Low-Q: Proton Electric and Magnetic Form Factors The Mainz high precision proton form factor measurement

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PAVI09: From Parity Violation to Hadronic Structure and more...
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Outline

1. Introduction: The elastic electron-proton cross section
2. The Mainz high-precision p(e,e')p measurement
   - Design considerations
   - Covered kinematical region
3. First results
   - Background and Background subtraction
   - Cross sections
   - Radii and their model dependence
4. Open Issues
5. Conclusion

Cross section and form factors for elastic e-p scattering

The cross section:

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

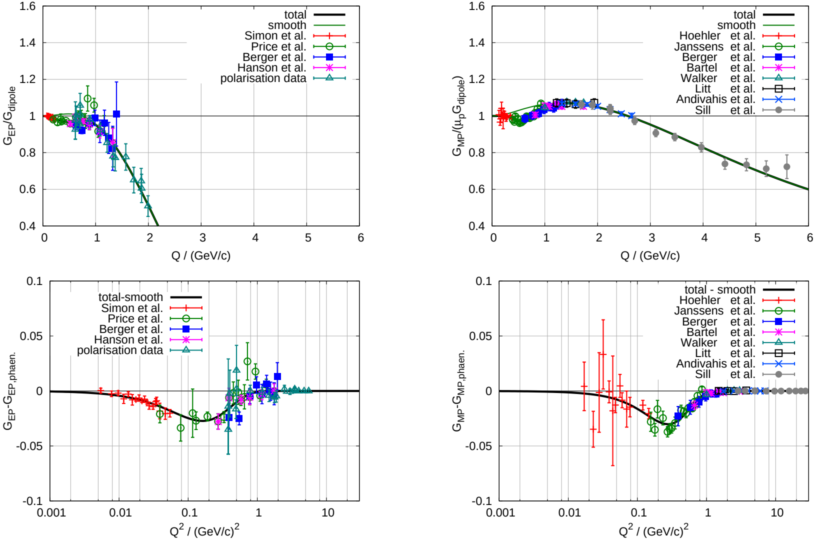
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2\left(1+\tau\right)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

Fourier-transform of G_E, G_M $\longrightarrow$ spatial distribution (Breit frame)

$$\left\langle r_E^2\right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\right|_{Q^2=0} \qquad \left\langle r_M^2\right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}\left(G_M/\mu_p\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

Motivation



(see J. Friedrich and Th. Walcher, Eur. Phys. J. A **17** (2003) 607)

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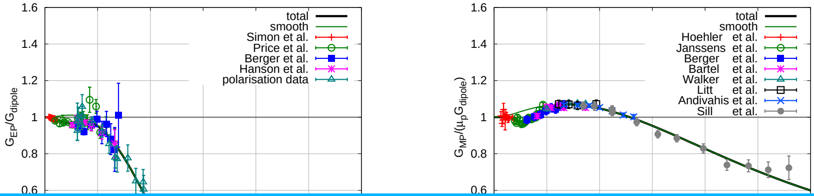
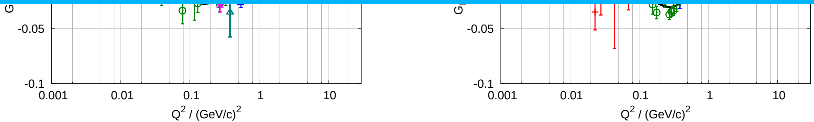


Discrepancy of existing values for proton electric radius:

- 0.809(11) fm: standard dipole at HEPL (Hand et al. 1963)
- 0.862(12) fm: low Q^2 at Mainz (Simon et al. 1979)
- 0.847(09) fm: dispersion relation (Mergell et al. 1996)
- 0.890(14) fm: Hydrogen Lamb shift (Udem et al. 1997)



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The Mainz high-precision p(e,e')p measurement at MAMI

Three spectrometer facility of the A1 collaboration:

Design goal: High precision

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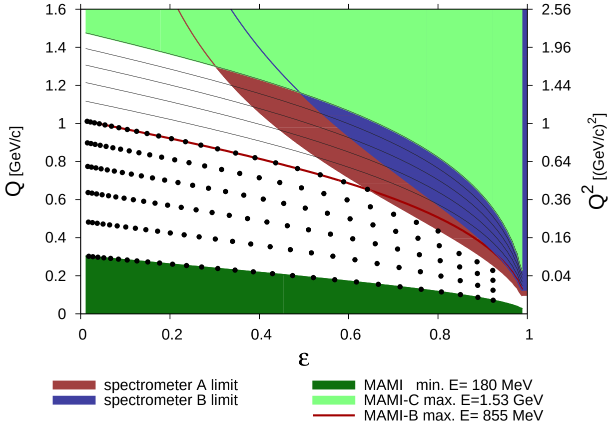
  - Overlapping acceptance
  - Where possible: Measure at the same scattering angle with two spectrometers

Measured settings and future expansion

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{\varepsilon(1+\tau)} \left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$



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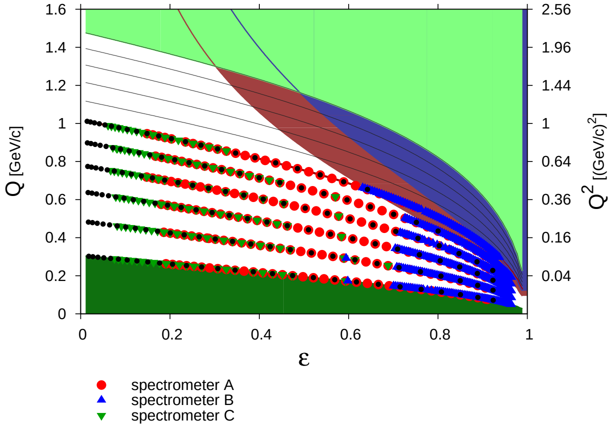


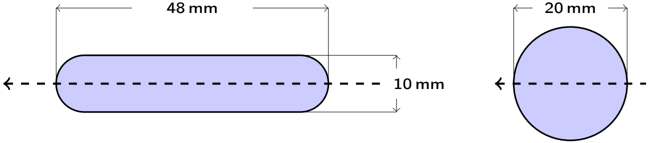
- ∼90 GByte on disc
- ∼1000 Settings
- $> 10^9$ events.

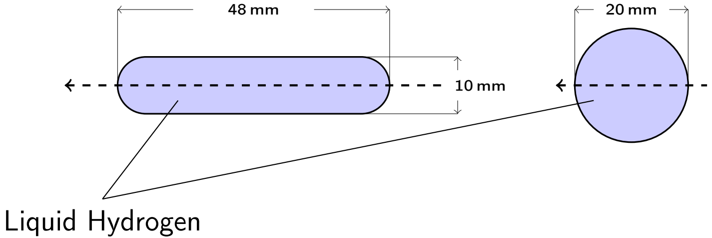
First results

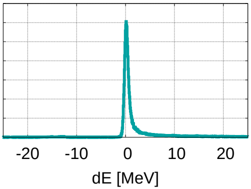
- Analysis is not complete!
  - Redundancy is not fully exploited.
  - Not all of the data completely analyzed!

All results are very preliminary! DO NOT QUOTE!

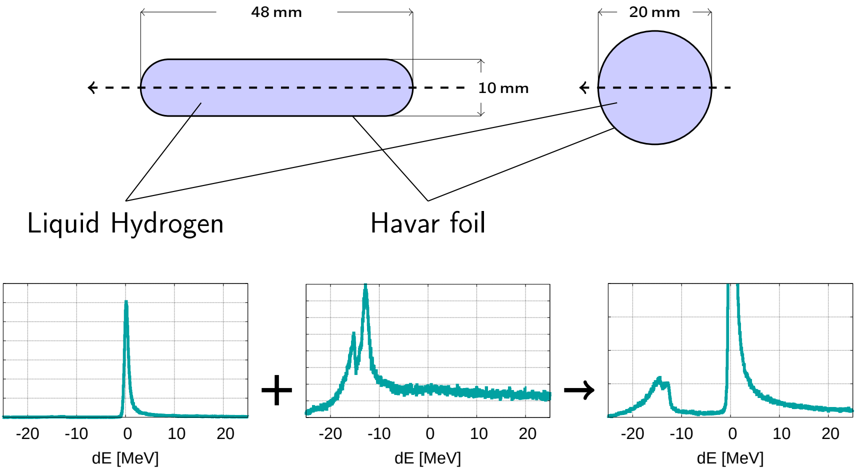
Background




48 mm
20 mm
10 mm
Liquid Hydrogen


-20
-10
0
10
20
dE [MeV]

Background


48 mm
20 mm
10 mm
Liquid Hydrogen
Havar foil
-20
-10
0
10
20
dE [MeV]
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$\Longrightarrow$Simulate and subtract background:

- Simulate elastic reactions off wall nuclei (Fe, Co, Cr, ...)
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$\Longrightarrow$Simulate and subtract background:

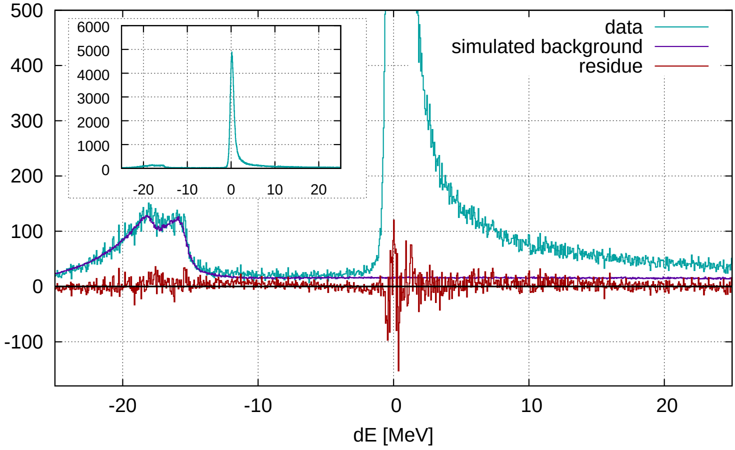
- Simulate elastic reactions off wall nuclei (Fe, Co, Cr, ...)
- Simulate quasi-elastic reaction (Fermi gas, de Forest)
- Simulate elastic scattering off hydrogen

$\Longrightarrow$Fit amplitudes to data

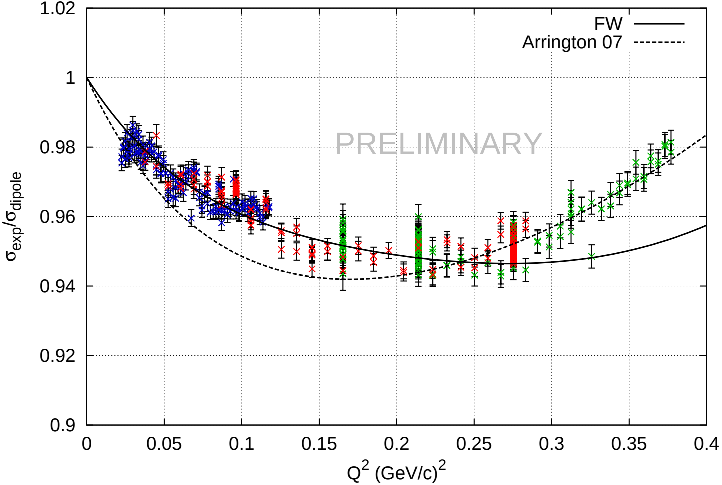
Data⟺Simulation matching

Simulation:

- Model for energy loss and small angle scattering
- Input: momentum, angular, vertex resolution of spectrometer



Cross sections: 450 MeV



[FW: Eur. Phys. J. A **17** (2003) 607, Arrington 07: Phys. Rev. C **76** 035205 (2007)]

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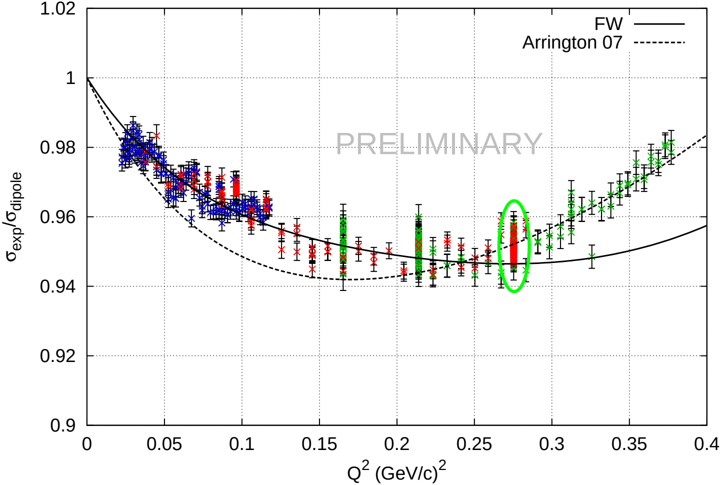

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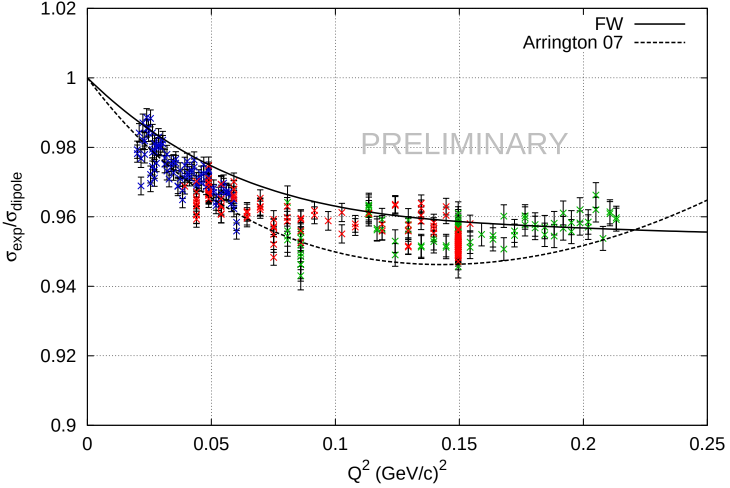
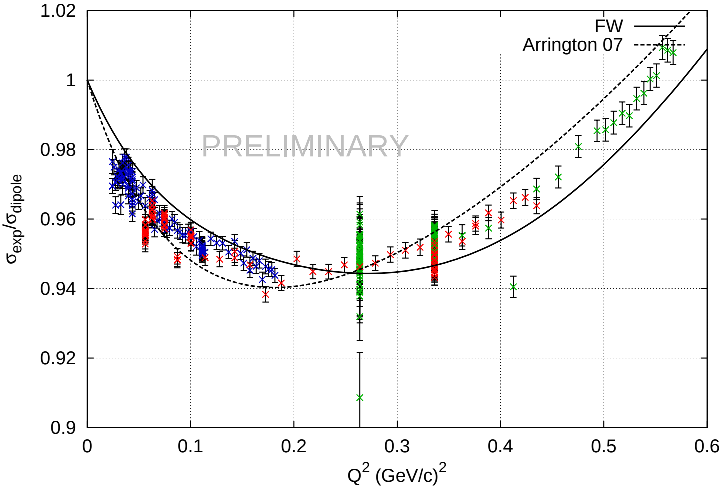
- Error bars: data spread $\longrightarrow 0.2 - 0.4\%$ (stat.: $0.1 - 0.3\%$)

Cross sections: 315 MeV



Cross sections: 585 MeV



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   - Feasible due to fast computers.
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For radii extraction: Needs a fit anyway!
Classical Rosenbluth: Extracted G_E and G_M highly correlated!
$\Longrightarrow$Error propagation very involved.

Some models

Dipole (different b for G_E and G_M):

$$G_D\left(Q^2, b\right) = \frac{1}{\left(1 + \frac{Q^2}{b}\right)^2}$$

Double Dipole (as in Friedrich/Walcher phenomenological fit [Eur. Phys. J. A **17** (2003) 607]):

$$G_{DD}\left(Q^2, a, b_1, b_2\right) = aG_D\left(Q^2, b_1\right) + (1 - a)\, G_D\left(Q^2, b_2\right)$$

More models

Polynomial × standard Dipole

$$G_P\left(Q^2, a_1, a_2, a_3, a_4\right) = \left(1 + a_1 Q^2 + a_2 Q^4 + a_3 Q^6 + a_4 Q^8\right) \cdot G_D\left(Q^2, 0.71\right)$$

Continued fraction expansion (I. Sick) [Phys. Rev. C **76** 035201 (2007)] :

$$G_C\left(Q^2, a_1, a2, ...\right) = \frac{1}{1 + \frac{a_1 Q^2}{1 + \frac{a_2 Q^2}{1 + \ldots}}}$$

Electric and magnetic rms radii

This fit was done at an early stage of the analysis!

- 840 data points ($< 2/3$ of all)
- Data normalization floating: 12 additional parameters
- Without correction for Coulomb distortion (Calculation by Friedrich: For 180 MeV: average effect $\approx 0.7\%$, variation over $Q^2 \approx 0.5\%$)
- Errors determined by perturbing the measured data according to error bars ($\pm 5\%$ normalization uncertainty) and refitting

Model	r_E [fm]	r_M [fm]	$\frac{\chi^2}{d.o.f.}$	number of fit parameters
Dipole	0.8706 ± 0.0011	0.8145 ± 0.0005	1.826	2
Double Dipole	0.8646 ± 0.0015	0.845 ± 0.002	1.467	6
Polynomial	0.879 ± 0.003	0.841 ± 0.008	1.083	8
Cont. Frac. 1	0.892 ± 0.004	0.845 ± 0.018	1.0599	10
Cont. Frac. 2	0.891 ± 0.003	0.866 ± 0.018	1.0595	10

Model dependence of radius extraction

Check extraction of radii with Monte-Carlo data:

- Monte-Carlo data from given parametrization (known radii!)
- Error distribution of this simulated data according to errors from real data
- Fit with different models

Assumption: $\pm 5\%$ normalization error

Model dependence of extracted rms radii

Difference of extracted radii to radii of input model, all radii in am

Input			Analysis							
			Dipole		Double Dipole		Polynomial		Cont. Frac.	
	r_E	r_M	r_E	r_M	r_E	r_M	r_E	r_M	r_E	r_M
Std. dipole	811	811	0 ± 1	0 ± 0.5	0 ± 2	0 ± 1	0 ± 3	0 ± 8	2 ± 6	7 ± 11
Arrington 07	878	858	-9 ± 1	-57 ± 0.5	-8 ± 2	-30 ± 2	-4 ± 3	-12 ± 7	0 ± 4	-3 ± 8
Arr. 03 (P)	829	837	29 ± 1	-34 ± 0.5	17 ± 2	-8 ± 2	3 ± 3	0 ± 7	0 ± 7	-2 ± 9
Arr. 03 (R)	868	863	0 ± 1	-55 ± 0.5	-1 ± 3	-14 ± 10	0 ± 2	-8 ± 7	0 ± 5	3 ± 8
FW	860	805	-1 ± 1	2 ± 0.5	-18 ± 2	32 ± 2	-3 ± 3	8 ± 8	2 ± 5	7 ± 18

[Arr. 03: Phys.Rev. C **69** 022201 (2004)]

Open Issues

- Better event generator for simulation
  (explicit calculation of Feynman diagrams of bremsstrahlung)
- Coulomb distortion, two photon effects etc.
- More models, e.g. models in r-space.
- Include existing, external data for higher Q^2.
- As many crosschecks as possible or as ideas come up.

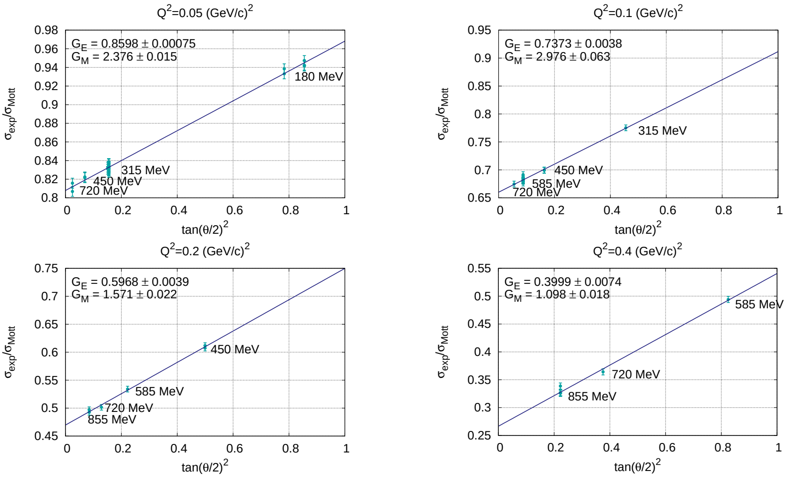
Conclusion

- The high precision data from MAMI promise a knowledge of separated electric and magnetic form factors in the Q^2 range from 0.003 to 1 $(\mathrm{GeV}/c)^2$.
- They will evidently also allow to calculate the ratio G_{Ep}/G_{Mp}.
- The expected errors are: $< 0.4\%$ statistical errors, $< 0.7\%$ systematical errors for the cross sections.
- Model dependence estimated for early analysis and reduced dataset:$< 1.5\%$ for radius, $< 0.5\%$ for G_E, G_M

Backup Foils

⟶Backup foils⟵

Classical Rosenbluth Separation



Continued Fraction Ambiguity