LOW-ENERGY QCD and STRANGENESS in the NUCLEON

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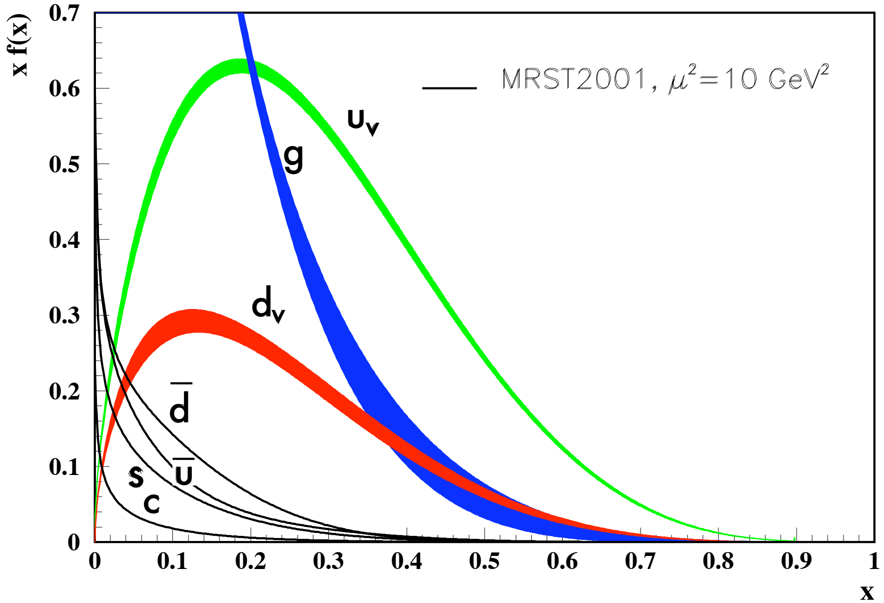
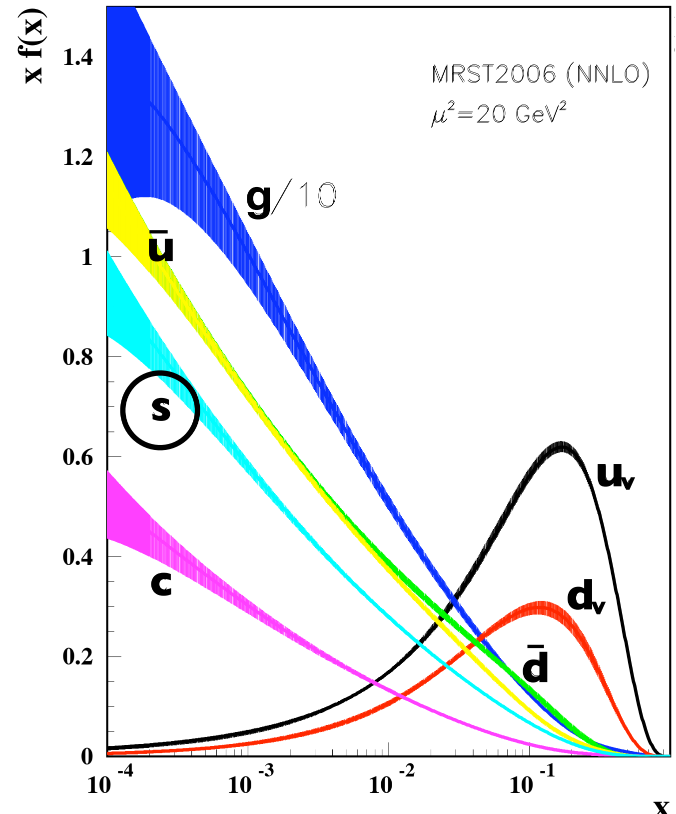
- **Strategies in Low-Energy QCD**:
  Lattice QCD and Chiral Effective Field Theory

- **Scalar Sector**:
  Nucleon **Mass** and **Sigma Term**

- **Vector Currents**:
  Role of **Strangeness** in e.m. **Form Factors**

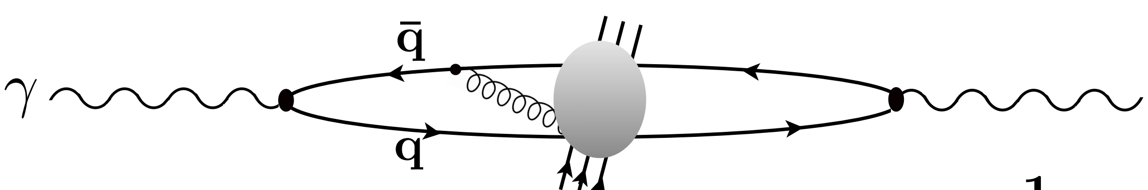
Prelude:
STRANGE SEA in Deep Inelastic Scattering





- Strange quark-antiquark pairs from (perturbative) QCD evolution at high Q^2 and small Bjorken-x

- Lab frame picture: highly excited interacting photon-nucleon system



$$\mathbf{x} = \frac{\mathbf{Q^2}}{\mathbf{2M\nu}}$$

large coherence length $\Delta\ell \sim \frac{1}{\mathrm{Mx}}$

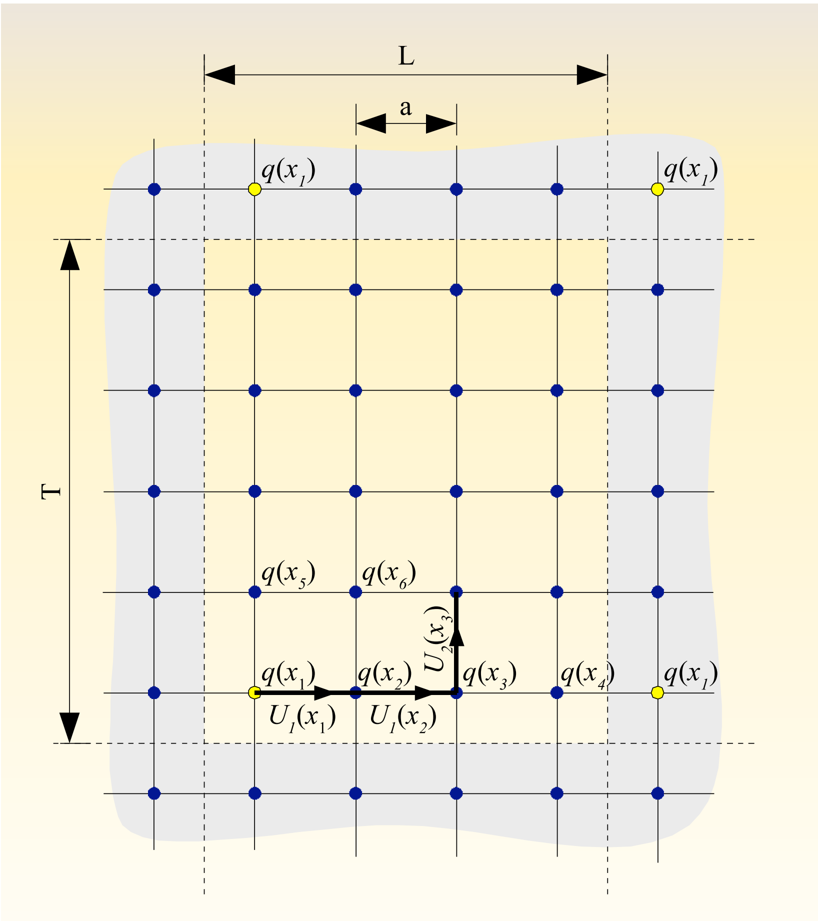
- **NOT** representative of sea quarks in the nucleon ground state

1.

LOW-Energy QCD: Concepts and Strategies

Lattice QCD ↔ **Chiral Effective Field Theory**

LATTICE QCD



- quarks $\psi(\mathbf{x}) = (\mathbf{u}(\mathbf{x}), \mathbf{d}(\mathbf{x}), \mathbf{s}(\mathbf{x}), \ldots)^{\mathbf{T}}$
  on nodes

- gauge fields (gluons)
  $$\mathbf{U}_\mu(\mathbf{x} \to \mathbf{y}) = \mathcal{P} \exp\left(-\int_{\mathbf{x}}^{\mathbf{y}} \mathbf{dx}' \mathcal{A}_\mu(\mathbf{x}')\right)$$
  on links

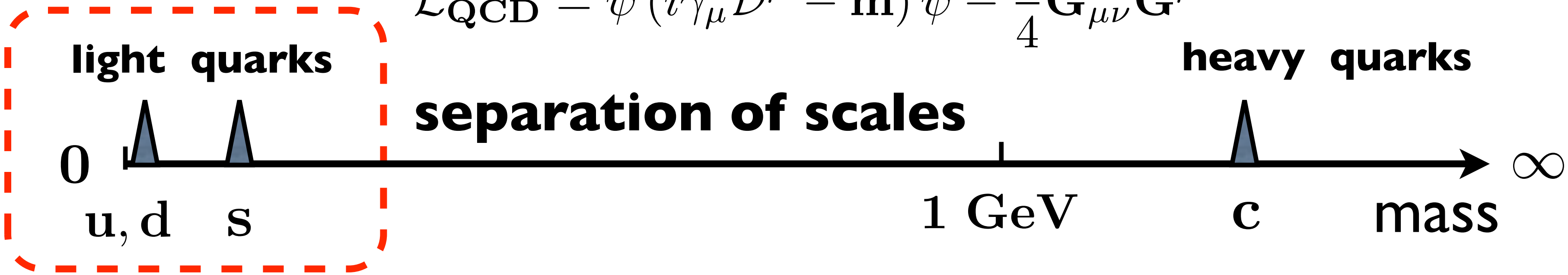
- for any operator $\hat{\mathcal{O}}$:
  $$\langle \hat{\mathcal{O}} \rangle = \frac{\int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}\mathcal{A}\ \hat{\mathcal{O}}\, \mathbf{e}^{-\mathbf{S}}}{\int \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, \mathcal{D}\mathcal{A}\ \mathbf{e}^{-\mathbf{S}}}$$
  $$\propto \int \mathcal{D}\mathbf{U}\ \hat{\mathcal{O}}_{\mathbf{eff}}[\mathbf{U}]\ \mathbf{e}^{-\mathbf{S_g}[\mathbf{U}]}\, \mathbf{det}\mathcal{M}[\mathbf{U}]$$

- **limits / extrapolations required**:

lattice spacing:	lattice volume:	quark masses:
$\mathbf{a} \to \mathbf{0}$	$\mathbf{L} \to \infty$	$\mathbf{m_q} \to \mathbf{m_q^{phys}}$

Hierarchy of **QUARK MASSES** and **SCALES** in **QCD**

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left(i\gamma_\mu \mathcal{D}^\mu - \mathbf{m}\right)\psi - \frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}$$



$\mathbf{m_d \simeq 3 - 7\ MeV}$	$\mathbf{m_u/m_d \sim 0.3 - 0.6}$
$\mathbf{m_s \simeq 70 - 120\ MeV}$	$(\mu \simeq \mathbf{2\ GeV})$

- **LOW-ENERGY QCD**: realized as **EFFECTIVE FIELD THEORY** with spontaneously broken **CHIRAL SYMMETRY**

$$\mathbf{SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V}$$

- **Expansion in powers** of $\mathbf{m_q}$ and **low momentum**

$m_c \simeq 1.25\ GeV$
$m_b \simeq 4.2\ GeV$
$m_t \simeq 174\ GeV$

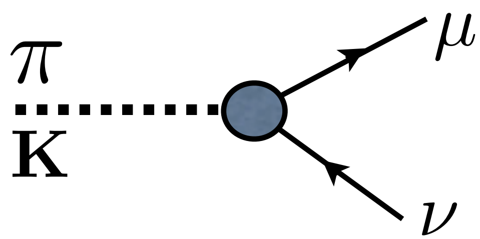
Non-**R**elativistic **QCD**:
HEAVY QUARK EFFECTIVE THEORY

Expansion in powers of $1/\mathbf{m_Q}$

Spontaneously Broken CHIRAL SYMMETRY

- **ORDER PARAMETERS**:
  DECAY CONSTANTS

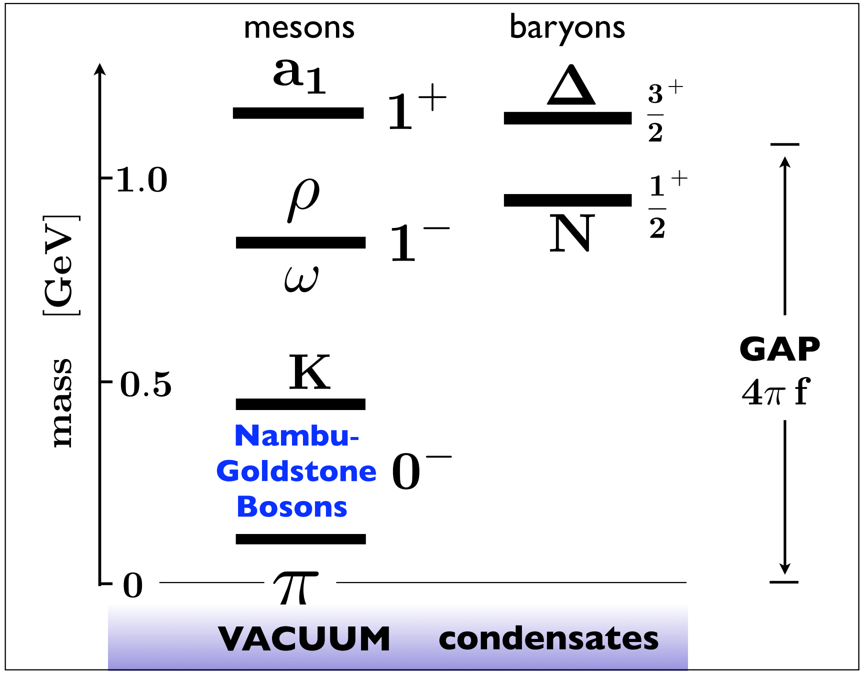
axial current



$$\mathrm{f}_\pi = 92.4\,\mathrm{MeV}$$

$$\mathrm{f_K} = 113.0 \pm 1.3\ \mathrm{MeV}$$

chiral limit: $\mathrm{f} = 86.2\ \mathrm{MeV}$



- **Symmetry Breaking**:

explicit SB

$$\mathrm{m}_\pi^2\,\mathrm{f}^2 = -\mathrm{m_q}\,\langle\bar{\psi}\psi\rangle + \mathcal{O}(m_q^2)$$

spontaneous SB

Gell-Mann, Oakes, Renner

SYMMETRY BREAKING SCALE ⟷ **MASS GAP**

$$\Lambda_\chi = 4\pi\,\mathrm{f} \sim 1\,\mathrm{GeV}$$

CHIRAL SU(3) EFFECTIVE FIELD THEORY

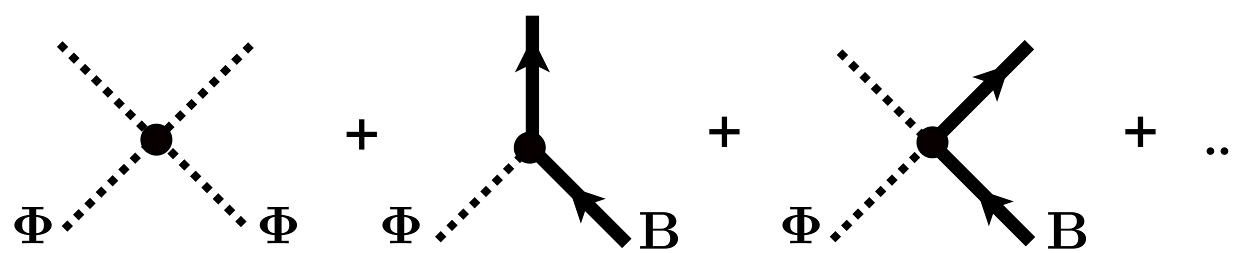
- Interacting systems of **NAMBU-GOLDSTONE BOSONS** (**pions, kaons**) coupled to **BARYONS**

$$\mathcal{L}_{eff} = \mathcal{L}_{mesons}(\Phi) + \mathcal{L}_B(\Phi, \Psi_B)$$

- Leading **DERIVATIVE** couplings (involving $\partial^\mu \Phi$) determined by spontaneously broken **CHIRAL SYMMETRY**

Φ : **pseudoscalar meson octet**

B : **baryon octet**



Low-Energy Expansion: **CHIRAL PERTURBATION THEORY**

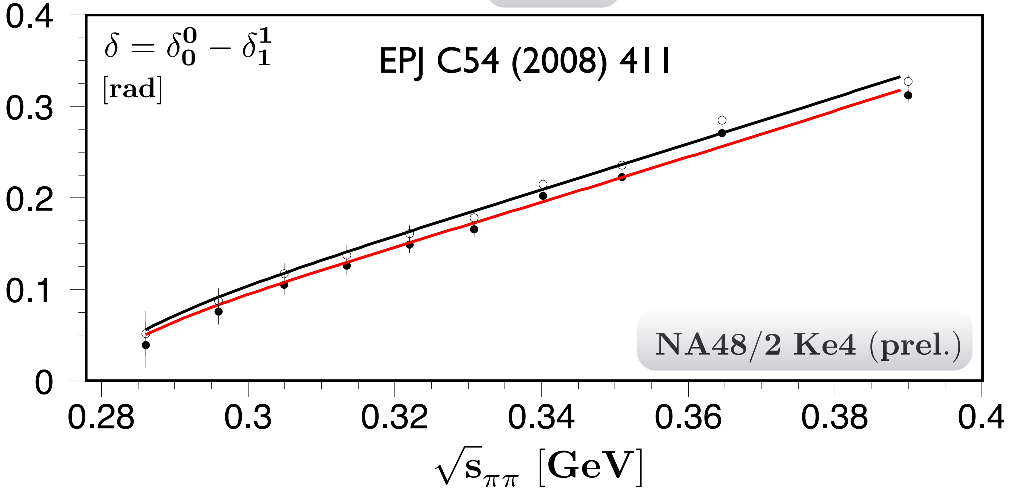
small parameter: $\dfrac{\mathbf{p}}{4\pi\, \mathbf{f}_\pi}$ $\dfrac{\text{energy / momentum / quark mass}}{\text{chiral s.b. scale of order } 1\ \mathrm{GeV}}$

CHIRAL EFFECTIVE FIELD THEORY

- . . . works **quantitatively** for low-energy **pion-pion** interactions, also quite well for processes involving **kaons** and for **pion-nucleon** interactions

- Precision measurements of $\pi\pi$ scattering lengths a_0, a_2

$$K^\pm \to \pi^+\pi^- e^\pm \nu$$

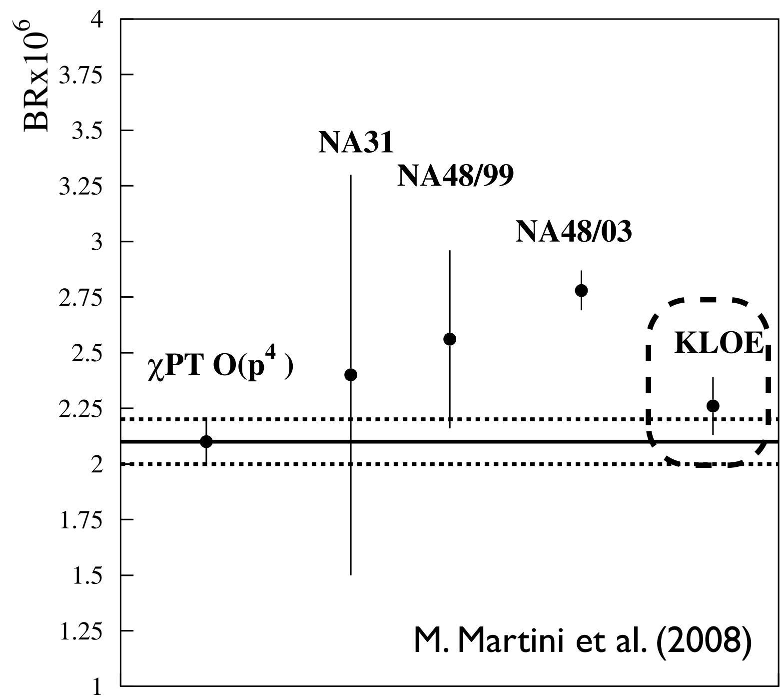


	Theory (ChPT)	Exp (NA48/2)
a_0	0.220 ± 0.005	0.218 ± 0.013
a_2	-0.044 ± 0.001	-0.0457 ± 0.0084

(in units of m_π^{-1})

G. Colangelo et al., Nucl. Phys. B 603 (2001) 125

- Precision measurement of $K_S \to \gamma\gamma$ (new KLOE result)



$$B(K_S \to \gamma\gamma) = (2.26 \pm 0.12 \pm 0.06) \times 10^{-6}$$

in perfect agreement with **ChPT**

2.
NUCLEON MASS and SIGMA TERMS

- Determination of **PION-NUCLEON SIGMA TERM**
- Contribution of **STRANGE SEA** to **SCALAR** $\bar{q}q$ **DENSITY** in the nucleon
- **Headline News:**

Hadronic Uncertainties in the Elastic Scattering of Supersymmetric Dark Matter

John Ellis,[1,*] Keith A. Olive,[2,†] and Christopher Savage[2,‡]

... ignorance of the $\langle N|\bar{q}q|N\rangle$ matrix elements linked to the π-nucleon σ term, ...
... impacting the interpretations of experimental searches for cold dark matter.
We plead for an experimental campaign to determine better the π-nucleon σ term.

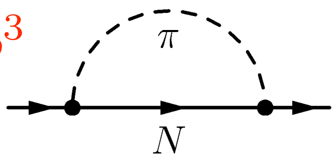
Phys. Rev. D77 (2008) 065026

MASS of the NUCLEON:
LATTICE QCD + CHIRAL PERTURBATION THEORY

$$\mathbf{M_N} = \langle \mathbf{N} | \frac{\beta(\mathbf{g})}{2\mathbf{g}} \mathbf{Tr}(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) + \sum_{\mathbf{i}} \mathbf{m_i}\,\bar{\mathbf{q}}_\mathbf{i}\mathbf{q}_\mathbf{i} | \mathbf{N} \rangle = \mathbf{M_0} + \mathbf{\Delta M}(\mathbf{m}_\pi)$$

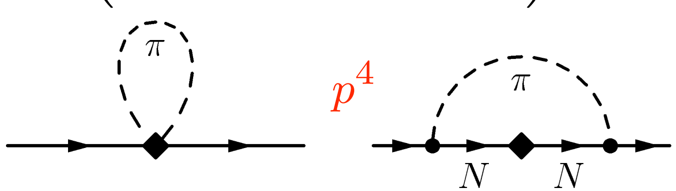
$$M_N = M_0 - 4\,c_1 m_\pi^2 - \frac{3\,g_A^2}{32\pi f_\pi^2}\, m_\pi^3$$

p^3



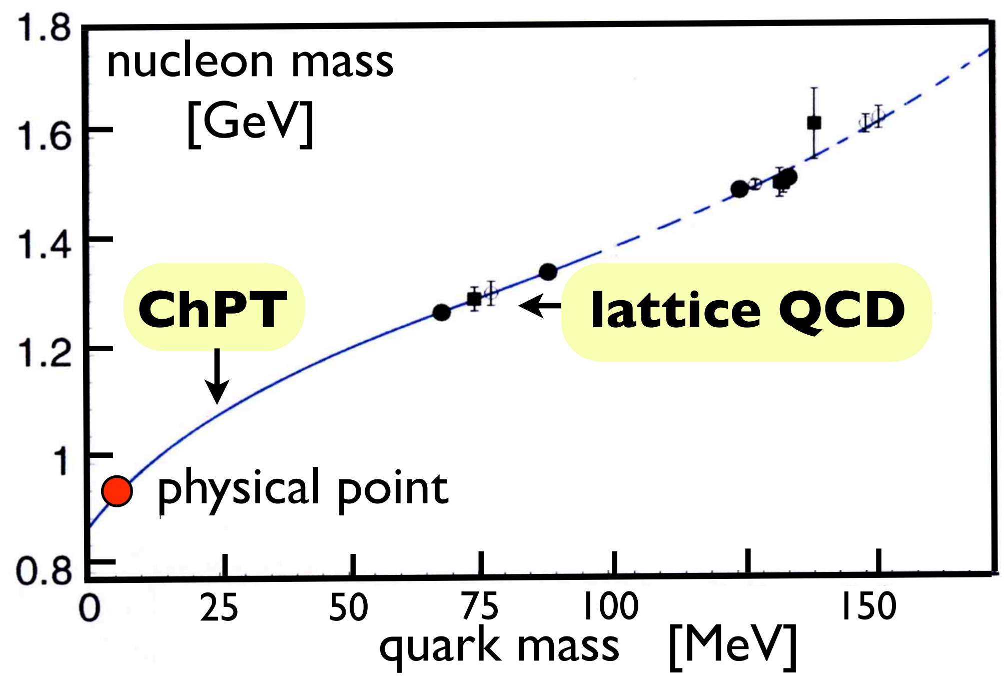
$$+ \left[4\,e_1^r(\lambda) + \frac{3\,c_2}{128\pi^2 f_\pi^2} - \frac{3\,g_A^2}{64\pi^2 f_\pi^2\, M_0} - \frac{3}{32\pi^2 f_\pi^2} \left(\frac{g_A^2}{M_0} - 8\,c_1 + c_2 + 4\,c_3 \right) \ln \frac{m_\pi}{\lambda} \right] m_\pi^4$$

p^4



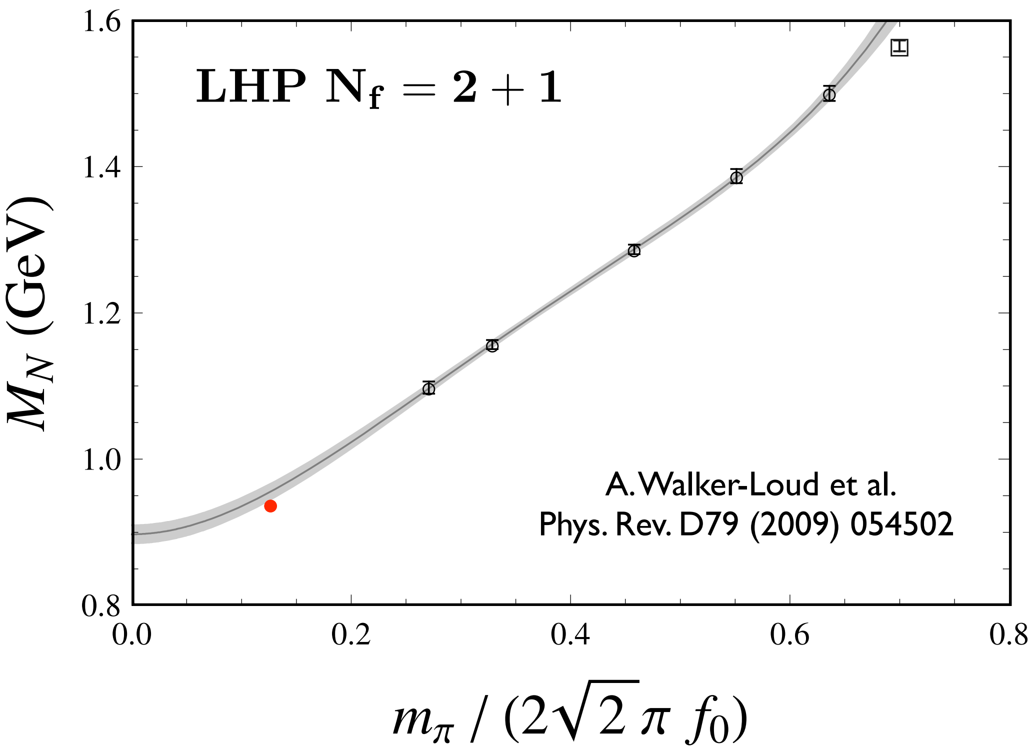
$$+ \frac{3\,g_A^2}{256\pi f_\pi^2\, M_0{}^2}\, m_\pi^5 + \mathcal{O}(m_\pi^6)$$

... five years ago:

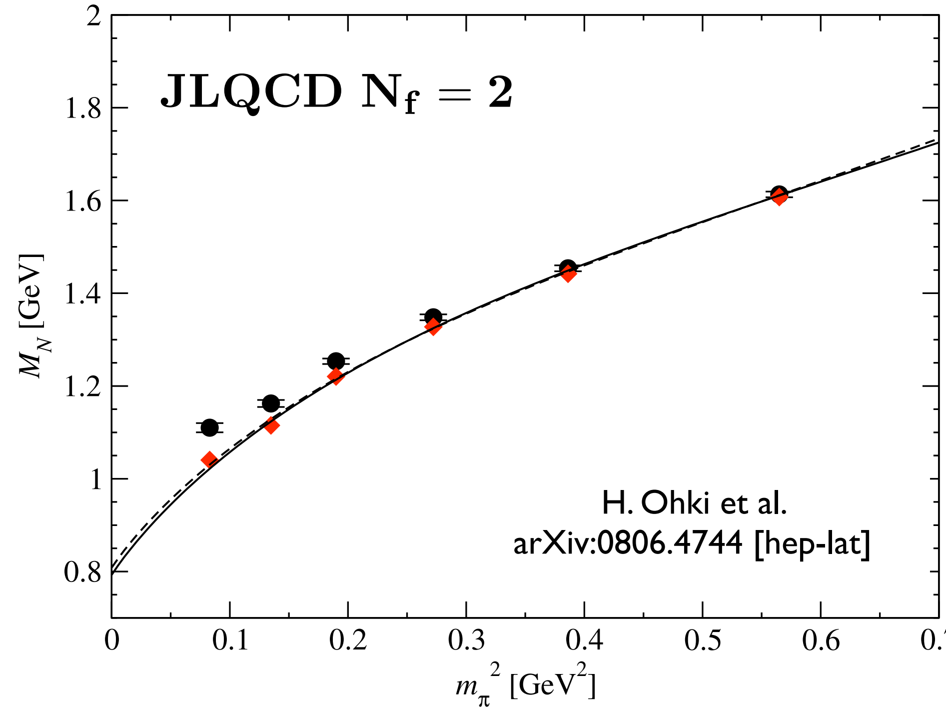


M. Procura, T. Hemmert, W.W.: Phys. Rev. D69(2004)034505

MASS of the NUCLEON: LATTICE QCD + CHIRAL PERTURBATION THEORY



- Full QCD simulations
- Pion masses now down to **~ 300 MeV**



- ChPT at NNLO used for chiral extrapolation
- Finite volume corrections included

SIGMA TERMS

- Pion-nucleon sigma term:

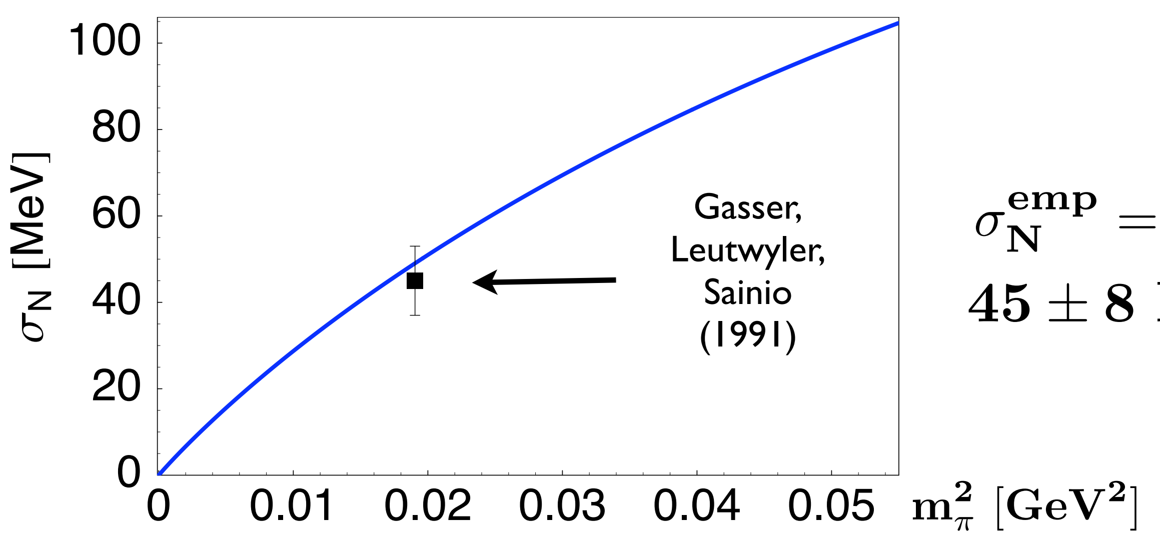
$$\sigma_N = \bar{m}\,\langle N|\bar{u}u + \bar{d}d|N\rangle = \bar{m}\,\frac{\partial M_N}{\partial \bar{m}} \simeq m_\pi^2\,\frac{\partial M_N}{\partial m_\pi^2} \qquad \left(\bar{m} = \frac{m_u + m_d}{2}\right)$$

- Previous status:

from chiral interpolation using older lattice data

M. Procura et al.
Phys. Rev. D73 (2006) 114510

$$\sigma_N = 49 \pm 3\ \mathrm{MeV}$$



$$\sigma_N^{emp} = 45 \pm 8\ \mathrm{MeV}$$

- **Strangeness content**:

$$\sigma_N = \frac{\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle}{1-y}$$

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} \simeq 1 - \frac{\bar{m}}{m_s}\,\frac{M_\Xi + M_\Sigma - 2\,M_N}{\sigma_N} \sim 0.2 - 0.3$$

assuming flavour SU(3) mass relations

. . . but with large uncertainties

B. Borasoy, U.-G. Meißner
Ann. of Phys. 254 (1997) 192

SIGMA TERMS ... today

- From most recent dynamical QCD simulations + chiral extrapolations (2+1 flavors)

LHP A.Walker-Loud et al., Phys. Rev. D79 (2009) 054502

HSC H.W. Lin et al., arXiv:0810.1661 [hep-lat]

PACS-CS S.Aoki et al., arXiv:0807.1661 [hep-lat]

S. Dürr et al., Science 322 (2008) 1224

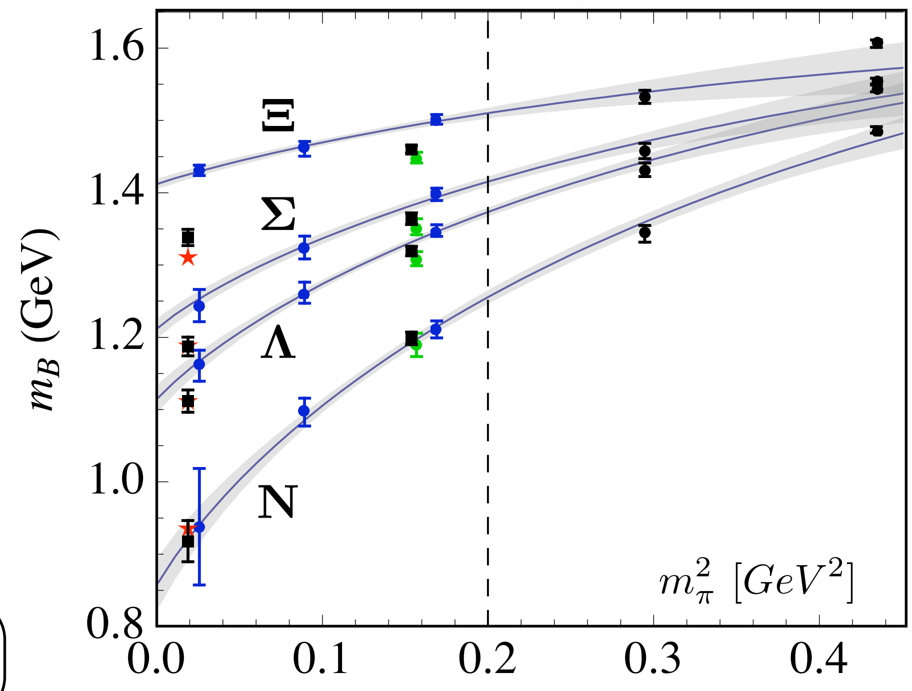
- Combined analysis of baryon octet (and decuplet)

R.D.Young, A.W.Thomas
arXiv:0901.3310 [hep-lat]

▶ $\sigma_N = 47\,(9)(1)(3)\ \mathrm{MeV}$

(statist.) (lattice artifacts) (chiral extrap.)

▶ consistent with earlier results, with recent JLQCD (2 flavors) and with phenomenology



- **Strange quark** contribution

$$\sigma_{Ns} = m_s \langle N|\bar{s}s|N\rangle = m_s \frac{\partial M_N}{\partial m_s}$$

▶ $\sigma_{Ns} = 31\,(15)(4)(2)\ \mathrm{MeV}$

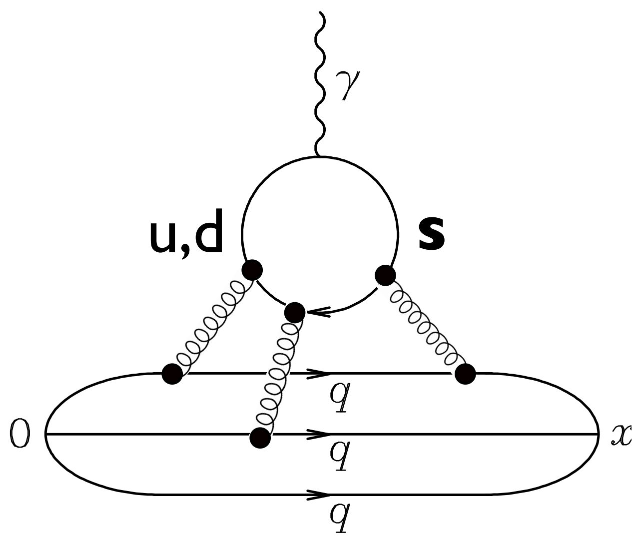
much smaller than previously expected ! $y \sim (5 \pm 4) \cdot 10^{-2}$

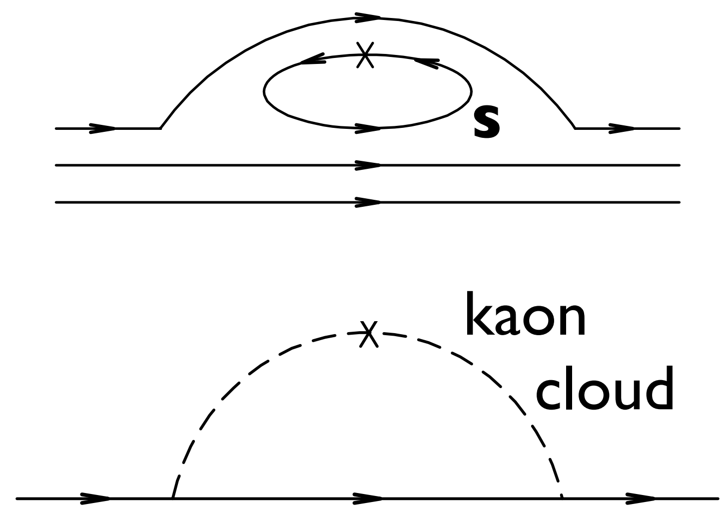
3.

STRANGENESS VECTOR CURRENT

$$\langle \mathrm{N} | \bar{\mathrm{s}} \gamma_\mu \mathrm{s} | \mathrm{N} \rangle$$

- **Strangeness** el. - mag. **Form Factors** and **Magnetic Moments** of the Nucleon





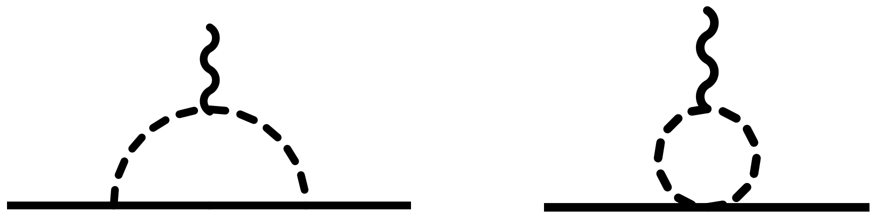
- First systematic study of $\mathrm{G}_\mathrm{M}^\mathrm{s}(\mathbf{0})$: D. Leinweber, A.W. Thomas; Phys. Rev. D62 (2000) 074505

ISOVECTOR ANOMALOUS MAGNETIC MOMENT

Lattice QCD combined with Chiral Perturbation Theory

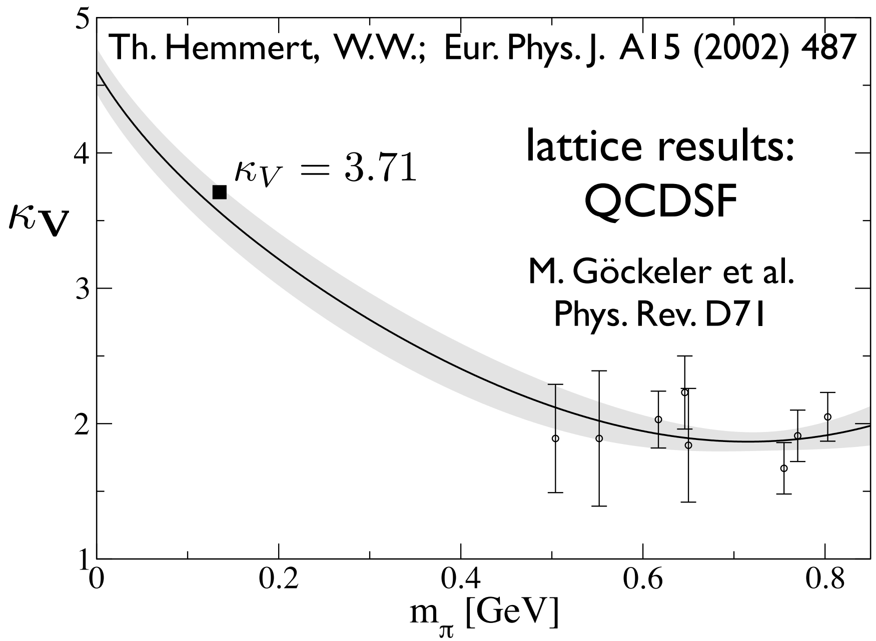
- Pion cloud effects in

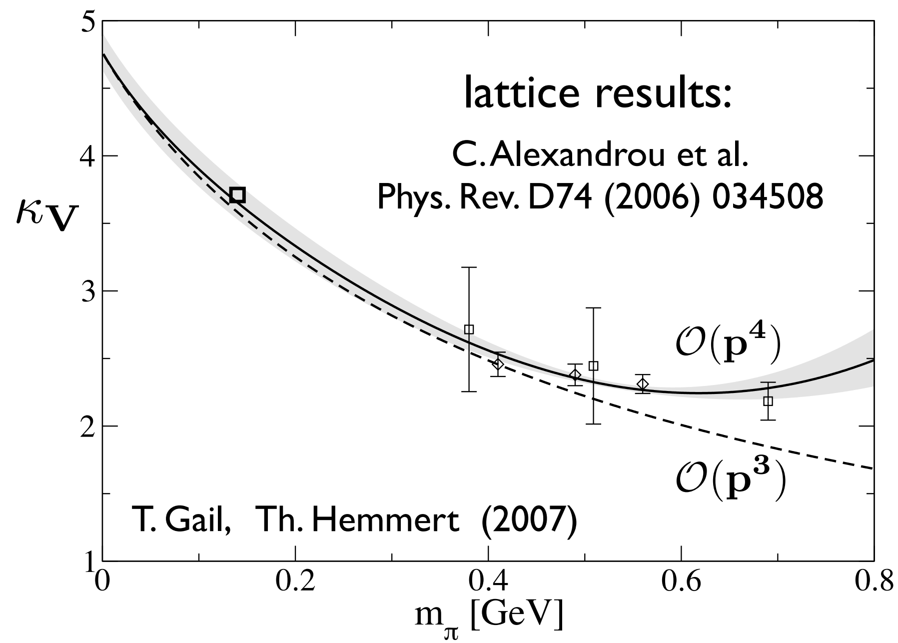
$$\kappa_V = \lim_{Q^2 \to 0} F_2(Q^2)$$



- Chiral extrapolations

(with update - right - including information from Q dependence of magnetic form factor)

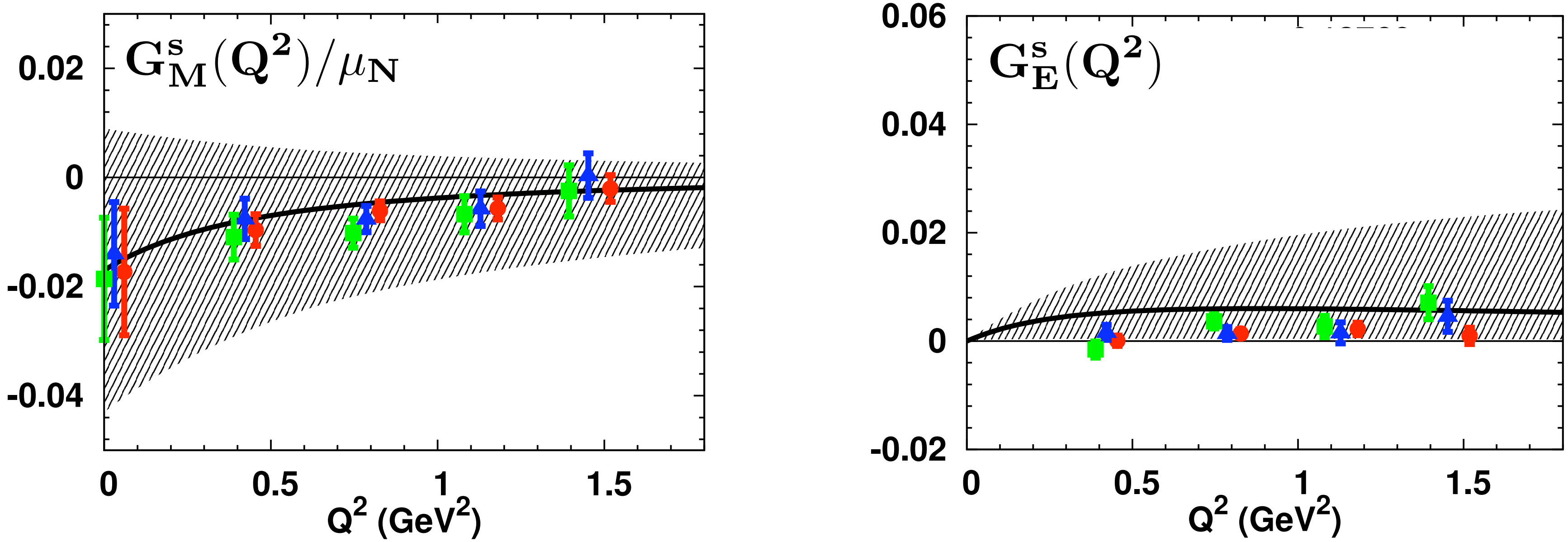




STRANGENESS EM FORM FACTORS

- Lattice QCD + chiral extrapolations (2+1 flavors)

T. Loi et al. (χQCD Collaboration); arXiv: 0903:3232 [hep-ph]



$$G_M^s(Q^2 = 0) = -0.017\,(25)(07)\;\mu_N$$

- Indirect results: sea quark effects from Lattice QCD + charge symmetry

D.B. Leinweber et al. (Adelaide - JLAB)

$$G_M^s(0) = -0.046 \pm 0.019\;\mu_N$$

PRL 94 (2005) 212001

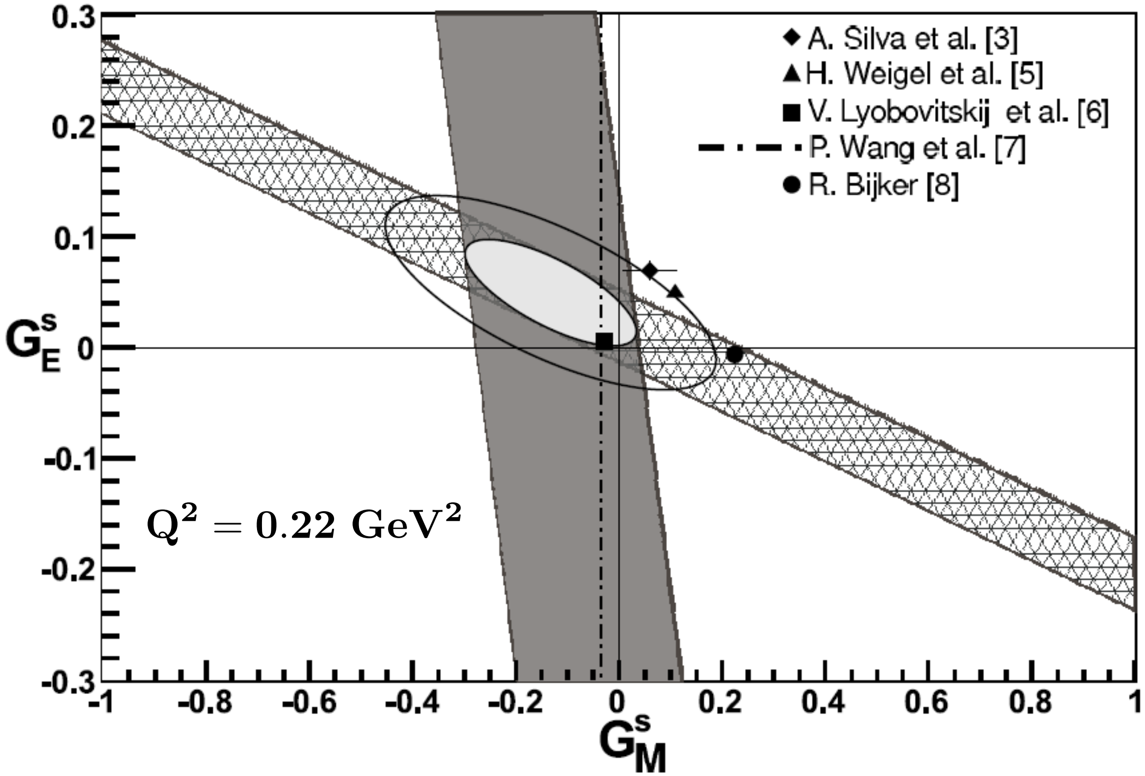
$$G_E^s(0.1\,GeV^2) = +0.001 \pm 0.008$$

PRL 97 (2006) 022001

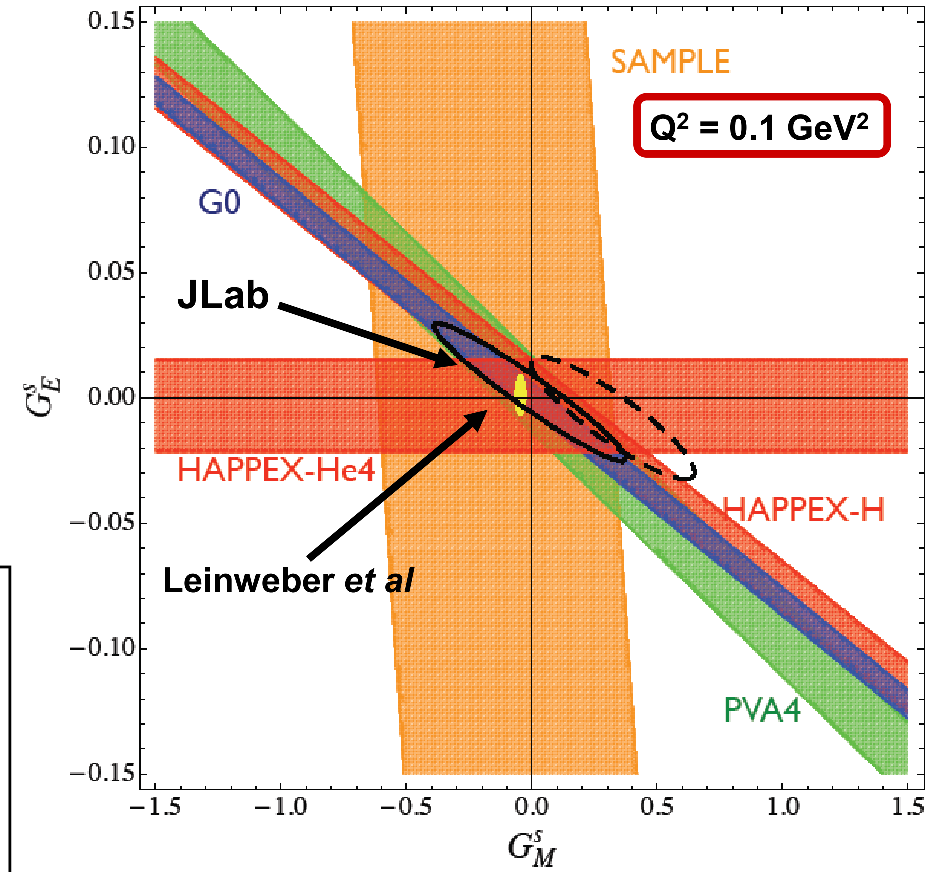
$$G_M^s(Q^2 = 0.23\,GeV^2) = -0.034 \pm 0.021\;\mu_N$$

P. Wang et al. arXiv:0807.0944 [hep-ph]

THEORY
status summary
in comparison with
EXPERIMENTS



$G_M^s = -0.14 \pm 0.11 \pm 0.11\ \mu_N$; $G_E^s = 0.050 \pm 0.038 \pm 0.019$



- Global analysis

R.D.Young et al., PRL 99 (2007) 122003

- PVA4 (Mainz)

S. Baunack et al.,
arXiv:0903.2733 [nucl-ex]

SUMMARY

- **LATTICE QCD** and **CHIRAL EFFECTIVE FIELD THEORY** in combination have reached the level of quantitatively accurate tools for **LOW-ENERGY QCD**
  - full QCD with almost physical quark masses
  - reliable chiral extrapolations now feasible

- **STRANGENESS CONTENT** of the **NUCLEON GROUND STATE** turns out to be unexpectedly **SMALL** (few %)
  - not only in **VECTOR** (electromagnetic) currents
  - but also in **SCALAR** densities (sigma terms)

Special thanks to Philipp Hägler and Tony Thomas