Quantum Information Processing with Nuclear Spin-Based Devices

T. F. Havel*, P. Cappellaro*, C. Ramanathan*, and D. G. Cory*

* MIT, Department of Nuclear Science and Engineering
150 Albany Street, Cambridge, MA 02139, USA
tfhavel@mit.edu, pcapell@mit.edu, sekhar@mit.edu, dcory@mit.edu

ABSTRACT

This talk will illustrate the benefits offered by QIP with two devices which use quantum entanglement among nuclear spins to perform high-precision measurements: (1) a spin gyroscope that operates by detecting the frequency shift in the spins precession rate in the rotating frame, which is potentially more accurate than mechanical or optical devices; (2) a quantum amplifier which correlates the states of a macroscopic number of nuclear spins with the state of a single target spin, so that a collective measurement of the state of the amplifier's spins reveals that of the target spin. Some potential implementations of these devices based on nuclear spins in naturally occurring materials will also be described.

Keywords: entanglement, metrology, single-spin measurement, gyroscope

1 INTRODUCTION

As nanotechnology progresses towards the atomic scale, quantum effects will become unavoidable. Although the Heisenberg uncertainty relation limits the precision with which one can measure the state of a quantum mechanical device, it has been demonstrated that it is also possible to take advantage of these effects so as to perform metrology, communications and computational tasks that are impossible within the approximation of classical physics. The theory and implementation of such quantum devices is known as Quantum Information Processing (QIP) [1].

This talk will illustrate the benefits offered by QIP with two devices which use quantum entanglement among nuclear spins to perform high-precision measurements: (1) a spin gyroscope that operates by detecting the frequency shift in the spins precession rate in the rotating frame, which is potentially more accurate than mechanical or optical devices; (2) a quantum amplifier which correlates the states of a macroscopic number of nuclear spins with the state of a single target spin, so that a collective measurement of the state of the amplifiers spins reveals that of the target spin. Some potential implementations of these devices based on nuclear spins in naturally occurring materials will also be described.

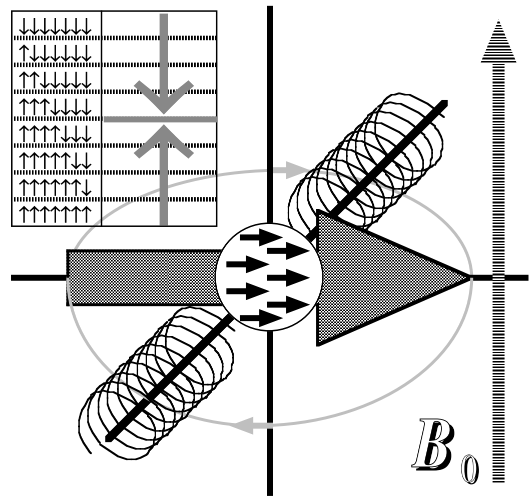

Figure 1: A spin gyroscope uses an ensemble of spins with a net magnet moment large enough to be detected by a macroscopic coil. In a magnetic field, rotation of the ensemble about the field's axis produces a phase shift in the otherwise constant rate of rotation of the spins' magnetic moment, which can be integrated to yield a continuous record of these rotations. The table in the upper left depicts the use of multiple quantum states of a spin $> 1/2$ to improve the sensitivity of the gyroscope (see text).

For introductions to nuclear spin based QIP, see Refs. [2-4].

2 A SPIN GYROSCOPE

A spin gyroscope, based on gaseous ^{129}Xe (Xenon), has previously been demonstrated by Mehring et al. [5]. In a co-moving frame, the precession frequency of the spins in a magnetic field is shifted by the frame's instantaneous rate of rotation about the field's axis. This in turn can be integrated to yield a net phase shift and hence a continuous record of the gyroscope's orientation changes. The frequency shift is independent of the precession frequency of the spins, so the precision of the phase measurement is bounded from below by the product of the spins' precession frequency and the local oscillator's frequency stability. It is also limited by

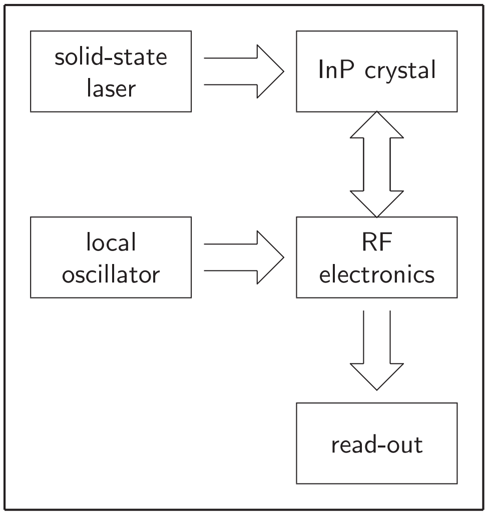

Figure 2: Simple block diagram of the solid-state spin gyroscope.

the intrinsic line width of the spins, which is inversely proportional to their T_2 relaxation time or decoherence rate. The proposed MIT device depicted in Figure 1 is based on the nuclear spins of In P (Indium Phosphide) in the solid state, where the relaxation times can be of order minutes below its Debye temperature (450 K). An intriguing possibility with this material is the use the multiple quantum coherence of the spin = 9/2 nucleus ^{115}In to further improve the precision. The device should also be suitable for fabrication by lithographic techniques.

2.1 Technical Specifications

A block-diagram of the proposed spin gyroscope is shown in Figure 2. Because a low precession rate of at most a few kilo-Hertz is required, a low magnetic field must be used and the thermal equilibrium magnetization of the spins in such a field will be almost undetectable. The spins must therefore be optically pumped in order to achieve an adequate signal-to-noise ratio, which in InP is readily achieved using a diode laser [6]. A minor complication due to the lack of motional line narrowing in solids can also be solved by refocusing all the two-body terms present in the spin-spin Hamiltonian using standard multi-pulse NMR techniques [7].

Phosphorus has a single spin = 1/2 isotope, ^{32}P, but we believe that a better device can be built using the spin = 9/2 of ^{115}In even though it has only half the gyromagnetic ratio and a substantial quadrapole moment (810 mbarn). The symmetry of the zinc blend (body-centered cubic) structure of InP effectively eliminates the quadrapolar couplings, whereas the higher spin allows us to more than compensate for the lower magnetic moment through the use of multiple quantum coherences.

MQC are quantum mechanical (phase coherent) correlations between nonadjacent energy levels in the 10-level ladder of Indium, which are not associated with a magnetic dipole and hence cannot be directly observed or manipulated. NMR multiple pulse techniques for re-introducing the quadrapolar coupling, or controlling the Indium spin via the phosphorous, may nevertheless allow them to be created from the observable single-quantum coherences between adjacent energy levels, and to map them back to single quantum coherences again where the accumulated phase due to motion can be read out. In particular, the phase changes of the 9-quantum coherence between the lowest and highest energy levels will accumulate nine times faster than the single quantum, which offers an improvement of 4.5 over using the Phosphorous (cf. upper left-hand diagram in Figure 1). Rather than decoupling the Phosphorous, however, it may be possible to also utilize yet higher-order two-spin coherences involving it, to obtain yet better precision.

3 ENTANGLEMENT-ASSISTED SINGLE-SPIN MEASUREMENT

A quantum amplifier may be viewed as a macroscopic array of spins = 1/2 which starts out with all spins pointing "down", but winds up with all spins pointing "up" if and only if the "target" spin to be measured is in the "up" state. In the case that the target spin starts out in a superposition $\alpha\,|\downarrow\rangle_{\mathsf{tgt}} + \beta\,|\uparrow\rangle_{\mathsf{tgt}}$ of the "down" and "up" states, therefore, the mapping produces a mutually entangled "cat state" of all the spins, i.e.

$$\begin{aligned} & |\downarrow\downarrow\,\cdots\downarrow\rangle_{\mathsf{amp}}\left(\alpha\,|\downarrow\rangle_{\mathsf{tgt}} + \beta\,|\uparrow\rangle_{\mathsf{tgt}}\right) \qquad (1) \\ \mapsto\;\; & \alpha\,|\downarrow\downarrow\,\cdots\downarrow\rangle_{\mathsf{amp}}|\downarrow\rangle_{\mathsf{tgt}} + \beta\,|\uparrow\uparrow\,\cdots\uparrow\rangle_{\mathsf{amp}}|\uparrow\rangle_{\mathsf{tgt}}\;. \end{aligned}$$

A standard von Neuman measurement on all the amplifier spins will then "collapse" both the amplifier spins and the target spin either into the all "down" or all "up" states with probabilities $|\alpha|^2$ and $|\beta|^2$, resp. The advantage of performing the mapping before attempting the measure the target spin directly, however, is that the

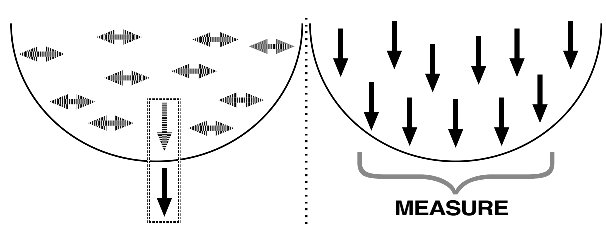

Figure 3: Schematic illustration of the quantum amplifier concept. A macroscopic array of spins in a highly entangled state (the amplifier) is allowed to interact with a single target spin, after which a macroscopic measurement of the amplifier's spins reveals them all to be in the same state as the target spin.

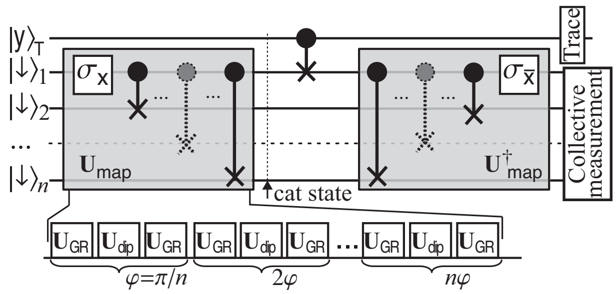

Figure 4: Circuit diagram of a quantum amplifier implementation which does not require the spins of the amplifier to be independently addressable. Instead of the sequence of c-NOT gates shown in the boxes (see text), a cat-like state among the amplifier spins is created by alternately applying the natural dipolar coupling and "grade-raising" operations (see text). Then providing the target spin is pointing "up", a single c-NOT operation between the target spin and the amplifier spin closest to it in space perturbs this massively entangled state so that the inverse map leads back to a state in which most of the amplifier spins are "up", and hence a macroscopic measurement can readily distinguish it from the initial "down" state.

collective magnetic moment of a macroscopic number of amplifier spins will be large enough to be easily detected.

There are many different unitary transformations which could be used to achieve a mapping between states of the form given above. The simplest conceptually would be a sequence of two-spin "c-NOT" (controlled-NOT) gates, each of which flips one of the amplifier spins if the target is "up", and otherwise does nothing (see Figure 4). Unfortunately, this implementation would require that the amplifier spins be independently addressable, a stringent requirement that is not otherwise necessary. On the other hand, it is relatively easy to let all the amplifiers spins interact with their neighbors and thereby become entangled. In the most favorable case one obtains a cat-state of the amplifier spins by themselves, which is extremely fragile in that a flip of just one of its spins conditional on the target spin's state will cause the inverse of the entangling evolution to yield a state in which all the amplifier spins have been flipped. An initial study of the feasibility of such a scheme may be found in Ref. [8].

3.1 Technical Specifications

An ideal (i.e. naturally occurring!) system in which to test the quantum amplifier idea is provided by crystals of the molecule adamantane (see Figure 5). The hydrogen atoms in this crystal all have spin = 1/2, and serve as the amplifier spins. The carbon atoms, on the other hand, are largely $^{12}\mathsf{C}$, which has spin = 0, together with about 1% of $^{13}\mathsf{C}$, which like the hydrogens has spin = 1/2, and plays the role of the target spin(s). At temperatures near 1K, hydrogen spins can be polarized to a significant extent via their interaction with the unpaired electron spins present in a (dilute) stable free radical dopant. The protons can then be entangled with one another via the application of an alternating sequence of evolutions under the spins' natural dipolar coupling ($\mathbf{U}_{\mathsf{dip}}$) and "grade raising" ($\mathbf{U}_{\mathsf{GR}}$) operations (as in Figure 4).

The latter may be implemented via a rapid sequence of RF (radio-frequency) pulses separated by free evolutions under the dipolar coupling (see Figure 6). Although it would be outside the scope of this paper to fully explain how this pulse sequence works, it is fairly easy to explain what a grade-raising operation on a pair of spins does: it simply flips that pair of spins in every term of the superposition describing the quantum state of the amplifier, e.g.

$$\cdots + |\cdots\uparrow\downarrow\cdots\rangle + \cdots \quad\mapsto\quad \cdots + |\cdots\downarrow\uparrow\cdots\rangle + \cdots \qquad (2)$$

Because the amplifier spins are indistinguishable and the interactions between nearest neighbors in the crystal lattice are the strongest, the pulse sequence actually performs such a grade raising operation in parallel on all pairs of nearest neighbor spins. The intervening dipolar evolutions, on the other hand, act to swap the states of neighboring spins, and thereby spread the entanglement created by the grade-raising operations across the whole amplifier. Schematically, we have

$$\begin{aligned} |\downarrow\downarrow\downarrow\cdots\rangle_{\mathsf{amp}} &\xrightarrow{\mathbf{U}_{\mathsf{GR}}} |\downarrow\downarrow\downarrow\cdots\rangle_{\mathsf{amp}} + |\uparrow\uparrow\downarrow\cdots\rangle_{\mathsf{amp}} + \\ &\qquad |\downarrow\uparrow\uparrow\cdots\rangle_{\mathsf{amp}} + \cdots \\ &\xrightarrow{\mathbf{U}_{\mathsf{dip}}} |\downarrow\downarrow\downarrow\cdots\rangle_{\mathsf{amp}} + |\uparrow\uparrow\downarrow\cdots\rangle_{\mathsf{amp}} + \\ &\qquad |\downarrow\uparrow\uparrow\cdots\rangle_{\mathsf{amp}} + |\uparrow\downarrow\uparrow\cdots\rangle_{\mathsf{amp}} + \cdots \end{aligned} \qquad (3)$$

The final trick needed to make this scheme work is to use quantum interference to destroy clusters of entanglement among small groups of spins, leaving only a highly entangled "cat-like" state behind. To explain

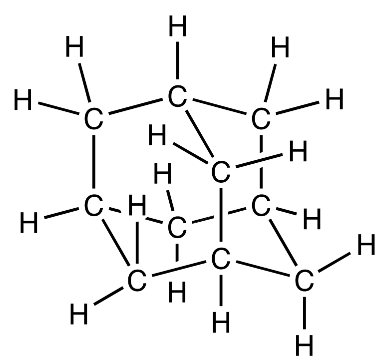

Figure 5: Chemical diagram of the molecule Adamantane proposed for an experimental demonstration of the quantum amplifier concept.

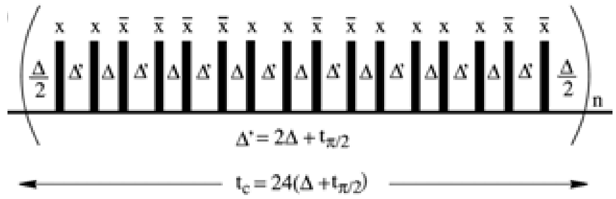

Figure 6: Diagram of the pulse sequence which performs a grade-raising operation (see text).

how this is done, we need to introduce the concept of "coherence order", which is essentially the number of spin flips by which two given terms in a superposition state differ. There is, therefore, only one state of an n spin system with coherence order n, and that is the cat state. In addition, the rate at which phase differences between the terms of a superposition accumulate under spin precession (or any other rotation about the magnetic field access) is proportional to their coherence orders [4]. Therefore, by allowing a small amount of precession $\varphi = \pi/n$ during each of the $n/2$ operations $\mathbf{U}_{\mathsf{GR}}\mathbf{U}_{\mathsf{dip}}\mathbf{U}_{\mathsf{GR}}$, or equivalently shifting the phase of the RF transmitter by that amount after each such operation, the phases of all contributions to every term of coherence order less than the maximum n are essentially random, so that they add up to zero. This in terms means that after the minimum $n/2$ operations needed to create an n-quantum coherence we will have something very close to the desired cat state, as desired.

REFERENCES

[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge Univ. Press, 2001).

[2] T. F. Havel, D. G. Cory, S. Lloyd *et al.*, Am. J. Phys. 70, 345-362 (2002).

[3] R. Laflamme, E. Knill, D. G. Cory *et al.*, Los Alamos Sci. 27, 2-37 (2002).

[4] L. M. K. Vandersypen and I. L. Chuang, Rev. Mod. Phys. 76, 1037-1069 (2004).

[5] M. Mehring and G. Wäckerle, Adv. Magn. Opt. Reson. 20, 67-186 (1997).

[6] C. A. Michal and R. Tycko, Phys. Rev. Lett., 81, 3988-3991 (1998).

[7] C. P. Slichter, Magnetic Resonance (Springer-Verlag, New York, 1982).

[8] P. Cappellaro, J. Emerson, N. Boulant, C. Ramanathan, S. Lloyd, D. G. Cory, Phys. Rev. Lett, in press.