \ \cdots \ Q(t-n)]
\]

\[
T = [T(t) \ T(t-1) \ \cdots \ T(t-n)]
\]

\[
T_w = [T_w(t) \ T_w(t-1) \ \cdots \ T_w(t-n)],
\]

\(^5\) Some HVAC distribution systems contain significant thermal storage capacity, and, moreover, differences between the heating and cooling distribution systems and terminal unit heat transfer modes may require separate terms to adequately model heating and cooling operations.

\(^6\) Several exterior surface or sol-air temperatures may be included as forcing functions.

\(^7\) The linearized star model (Davies 1973; Walton 1983; Seem et al. 1989) approximates \(T\) as a weighted mean of air and surface temperatures.
The transient response of load $Q$ (Equation 6) is known as a comprehensive room transfer function, or CRTF (Seem 1987). We will call the corresponding expression for zone temperature $T$ (Equation 8) the inverted CRTF, or iCRTF. Equations 6 and 8 are the two basic forms of the desired thermal response model postulated in Figure 1.

There are additional conditions the coefficients must satisfy for the CRTF and iCRTF models to be physically plausible. To see this, it is necessary to relate Equations 6 and 8 to their z-transforms. The system can be represented by $1 + W$ transfer functions for the zone thermal load response $Q$ and $1 + W$ transfer functions for the zone temperature response $T$, where $W$ is the number of walls exposed to different driving temperatures or sol-air conditions. In the z-transform domain we have:

$$G_{Q/T}(z) = \frac{Q(z)}{T(z)} = \sum_{k=0}^{n} \theta_{k} z^{-k}$$

$$G_{T/Q}(z) = \frac{T(z)}{Q(z)} = \sum_{k=0}^{n} \phi_{k} z^{-k}$$

$$G_{Q/T_w}(z) = \frac{Q(z)}{T_w(z)} = \sum_{k=0}^{n} \theta_{w,k} z^{-k}$$

$$G_{T/T_w}(z) = \frac{T(z)}{T_w(z)} = \sum_{k=0}^{n} \phi_{w,k} z^{-k}$$

Equations 9-12 make use of the z-transform shifting theorem (Ogata 1987) whereby

$$Z[\theta_{k} T(t-k)] = \theta_{k} z^{-k} T(z).$$

Equations 6 and 8 may be expressed in terms of the simple transfer functions (Equations 9-12). Thus, the CRTF becomes

$$Q(z) = G_{Q/T}(z)T(z) + \sum_{w} G_{Q/T_w}(z)T_w(z)$$

in z-transform notation and the iCRTF is

$$T(z) = G_{T/Q}(z)Q(z) + \sum_{w} G_{T/T_w}(z)T_w(z).$$

The roots of the denominators in Equations 9-12 correspond to the eigenvalues, $\lambda_{i}$, and, thus, to the time constants, $\tau_{i} = -1/\lambda_{i}$. The transient response of load $Q$ (Equation 6) is known as a comprehensive room transfer function, or CRTF (Seem 1987). We will call the corresponding expression for zone temperature $T$ (Equation 8) the inverted CRTF, or iCRTF. Equations 6 and 8 are the two basic forms of the desired thermal response model postulated in Figure 1.

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$$G_{T/T_w}(z) = \frac{T(z)}{T_w(z)} = \sum_{k=0}^{n} \phi_{w,k} z^{-k}$$

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The roots of the denominators in Equations 9-12 correspond to the eigenvalues, $\lambda_{i}$, and, thus, to the time constants, $\tau_{i} = -1/\lambda_{i}$.
of any system (CRTF or iCRTF) of interest as follows:

$$\ln \left( \frac{1}{r_i} \right) = \lambda_i \Delta t$$  \hspace{1cm} (14)

or, equivalently,

$$\ln (r_i) = \frac{\Delta t}{\tau_i}$$  \hspace{1cm} (15)

For diffusion processes, the characteristic frequencies, or poles, $\lambda_i$, must be real and negative, corresponding to time constants, $\tau_i$, that are real and positive. Note that the roots are monotonic in both $\tau$ and $\lambda$. It has been shown (Hittle and Bishop 1983) that for multilayer walls the surface flux and temperature response function’s pole locations must alternate along the real axis. That is, if the poles $[\lambda_{\phi1}]$ of $\Sigma \theta_i z^{-k}$ and the poles $[\lambda_{\theta1}]$ of $\Sigma \phi_i z^{-k}$ are each presented as ordered sets, then together they satisfy

$$\lambda_{\phi1} < \lambda_{\phi2} < \cdots < \lambda_{\phi n} < \lambda_{\theta1} < \lambda_{\theta2} < \cdots < \lambda_{\theta n}.  \hspace{1cm} (16)$$

Unless the coefficients of the transfer function denominators are somehow constrained, the parameter estimation process is likely to return one or more thermodynamically infeasible (complex or imaginary) poles. With some additional manipulation of Equation 16, a bounded nonlinear search algorithm (Gill et al. 1981) can be made to handle this constraint set.

**MODEL IDENTIFICATION**

The objective of model identification is to estimate, given noisy data, the parameters of Equations 6 and 8, subject to Equations 7 and 16. There are a number of different ways, in terms of problem formulation and solution algorithm, to address the adverse effects of observational noise and to handle the model constraints (Golub and Van Loan 1980; Wunsch 1996; Armstrong et al. 2000). Three formulations relevant to zone thermal response (CRTF and iCRTF) identification are developed below.

**Model Identification with U-Factor Constraints**

The discrete-time, linear, time-invariant simulation model (Equation 3) can, in principle, be obtained from observations of thermal response under a range of (zone heat rate and sol-air temperature) excitations by applying linear least squares.

However, unconstrained least-squares analysis minimizes the model-observation deviations without regard for the thermodynamic constraints previously noted. Another problem is that inverting a conduction transfer function (CTF) or CRTF model obtained by least squares often results in very poor forecasts or simulations. One could, instead, identify the inverted model from the data by least squares, but it would be desirable not to have to maintain two models that are inconsistent in terms of their equivalent thermal parameters, $UA$ and $mc_p$.

Therefore, we normalize temperatures by subtracting the current zone temperature or one of the sol-air temperatures from all other current and lagged temperatures. This eliminates the current zone (or sol-air) temperature term, reduces the order of the least squares problem by one, and results in a solution that satisfies the U-factor constraint (Armstrong et al. 2000).

**Model Identification with Constrained Poles**

The constraint represented by Equation 16 cannot be implemented in such a simple manner. However, a formulation that fits a standard bounded nonlinear search model is possible. From Equations 14 and 16 one can write the constraint as

$$r_{\theta1} > r_{\phi1} > r_{\phi2} > \cdots > r_{\theta n} > r_{\phi n}.  \hspace{1cm} (17)$$

We can now define a new search vector, $x = [x]$, each element of which has simple bounds, $0 < x < 1$, from which the roots can be evaluated recursively:

$$r_{\phi1} = x_{\phi1} r_{\phi max} \hspace{1cm} 0 < x_{\phi1} < 1$$
$$r_{\phi1} = x_{\phi1} r_{\phi1} \hspace{1cm} 0 < x_{\phi1} < 1$$
$$r_{\phi2} = x_{\phi2} r_{\phi2} \hspace{1cm} 0 < x_{\phi2} < 1$$
$$r_{\phi2} = x_{\phi2} r_{\phi2} \hspace{1cm} 0 < x_{\phi2} < 1$$

where

$$r_{\theta max} = \exp(\Delta t/\tau_{max}).$$

Besides reducing the multi-variable constraints to a set of simple bounds on individual variables, there are two other useful properties of this formulation. First, note that $x_{\phi i}$ corresponds to the largest time constant, and a reasonable upper limit, $\tau_{max}$, on this time constant can almost always be easily estimated. Second, minimum spacing of time constants can be imposed by using a lower bound that is greater than zero or an upper bound that is less than one. Without pole constraints, least squares identification will typically return parameters that correspond to an unstable model (simulation grows without bound), complex or imaginary eigenvalues (non-monotonic frequency response), or negative time constants (noncausality). None of these three properties is possible in a diffusion system.

**Model Identification with Uncertainty**

Evaluation of thermal response is needed in each of the two modes (Equations 6 and 8). Ordinary least squares anal-
ysis minimizes the Euclidean norm of errors in the one-step-ahead forecast of a selected response. Thus, if ordinary least squares (OLS) is applied to heat-rate observations in order to identify a CRTF model (Equation 6), temperatures forecast by the corresponding iCRTF model (Equation 8) are likely to have a relatively large error norm. Conversely, if OLS is applied to zone temperature observations in order to identify an iCRTF model, heat rates forecast by the corresponding CRTF model are likely to have a relatively large error norm. Total least squares (Golub and Van Loan 1980) may provide the better model parameter estimates.

The exogenous temperature errors are of less concern than the zone temperature and heat rate errors because we are not using the models to estimate or forecast the exogenous temperatures. This suggests an alternate formulation:

\[ Q + \theta_0 T = B^{1:n}(\phi, Q) + B^{0:n}(\theta_w, T_w) - B^{1:n}(0, T) \]  

(19)

in which the objective function for \( m \) observations (\( t = 1:m \) is

\[ J_g = ||e(1:m)|| = ||Q(1:m) + \theta_0[T(1:m) - x_g(1:m)]b_g || \]  

(20)

where

\( e(1:m) = \) vector of hybrid model residuals

\( ||v|| = \) the mean square norm of a vector or time series \( v \)

\( x_g(t) = \) \( [Q(t-k, k = 1:n)T(t-k, k = 1:n)T(t-k, k = 0:n)\cdots T_w(t-k, k = 0:n)] \)

\( b_g = [\phi(1:n) \theta(1:n) \theta(1:n) \theta_2(0:n) \cdots \theta_w(0:n)]. \)

In this formulation the zero-lag zone temperature coefficient, \( \theta_0 \), is estimated by nonlinear least squares. At each iteration, the left-hand-side observation vector, \( Q + \theta_0 T \), is generated and the remaining coefficients are estimated by OLS. The foregoing objective function, Equation 20, in combination with the nonlinear formulation that constrains poles, is another approach to be tested.

**MODEL APPLICATION**

Now we are in a position to fill in some of the Figure 1 details. Precooling and model-based controls typically treat zone temperature \( T \) as a given trajectory (Kintner-Meyer 1994) and simulated \( Q \) is added to the command given by a local-loop controller. Conversely, for optimal start, a \( Q \) trajectory is given and the response of interest is temperature as start of occupancy approaches. For curtailment, both modes are important. In deciding whether a given level of curtailment is feasible a \( Q \) trajectory (corresponding to the reduction in chiller power called for by the ISO) is given and the response of interest is comfort, i.e., the zone temperature trajectory estimated by the iCRTF. Once committed to curtailment, the verification of demand reduction can be based on the original CRTF model; the model tells us how much heating or cooling would have been required if the normal room temperature setpoint had been maintained.

The heat-rate response (CRTF) mode of model application is illustrated in Figure 2a. Inputs include all of the exogenous conditions of weather (temperature, solar gains on the main opaque and window surfaces of the envelope) and building operation (light and plug and certain other—mostly non-HVAC—electrical gains, metabolic gains). Zone temperature is an input and net heat rate over all interior surfaces is the output of interest. The transfer functions of Equations 9 and 11 give this response by superposition. The temperature-response (iCRTF) mode of model application is illustrated in Figure 2b. Inputs include the same exogenous conditions of weather and building operation. Net heat rate is an input and zone air temperature\(^9\) is the output of interest. The transfer functions of Equations 10 and 12 give this response by superposition.

Two applications follow. In these cases the system is modeled with just one external temperature excitation, \( T_1 \), or two external excitations, \( T_1 \) and \( T_2 \). In both cases it is shown that the transient responses are adequately characterized by a CRTF of low order, that is, \( n \) equal to 2 or 3.

**TEST ROOM RESULTS**

\( T_{op} = (T_{sup} + T_{air})/2 \). The ideal sensor measures what the occupant perceives, \( T_{occ} \), which is a weighted average from head to toe at whatever location the occupant chooses to occupy at a given time. A single fixed sensor can only provide an estimate, \( T_{sense} \), of occupant-perceived temperature. In the most general linear model, \( T_{occ} = C_s x + D_s u \) and \( T_{sense} = C_s x + D_s u \). These relations show that there is at least the possibility of obtaining a better estimate of \( T_{occ} \) from \( T_{sense} \) via a state observer.

\( ^9 \) Any of several response temperatures may be defined, such as star temperature (Davies 1973; Seem 1987) or operative temperature, \( T_{op} = (MRT + T_{air})/2 \). The ideal sensor measures what the occupant perceives, \( T_{occ} \), which is a weighted average from head to toe at whatever location the occupant chooses to occupy at a given time. A single fixed sensor can only provide an estimate, \( T_{sense} \), of occupant-perceived temperature.
A test room was instrumented to provide training and testing data. An axial fan drew air from a lab area (4850 ft² [451 m²]) via a supply duct and distributed it to the test room (740 ft² [68 m²]); zone air was allowed to return to the main lab through one or both sets of double doors. Our model treats the test room as having two exterior walls, one separating it from the large lab area and the other separating it from an adjacent room to the east of the test room. A lighting load of 2013 Btuh (2013 W) was the dominant source of internal gain. The supply airflow rate was measured and the corresponding thermal capacitance rate was calculated to be 1620 Btuh/°F (855 W/K). Eight temperature loggers, accurate to ±0.1°F (±0.06°C) in the room temperature (65-80°F [18-27°C]) range, were deployed to assess spatial variations; additional loggers monitored supply and exit airstream temperatures.

Model Identification Results

Parameters were determined by least-squares fit using each of the three response models: CRTF, iCRTF, and hybrid. The models have identical terms but different objective functions; the identified coefficients can, therefore, be quite different. Least-squares solutions for the three methods are shown in Tables 1-3, where \( T \) is in °F, \( Q \) is in Btuh, and the model time step is 15 minutes. Temperature \( T_1 \) refers to the large lab area and \( T_2 \) to the adjacent room; \( T_1 \) and \( T_2 \) are the two exogenous temperatures to which the test room responds. Each model was used to simulate the two responses of interest: zone temperature and zone heat rate. Results were assessed by residual analysis and cross-validation. In all time-series plots of \( Q \) and \( T \), the forecast response is solid and the observed response is dashed. Responses are shown in Figures 3-8.

Residual analysis indicated that the hybrid model identification procedure reduced the bad behavior of inverted models. The bias of predictions over the testing set was also somewhat smaller with the hybrid procedure. The behavior of the three fitting procedures can be viewed in terms of the equivalent thermal parameters as well as in terms of observation-prediction residuals.

### Residual Norms

The deviation of predicted from observed heat rate with CRTF model identification is on the order of 10% of rms heat rates measured during training and testing periods. The norms are 186 Btu/h (54 W) for training and 445 Btu/h (130 W) for testing. When zone temperature is treated as the response, the

### Table 1. Coefficients, Confidence Intervals (CI), and Time Constants from CRTF Identification

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th></th>
<th>coef</th>
<th>/CI</th>
<th>Root</th>
<th>Tau(hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q(t-0) )</td>
<td>-1.0000</td>
<td>9.6</td>
<td>1.0205</td>
<td>n.a.</td>
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<tr>
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<td>1.819</td>
<td>0.34</td>
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<tr>
<td>( Q(t-2) )</td>
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<td>1.4</td>
<td>0.1819</td>
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<td>( T_1(t-0) )</td>
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<td>210</td>
<td>0.3</td>
<td></td>
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<tr>
<td>( T_2(t-0) )</td>
<td>-1351</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( T_2(t-1) )</td>
<td>2371</td>
<td>0.7</td>
<td>1.3351</td>
<td>n.a.</td>
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<tr>
<td>( T_2(t-2) )</td>
<td>977</td>
<td>0.2</td>
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</tbody>
</table>

*The confidence interval (CI) is the one-standard-deviation band of uncertainty associated with a coefficient’s least-squares estimate; unless required by physics, a coefficient smaller in magnitude than its CI is discarded.

### Results Summary

### Table 2. Coefficients, Confidence Intervals, and Time Constants from iCRTF Identification

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th></th>
<th>coef</th>
<th>/CI</th>
<th>Root</th>
<th>Tau(hr)</th>
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<td>102271</td>
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<td>( T_1(t-1) )</td>
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<td>( T_1(t-2) )</td>
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### Table 3. Coefficients, Confidence Intervals, and Time Constants from Hybrid Identification

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<thead>
<tr>
<th>Term</th>
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<th>coef</th>
<th>/CI</th>
<th>Root</th>
<th>Tau(hr)</th>
</tr>
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<tr>
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<td>n.a.</td>
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<td>( T_2(t-1) )</td>
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<td>0.2</td>
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<td>( T_2(t-2) )</td>
<td>1544</td>
<td>0.3</td>
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**Figure 3** Forecast (solid line) of temperature by inversion of identified CRTF mode compared to measurement (dashed line). Training data are to the left of the vertical line and validation data are to the right.

**Figure 4** Forecast of heat rate by identified CRTF model.

**Figure 5** Forecast of temperature by identified iCRTF model.

**Figure 6** Forecast of heat rate by inversion of identified iCRTF model.

**Figure 7** Temperature by model identified using weighted sum of zero-lag observations.

**Figure 8** Heat rate by model identified using weighted sum of zero-lag response observations.
norms are 0.14°F (0.08°C) for testing and 0.33°F (0.18°C) for training. These error norms are quite reasonable, but the large deviations seen in Figures 3 and 4 during transients are unacceptable. The deviation of predicted from observed zone temperature with iCRTF model identification was on the order of 5% of the rms temperature variations measured during training and testing periods. The norms are 0.02°F (0.01°C) for training and 0.04°F (0.02°C) for testing. When the heat rate was simulated by the inverted model, the norms were acceptably large: 6328 Btu/h (1854 W) for testing and 11650 Btu/h (3413 W) for training. Hybrid identification gave the best of both worlds in terms of overall error norms (0.03°F [0.02°C] and 346 Btu/h [101 W] for training, 0.05°F [0.03°C] and 498 Btu/h [146 W] for testing) and deviations during transients and reduced bias during the test period.

**UA**

The zone load coefficients inferred from the three identification procedures were 15634 (Btu/h)/°F (8245 W/K) for the CRTF, 8317 (Btu/h)/°F (4386 W/K) for the iCRTF, and 4980 (Btu/h)/°F (2626 W/K) for the hybrid formulation. The calculated UA was only about 2000 (Btu/h)/°F (1055 W/K). This is partly explained by the large return air opening that provides a bidirectional free-convection path when the fan is off.

**Time Constants**

Two of the four time constants obtained by CRTF identification violated diffusion-system constraints. A similar result was obtained with iCRTF identification. In both cases the resulting models were unstable. The hybrid identification returned time constants that were all positive real and, furthermore, obeyed the alternating magnitude constraint.

**ZUBKOVA TEST RESULTS**

A 60-unit, five-story apartment building located at 22-1 Zubkova Street, Ryazan, Russia, serves as a second test case for model identification. This building was instrumented with 24 high-resolution room temperature loggers uniformly distributed among the five floors and four cardinal exposures. All building utilities, including cooking gas; domestic hot water, cold water, and waste water enthalpy flows; and space heating water and electricity, were monitored. A weather station on the roof measured wind speed, solar radiation, and outdoor temperature. Pre-cast panels for partitions as well as exterior walls and floor decks (Armstrong et al. 2000) give the building considerable thermal “inertia.”

**Test Conditions**

In many Russian apartment buildings the cold water enthalpy stream is significant. In the case of Zubkova, it is larger in magnitude than the heat input by electric power. Cold water, entering at 37-41°F (3-5°C) in winter, is mainly used in the toilets, often with leaky flush valves, and has a long residence time in the flush tanks. Waste water typically leaves the building at temperatures close to room temperature. The average, maximum, and minimum values of the daily mean outdoor temperature for December 4, 1996, to April 16, 1997, were 23.2°F, 51.6°F, and −17.5°F (−4.9°C, 10.9°C, and −27.5°C). Horizontal solar radiation averaged 20.0 Btu/h/ft² (63.2 W/m²), and wind speed averaged 8.5 mph (3.8 m/s). When district heat supply temperatures were significantly below the values established by the district heating outdoor temperature reset schedule, the room temperatures dropped to as low as 57°F (14°C). Tenants coped as best they could by drawing less cold tap water, drawing more hot water, and using their kitchen stoves to stay warm. Gas use more than doubles during periods of heat deficit—that is, at times when indoor temperatures are much below 64°F (18 °C).

**Model Identification**

The parameters of second- and third-order models, determined by least squares, are presented in Table 4, where \( T \) is in °C and \( Q \) is in kW. Residual error norms \( SE(Q) \) and \( SE(T) \) and equivalent thermal parameters \( UA \) and \( \tau \) are reported for the second- and third-order models in Table 5. Here \( T_1 \) refers to the outdoor dry-bulb temperature. The \( Q \) time constants and \( UA \) parameters of the building inferred from the two models are very similar. The \( T \) time constants for the second-order model are more in line with our expectations than the corresponding time constants of the third-order model, and, furthermore, the interleaving constraint on \( Q, T \) time constants is satisfied by the second-order model but not by the third. The tabulated results nevertheless show the distinct possibility of a meaningful third-order model. The auxiliary heat rate terms, when statistically significant, satisfy physical expectations and the first two \( (x_1 \text{ and } x_2) \) are at least statistically marginal.

The trend in the residuals is small but significant and probably results from the model’s use of total solar radiation on the horizontal instead of that incident on the walls. The ratios of incident and transmitted beam to horizontal beam radiation cannot be calculated without knowing the beam-diffuse split, which was not measured. These ratios vary systematically and substantially over the diurnal cycle and through the heating season.

The measured heat rate time series and corresponding model residuals are shown in Figure 9. The distribution of residuals (observations-predictions) is shown in the cumulative frequency plot (Figure 10), where the deviation from a normal distribution (dotted) is seen to be small except in the tails. One of the extreme values coincides with interruption of district heat service on January 5, 1997: we cannot say if the others are the result of errors in data or model inadequacy. The autocorrelation (Figure 11) shows a strong diurnal peak. This can again be attributed to the model using total solar radiation on the horizontal for both direct gain (windows) and sol-air temperature (exterior envelope) terms. The manual operation

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10. Use of radiation on at least the main SE-facing wall would probably improve the result.
of windows (exhaust rates were monitored but data were collected from the loggers only sporadically) and treatment of cold service water heating load as instantaneous may also contribute to the problem.

Results Summary

The Zubkova data set consistently produced physically valid second-order models with only the sum-of-temperature-coefficients constraint imposed. The distributions of residuals were Gaussian except for about 1% that may be considered outliers. The UA inferred by the models ranges within 10% of the calculated UA (Armstrong et al. 2000) and the value does not change substantially with model order. The inferred dominant time constants are shorter than the calculated value by factors of 4 to 8, but the interleaving of zone temperature and heat rate time constants is generally correct. The autocorrelation indicates a diurnal cycle in the residuals. This may be attributed to the use of solar radiation on the horizontal as an input when in fact the system is driven mainly by beam and diffuse radiation incident on the north and south walls. Other data limitations are the lack of albedo observations and lack of kitchen temperature measurements. Kitchen temperatures are important because kitchens generally get warmer than other rooms during cooking, and cooking generally occurs on a

Table 4. Coefficients and Confidence Intervals (CI) for Third-Order CRTF Model (Left) and Second-Order CRTF Model (Right) with a Two-hour Time Step. Auxiliary terms are $x_1 =$ constant, $x_2 =$ solar on horizontal, and $x_3 =$ the product of windspeed (m/s) and indoor-outdoor temperature difference (K).

| Term     | Units | Coefficient | CI    | |coeff|/CI | Coefficient | CI    | |coeff|/CI |
|----------|-------|-------------|-------|-------|------|-------------|-------|------|------|
| $Q(t - 0)$ kW | -1.0000 | n.a. | -1.0000 | n.a. | -1.0000 | n.a. |
| $Q(t - 1)$ kW | 1.0413 | 0.054 | 19.2764 | 1.0676 | 0.0553 | 19.3107 |
| $Q(t - 2)$ kW | -0.2657 | 0.0733 | 3.6228 | -0.2773 | 0.0487 | 5.6914 |
| $Q(t - 3)$ kW | 0.004 | 0.0507 | 0.0791 |
| $T(t - 0)$ °C | -50.8952 | 8.5922 | 5.9234 | -48.4847 | 8.6212 | 5.6239 |
| $T(t - 1)$ °C | 69.3284 | 2.8871 | 5.3797 | 72.474 | 12.7778 | 5.6719 |
| $T(t - 2)$ °C | -7.6737 | 3.1546 | 0.5833 | -25.4102 | 8.4852 | 2.9947 |
| $T(t - 3)$ °C | -12.2181 | 8.4002 | 1.4545 |
| $T_1(t - 0)$ °C | -1.0689 | n.a. | 0.158 | n.a. | n.a. |
| $T_1(t - 1)$ °C | -0.2401 | 1.0225 | 0.2348 | 2.5783 | 0.9387 | 2.7467 |
| $T_1(t - 2)$ °C | 3.8853 | 1.0091 | 3.8504 | -0.9994 | 0.5864 | 1.7044 |
| $T_1(t - 3)$ °C | -3.2555 | 0.5602 | 5.8112 |
| $x_1$ kW | 3.355 | 3.1904 | 1.0516 | 1.2471 | 3.2696 | 0.3814 |
| $x_2$ W/m² | -0.016 | 0.0122 | 0.2348 | 2.5783 | 0.9387 | 2.7467 |
| $x_3$ K·m/s | 3.8853 | 1.0091 | 3.8504 | -0.9994 | 0.5864 | 1.7044 |

Table 5. Model Standard Errors and Equivalent Thermal Parameters for the CRTF Models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Third-Order</th>
<th>Second-Order</th>
</tr>
</thead>
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<td>$\tau_{\theta_1}$ h</td>
<td>16.77</td>
<td>28.91</td>
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<tr>
<td>$\tau_{\theta_2}$ h</td>
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<td>$\tau_{\theta_3}$ h</td>
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<td>$\tau_{q_1}$ h</td>
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<tr>
<td>$\tau_{q_2}$ h</td>
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<td>$\tau_{q_3}$ h</td>
<td>0.484</td>
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<td>$UA$ kW/K</td>
<td>6.620</td>
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<tr>
<td>$SE(Q)$ kW</td>
<td>16.98</td>
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<tr>
<td>$SE(T)$ K</td>
<td>1.30</td>
<td>1.27</td>
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</table>

Figure 9 Observed (dotted) and predicted (solid) net enthalpy rate of utilities plus direct solar gains. The residual is the solid trace centered on zero.
diurnal schedule. Aggregate metabolic heat rate (estimated by \( x_1 \)) and other occupant behaviors, such as window opening, may also be significant.

**DISCUSSION**

The general model form and methods for its reliable identification have been shown to give good results for two real-world test cases. However, it would be surprising if such a simple model, particularly with its most basic properties of linearity and time-invariance, did not run into difficulty in some applications. We can anticipate what we believe are the most important of these.

Fan operation and other causes of change in indoor air movement\(^{11}\) will make the time-invariance assumption untenable in some cases. One possible approach in these cases is to define auxiliary inputs related to fan operation that allow a more elaborate, but time-invariant, model to predict response over a complete data set, as well as two simpler models, one for fan-on and one for fan-off operation. Variable airflow rates may require more elaborate treatment in some buildings.

It is still a challenge to obtain data of acceptable quality (accuracy and completeness) for identification of reliable and useful models. Data sets that include intake and exhaust rates, or related variables such as zone pressures, are needed when infiltration loads are significant. Separate observations of beam, diffuse, and ground-reflected radiation are needed when solar gains, incident-angle-dependent transmission, snow cover, and seasonally or diurnally varying shading are significant (Subbarao 1985). Finally, accurate observation of instantaneous room air temperature is often difficult but critical to reliable model identification.

CRTF models apply rigorously only to *sensible* heating and cooling loads. Transient latent cooling load responses of the building envelope and its contents to changes in zone humidity conditions may be important for the curtailment application (Fairey and Kerestecioglu 1985). However, the mass diffusion process may be too nonlinear for the successful application of approximate transient linear models. Future research should assess the importance of latent load to the curtailment control and precooling applications and, if found to be important, should seek new ways to implement model-based control autonomously and reliably.

**CONCLUSION**

A discrete-time representation of the general linear diffusion system with multiple, arbitrary, discrete forcing functions was developed. Three thermodynamic constraints were identified. The constraints are crucial to on-line identification for the model-based control application because the observations used for model identification are inevitably noisy and often incomplete or biased. Without the constraints, a physically implausible model that behaves badly under certain\(^{12}\) conditions will often be accepted as the “best” model implied by the data.

Methods were developed for enforcing the constraints using the simple optimization tools of ordinary least squares and bounded-search nonlinear least squares. The steady-state conductance constraint is handled algebraically. This approach eliminates one unknown but, more importantly, has the great advantage of leaving the problem in ordinary (linear, uncondi-\(^{12}\)

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\(^{11}\) For example, natural ventilation cooling or ventilation by large attic fans.

\(^{12}\) Usually conditions of fast transients or temperature or heat flux extremes not encountered in the training.
strained) least squares form. The constraint on time constants is, unfortunately, a nonlinear constraint. To avoid the difficulties traditionally associated with nonlinear constraints, a way was found to reformulate the problem as an unconstrained nonlinear least squares problem. A further modification was devised to enforce the constraint of alternating roots by imposing simple bounds on the transformed unknowns.

A new response that is the sum of the zero-lag terms for $Q$ and $T$ was defined. This approach has the advantage of mathematical simplicity but the disadvantage of making the estimation problem nonlinear in one unknown, the ratio of $Q$ and $T$ zero-lag coefficients. In the case of the test room, which exhibited bad behavior when its conventionally identified CRTF model was inverted, the new procedure gave very satisfactory results.

Second-order models were identified for both test cases. The massive Zubkova building came close to giving a third-order model and it is likely that better observations of excitations, particularly the solar gains, would result in reliable identification of valid third-order models for such buildings.

**ACKNOWLEDGMENT**

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