Charge-symmetry-breaking nucleon form factors

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Outline

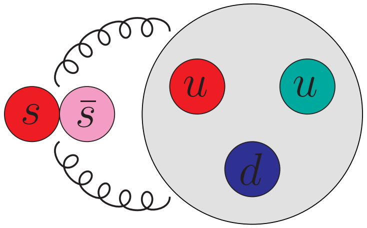
- Strangeness of the nucleon, parity violating electron scattering, and isospin violation
- Isospin violating form factors in chiral perturbation theory
- Resonance saturation
- Isospin mixing in Helium-4

BK, Lewis, Phys. Rev. C74 (2006) 015204

Viviani, Schiavilla, BK, Lewis et al., Phys. Rev. Lett. 99 (2007) 112002

Strangeness of the nucleon (1)

- strange quark–antiquark pairs in the proton:



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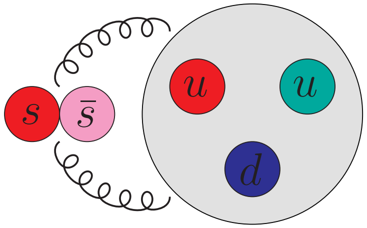
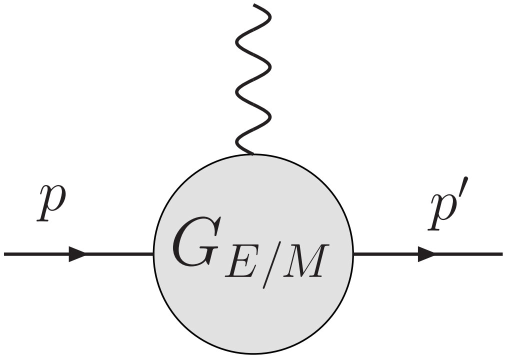


- How "strange" is the nucleon?

$\langle N \vert \bar{s}s \vert N \rangle$	contribution to mass $\Rightarrow$ σ-term
$\langle N \vert \bar{s}\gamma_\mu\gamma_5 s \vert N \rangle$	contribution to spin
$\langle N \vert \bar{s}\gamma_\mu s \vert N \rangle$	contribution to magnetic moment

- vector form factors:



- electric + magnetic form factors
- contribution of the three lightest quarks

$u,\ d,\ s$: $\boxed{G_{E/M}^{u,d,s}}$

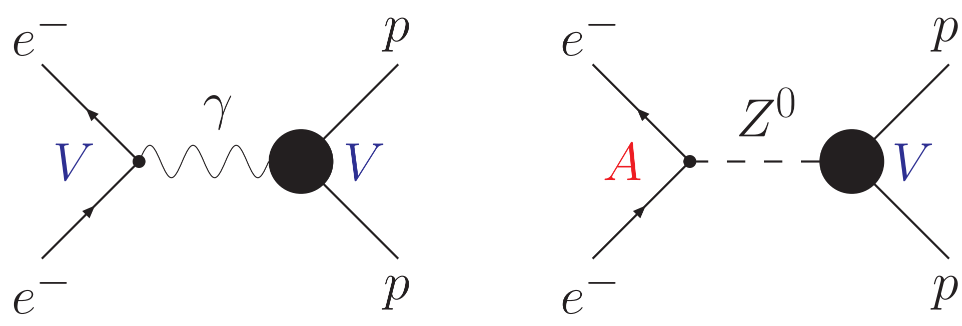
Strangeness of the nucleon (2)

- wanted: flavour decomposition of the vector current $\Rightarrow\ G^{s}_{E/M}$
- electromagnetic current: $J^{\mathrm{EM}}_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}(\bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s) \quad \Rightarrow$

$$G^{\gamma,p}_{E/M} = \frac{2}{3}G^{u}_{E/M} - \frac{1}{3}\big(G^{d}_{E/M} + G^{s}_{E/M}\big)$$

- different linear combination: weak vector current

$$G^{Z,p}_{E/M} = \Big(1 - \frac{8}{3}\sin^2\theta_W\Big)G^{u}_{E/M} - \Big(1 - \frac{4}{3}\sin^2\theta_W\Big)\big(G^{d}_{E/M} + G^{s}_{E/M}\big)$$



$\Rightarrow$ parity violating electron scattering SAMPLE, HAPPEX, A4, G0

Strangeness of the nucleon (3)

- third linear combination: isospin-(charge-)symmetry!

$$u \leftrightarrow d \; + \; p \leftrightarrow n \,, \qquad \text{i.e.} \quad G^{u,n}_{E/M} = G^{d,p}_{E/M} \quad \text{etc.}$$

$\Rightarrow$ use the neutron form factor as third input

$$\boxed{G^{Z,p}_{E/M} = \left(1 - 4\sin^2\theta_W\right) G^{\gamma,p}_{E/M} - G^{\gamma,n}_{E/M} - G^{s}_{E/M}}$$

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- without isospin conservation:

$$G^{u,d}_{E/M} = \frac{2}{3}\left(G^{d,p}_{E/M} - G^{u,n}_{E/M}\right) - \frac{1}{3}\left(G^{u,p}_{E/M} - G^{d,n}_{E/M}\right)$$

$\Rightarrow$ isospin violation generates “pseudo-strangeness”!

- experimental bounds on vector-strangeness tighter and tighter: should we be worried? $\Rightarrow$ quantify $G^{u,d}_{E/M}$ theoretically

Models for isospin-breaking form factors

(1) constituent quark model(s)

Dmitrašinović, Pollock, PRC 62 (1995) 1061

Miller, PRC (1998) 1492

(2) light-cone meson–baryon model

Ma, PLB 408 (1997) 387

see also Lewis@PAVI'06, EPJA 32 (2007) 409

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Problems with those:

- how to quantify accuracy/uncertainties?
  "how close" to the Standard Model?
- symmetries wrong?
  e.g. $G_M^{u,d}(t = 0) = 0$ in (1) due to symmetry in the quark model wave function; not required by Standard Model symmetries!

Isospin violation and chiral perturbation theory

- two sources of isospin violation in the standard model:

$$m_u \neq m_d \quad \text{(strong)} \qquad q_u \neq q_d \quad \text{(electromagnetic)}$$

- no fixed hierarchy between both effects:

$$\begin{aligned} M_{\pi^+} - M_{\pi^0} &\simeq 4.5\ \text{MeV}_{\text{em}} + 0.1\ \text{MeV}_{m_u \neq m_d} \\ m_n - m_p &\simeq -0.8\ \text{MeV}_{\text{em}} + 2.1\ \text{MeV}_{m_u \neq m_d} \\ \epsilon_{\pi^0\eta} &\simeq \big(\epsilon_{\pi^0\eta}\big)_{m_u \neq m_d} \qquad \text{(e.g. in } \eta \to 3\pi\text{)} \end{aligned}$$

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chiral perturbation theory (ChPT):

Weinberg (1979); Gasser, Leutwyler (1984, 1985)

- effective theory of the strong interactions
- dynamics and interaction of Goldstone bosons π (K, η)
- perturbation theory in quark masses
- inclusion of (virtual) electromagnetic effects

Urech, NPB 433 (1995) 234 . . .

ChPT: Generic power counting for nucleon form factors...

- generic expansion parameter $M_\pi/m_N \approx 0.15$

$$\underbrace{\mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}}_{\text{tree}} + \underbrace{\mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}}_{\text{1-loop}} + \ldots$$

- "conventional" Sachs form factors:

$$G_E(t) = F_1(t) + \frac{t}{4m_N^2}F_2(t) = \underbrace{Q}_{\mathcal{O}(p)} + \underbrace{\frac{1}{6}\langle r_E^2\rangle\, t}_{\mathcal{O}(p^3)} + \ldots$$

$$G_M(t) = F_1(t) + F_2(t) = \underbrace{\mu}_{\mathcal{O}(p^2)} + \underbrace{\frac{1}{6}\langle r_M^2\rangle\, t}_{\mathcal{O}(p^4)} + \ldots$$

- polynomial contributions (i.e. counterterms) to the electric (magnetic) radii appear at (sub)leading one-loop order

. . . and for isospin-breaking form factors

- “polynomial” isospin breaking suppressed by

$$m_d - m_u = \mathcal{O}(p^2) \qquad \text{or} \qquad e^2 = \mathcal{O}(p^2)$$

- therefore leading moments, generically:

$$G_E^{u,d}(t) = \underbrace{\rho_E^{u,d}}_{\mathcal{O}(p^5)} t + \mathcal{O}(t^2)$$

$$G_M^{u,d}(t) = \underbrace{\kappa^{u,d}}_{\mathcal{O}(p^4)} + \underbrace{\rho_M^{u,d}}_{\mathcal{O}(p^6)} t + \mathcal{O}(t^2)$$

- claim: calculate $G_E^{u,d}$ up to $\mathcal{O}(p^4)$, $G_M^{u,d}$ up to $\mathcal{O}(p^5)$

Why this is not quite *that* difficult

Lewis, Mobed, PRD 59 (1999) 073002

- claim: calculate $G_E^{u,d}$ up to $\mathcal{O}(p^4)$, $G_M^{u,d}$ up to $\mathcal{O}(p^5)$
- task: calculate all possible one- and two-loop diagrams with virtual pions and photons including all possible isospin breaking

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(1) no contribution from pion mass difference $M_{\pi^+}^2 - M_{\pi^0}^2 \;\propto\; e^2$:

charge symmetry: $u \leftrightarrow d$, $p \leftrightarrow n$

charge independence: general rotations in isospin space

$M_{\pi^+}^2 - M_{\pi^0}^2$ only breaks charge independence:

$$u \leftrightarrow d \;\;\Rightarrow\;\; \pi^+ \leftrightarrow \pi^- \;,\;\; \pi^0 \leftrightarrow \pi^0$$

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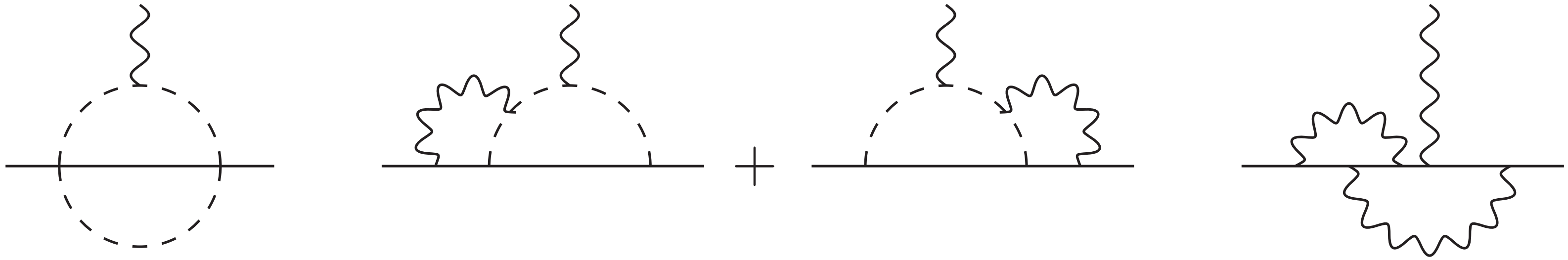
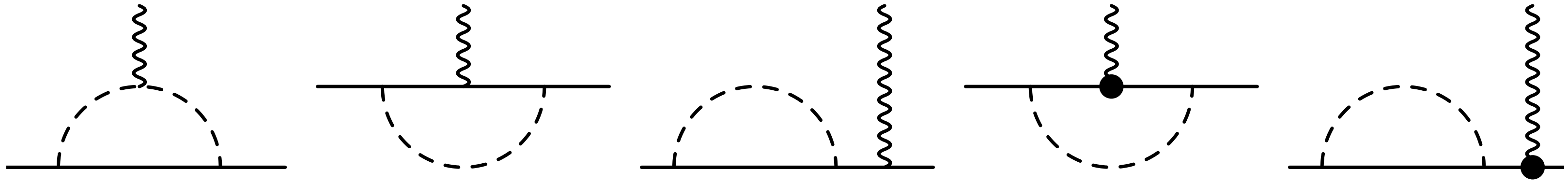
(2) no two-loop diagrams contribute (to $G_M^{u,d}$):



Isospin violating form factors in ChPT

- (non-trivial) diagrams:



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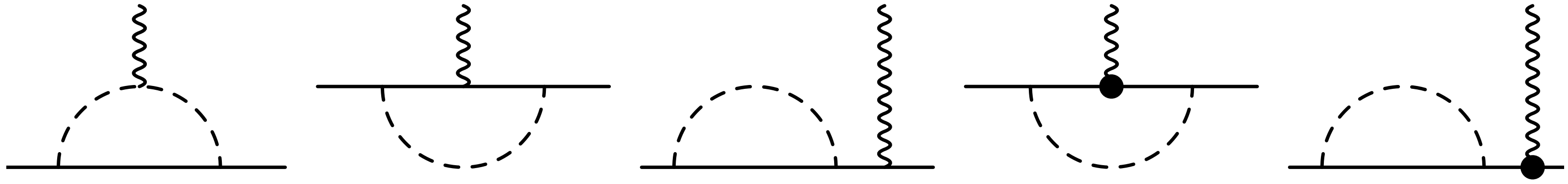


- radii: can be expressed in terms of $\Delta m = m_n - m_p$

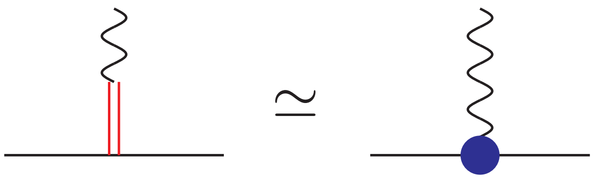
$$\rho_E^{u,d} = \frac{5\pi C}{6 M_\pi m_N} \, , \quad \rho_M^{u,d} = \frac{2C}{3 M_\pi^2} \left\{ 1 - \frac{7\pi}{4} \frac{M_\pi}{m_N} \right\} , \quad C = \frac{g_A^2 m_N \Delta m}{16\pi^2 F_\pi^2}$$

note: nonanalytic in M_π $\Rightarrow$ no undertermined counterterms!

- missing: unknown low-energy constant in $\kappa^{u,d}$
  $\Rightarrow$ resonance saturation

Resonance saturation

- low-energy constants parameterise effects of heavy (non-Goldstone boson) states:



$$\frac{1}{M_{\text{res}}^2 - t} \simeq \frac{1}{M_{\text{res}}^2}\left\{1 + \frac{t}{M_{\text{res}}^2} + \ldots\right\}$$

- modern form of "vector meson dominance"

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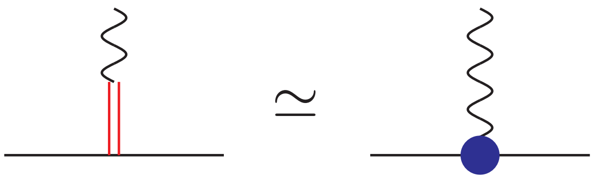

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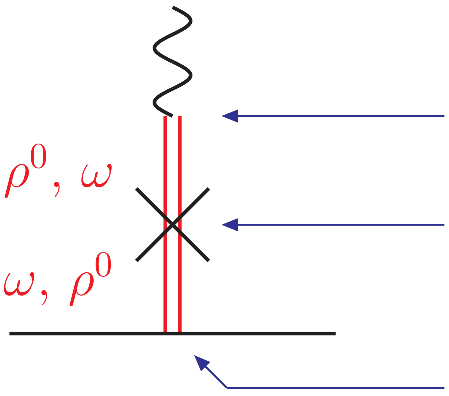
- modern form of “vector meson dominance”
- here: $\rho - \omega$ mixing



$F_\rho,\ F_\omega$ (decay constants) from $\rho^0,\ \omega \to e^+e^-$

$\Theta_{\rho\omega}$ (mixing angle) from pion form factor

$g_{\rho,\omega},\ \kappa_{\rho,\omega}$ from form factor dispersion analysis

Belushkin, Hammer, Meißner, PRC 75 (2007) 035202

- higher-order low-energy constants (in $\rho^{u,d}_{E/M}$) “for free”

Uncertainties in this procedure

- renormalisation scale dependence of low-energy constants:

$$\kappa^{u,d}(\lambda) \approx \kappa^{u,d}_{\text{res}} , \quad \lambda = ??$$

scale dependence only logarithmic

$$0.5 \text{ GeV} \le \lambda \le 1 \text{ GeV} \Rightarrow \delta\kappa^{u,d} = \pm 0.004$$

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- convergence of momentum dependence:
no significant two-loop contributions expected

Results (1)

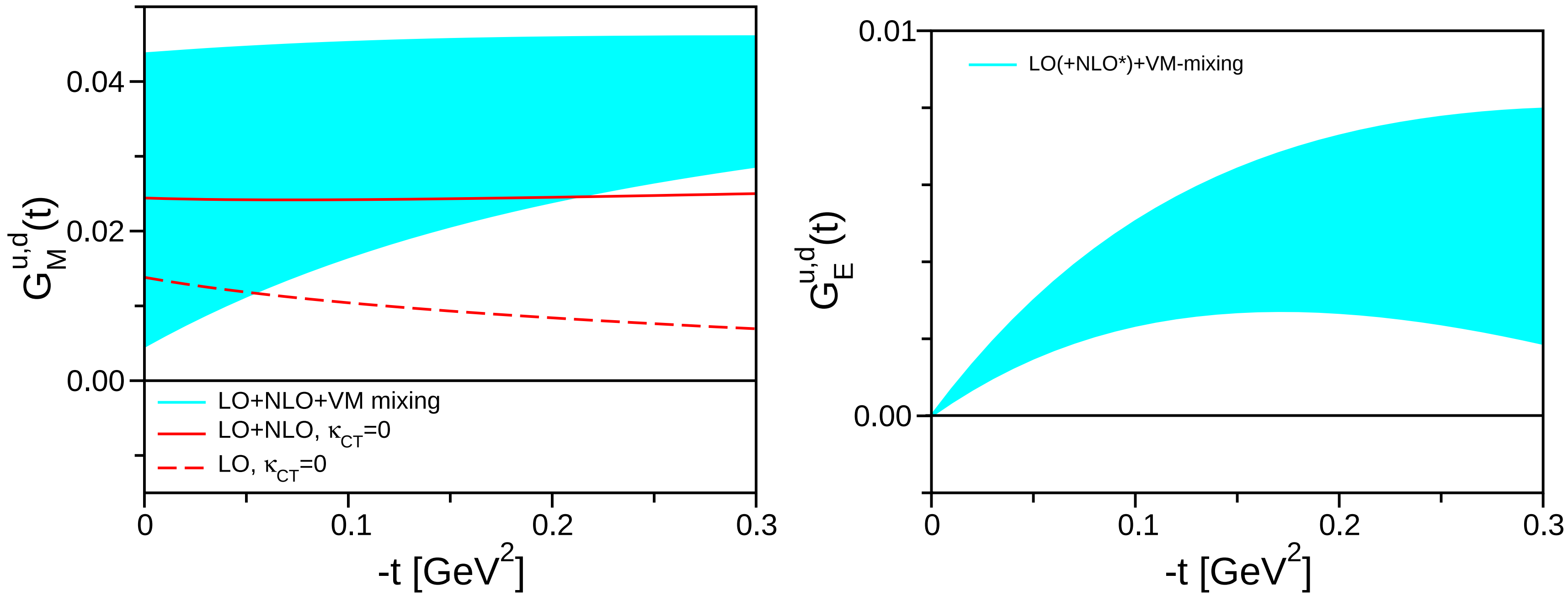
- compare loop- and counterterm contributions:

	$\kappa^{u,d}$	$\rho_E^{u,d}$	$\rho_M^{u,d}$
ChPT	0.025	0.03	0.01
$\rho^0 - \omega$ mixing	$-0.02 \ldots +0.02$	$-0.10 \ldots -0.07^*$	$-0.15 \ldots -0.03^*$
	> 0	< 0	< 0

- note: contributions marked * are subleading!
- uncertainties from badly known vector meson couplings (κ_ω)
- conclusion: strong vector meson contributions tend to upset power counting
- include full mixing amplitudes for predictions

Results (2)

- uncertainties in various couplings generate error bands:



- isospin breaking on the percent level
- t-dependence moderate
- unlike in some quark models, $G_M^{u,d}(0) \neq 0$!

Results (3)

- compare isospin breaking to strangeness at $t = -0.1$GeV^2:

experiment	electric/magnetic	G^s (measured)	$G^{u,d}$ (calculated)
SAMPLE	G_M	$0.37 \pm 0.20 \pm 0.26 \pm 0.07$	$0.02 \ldots 0.05$
A4	$G_E + 0.106\, G_M$	0.071 ± 0.036	$0.004 \ldots 0.010$
HAPPEX	$G_E + 0.080\, G_M$	$0.030 \pm 0.025 \pm 0.006 \pm 0.012$	$0.004 \ldots 0.009$

latest A4 results: Baunack et al., PRL 102 (2009) 151803

$$G_E^s(Q^2 = 0.23\text{GeV}^2) = (0.050 \rightarrow 0.045) \pm 0.038 \pm 0.019 \pm 0.002$$

$$G_M^s(Q^2 = 0.23\text{GeV}^2) = (-0.14 \rightarrow -0.18) \pm 0.11 \pm 0.11 \pm 0.01$$

- conclusion: isospin violation still smaller than other (experimental) uncertainties
- necessary correction for precision determinations of strange matrix elements

Isospin mixing in Helium-4 (1)

Viviani, Schiavilla, BK, Lewis et al., PRL 99 (2007) 112002

- parity-violating electron scattering on ^{4}He: $J^\pi = 0^+$ target $\Rightarrow$ no magnetic or axial vector transitions $\Rightarrow$ direct access to G_E^s
- isospin mixing in ^{4}He:
  nuclear form factors from isoscalar/isovector charge operators

$$\frac{1}{Z}\langle {}^4\mathrm{He}|\rho^{(0/1)}(\mathbf{q})|{}^4\mathrm{He}\rangle \equiv F^{(0/1)}(q)$$

- asymmetry:

$$A_{PV} = -\frac{G_F\, t}{4\pi\alpha\sqrt{2}}\left\{4\sin^2\theta_W + \Gamma\right\}, \quad \Gamma = -\underbrace{2\,\frac{F^{(1)}(q)}{F^{(0)}(q)}}_{\text{nuclear}} - \underbrace{\frac{G_E^{\not{s}} - G_E^s}{(G_E^p + G_E^n)/2}}_{\text{single nucleon}}$$

where $\quad G_E^{\not{s}} = \frac{1}{2}\left(G_E^{u,p} - G_E^{d,n} - G_E^{d,p} + G_E^{u,n}\right)$

Isospin mixing in Helium-4 (2)

$$A_{PV} = -\frac{G_F\, t}{4\pi\alpha\sqrt{2}}\left\{4\sin^2\theta_W + \Gamma\right\}$$

- experimental result for $t = -0.077$ GeV^2:

$$A_{PV} = \left[+6.40 \pm 0.23_{\mathsf{stat}} \pm 0.12_{\mathsf{syst}}\right] \times 10^{-6} \;\Rightarrow\; \Gamma = 0.010 \pm 0.038$$

HAPPEX, PRL 98 (2007) 032301

- various potential models including isospin violation for $F^{(1)}$:

$$\Gamma \;=\; \underbrace{-2\,\frac{F^{(1)}(q)}{F^{(0)}(q)}}_{\approx 0.003} \underbrace{-\frac{2G_E^{\not{s}}}{G_E^p + G_E^n}}_{0.008\pm 0.003} + \underbrace{\frac{2G_E^s}{G_E^p + G_E^n}}_{?} = 0.010 \pm 0.038$$

$$\boxed{G_E^s \;=\; -0.001 \pm 0.016}$$

$\Rightarrow$ central value of A_{PV} entirely due to isospin mixing

Conclusions

- combination of ChPT + resonance saturation allows for determination of isospin-violating nucleon form factors at low t
- relies on QCD symmetries + experimental data
- $G_{E/M}^{u,d}$ small, t-dependence moderate, $G_M^{u,d}(0) \neq 0$
- asymmetry on ^{4}He: significant corrections
  nucleon (as opposed to nuclear) isospin violation biggest effect
- necessary correction for precision determinations of strange matrix elements