Building a Better Snail: Lubrication and Adhesive Locomotion

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(Dated: March 16, 2005)

Many gastropods, such as slugs and snails, crawl via an unusual mechanism known as adhesive locomotion. We investigate this method of propulsion using two mathematical models, one for direct waves and one for retrograde waves. We then test the effectiveness of both proposed mechanisms by constructing two mechanical crawlers. Each crawler uses a different mechanical strategy to move on a thin layer of viscous fluid. The first uses a flexible flapping sheet to generate lubrication pressures in a Newtonian fluid which in turn propel the mechanical snail. The second generates a wave of compression on a layer of Laponite, a non-Newtonian, finite-yield stress fluid with characteristics similar to those of snail mucus. This second design can climb smooth vertical walls and perform an inverted traverse.

I. INTRODUCTION

Understanding adhesive locomotion, the crawling mechanism adopted by most marine and terrestrial gastropods, provides an interesting challenge for fluid dynamicists, biologists and roboticists alike. As a snail propels itself forward, the forces created by muscles in the foot interact with the substrate through a layer of viscous fluid (pedal mucus) secreted by the animal. Locomotion is a directly coupled to the stresses generated within this thin film and the dynamic and material properties of the fluid are intimately involved in the propulsion mechanism.

It has long been known that muscles along the foot of a snail drive deformations that propel the animal forward. Observations of crawling snails show that the moving foot is divided into alternating bands of translating waves and interwaves (see figure 1). These waves have been classified by Vlés [1] as direct waves, propagating in the direction of motion of the snail, or retrograde waves, propagating opposite to the snail motion. Rarer forms of movement have also been observed including diagonal waves, “crawl-step” events [2] and “galloping” in which the snail lifts a portion of its body off the substrate similar to an inchworm. However, simple direct or retrograde waves predominate in most species; direct waves are used primarily by terrestrial gastropods while retrograde waves are seen mainly in marine species. It has been well established, by measuring the distance between marked points on the foot [3] or by anatomical evidence obtained by flash-freezing crawling snails [2, 4], that these waves correspond to regions of lateral compression in the foot.

It was not until Lissmann’s work in 1946 [5] that biologists began to uncover the mechanical role of these waves in driving propulsion. Lissmann proposed that there are a number of forces involved in snail locomotion including what he termed “external sliding friction” in the form of both “drag” and “slip.” He recognized that differential friction between the foot and the ground in the wave and the interwave segments is required to propel the animal forward. However, Lissmann also postulated that the distinct friction within each segment is a result of the snail lifting portions of its foot, creating a thicker mucus layer in the wave regions. While this differential friction is indeed necessary for locomotion, subsequent studies have shown that the difference does not arise from geometrical changes in the interstitial thickness, rather from the unusual properties of the pedal mucus.

The first person to correctly elucidate the role of the pedal mucus in locomotion was Denny [6] who characterized the properties of the mucus of the banana slug, Ariolimax columbianus. He measured a finite yield stress in the fluid and proposed that the requisite differential friction arises naturally if the applied stresses in the mucus in the interwave region remain below the critical yield stress (thus the fluid acts as an adhesive) while stresses in the wave region are sufficient to drive a flow. He then quantitatively compared experimentally measured stresses in pedal mucus under crawling snails with order of magnitude estimates based on measured yield stresses of the

FIG. 1: Photo of the underside of Limax maximus, commonly known as the giant leopard slug, showing wave and interwave segments. Waves propagate in the direction of locomotion i.e. locomotion is driven by direct waves.
fluid [4]. In section II B we expand on Denny’s work and present a detailed model of the stresses generated within the mucus.

In addition to stimulating purely academic interest, adhesive locomotion is inspiring a new paradigm in robotics. Robotic design frequently looks to biology to gain insight into the mechanics of locomotion. One can easily argue that snail locomotion is slow, slimy and inefficient and hence hardly seems like something roboticists should strive to emulate. However, despite these shortcomings, adhesive locomotion presents two unique advantages. First, gastropods have only one foot so they are mechanically simple and very stable, making mechanical replication far simpler than that required for higher order organisms. Second, they are often found in habitats that are topologically complex; thus they have evolved means of maneuvering through these challenging terrains by adhering to substrates, enabling them to crawl straight up vertical walls and traverse ceilings and overhangs. This versatility, in conjunction with new soft actuators and other recent technological developments in material science, is beginning to redirect robotic designs toward compliant machines and crawlers.

In light of these advantages, a few research groups have designed and constructed mechanical “snails.” Notably, A. Ito et. al., have built a mechanical “snail” that consists of a series of electromagnets beneath a soft matrix [7]. A layer of viscous fluid is deposited on the soft substrate and a rigid “snail” is placed on top of the fluid. The magnets then are activated in sequence, creating an out-of-plane wave in the substrate which in turn drives the crawler. However, this design is not ideal as it is not the “snail” that generates the waves, rather the substrate. More recently, Mahadevan et. al. [8] carried out a series of experiments using soft hydrogel crawlers to investigate the origin of various crawling gaits. Forward motion in their experiments arises due to differential friction supplied by angled incisions in the gel. However, as in Ito’s setup, the substrate provides the propulsive energy rather than the crawler, in this case, through externally supplied vibrations.

In this paper, we pursue two paths in analyzing gastropod locomotion. First, we construct a mathematical model that describes the thin fluid layer between the foot and the substrate and demonstrates that, under certain conditions, the stresses in this layer can lead to locomotion. Two separate analyses are carried out for the case of retrograde and direct waves, respectively. Second, to verify the effectiveness of our proposed propulsion systems, we build two mechanical snails, one using direct waves and one using retrograde waves. The performance of the mechanical snails is compared with model predictions.

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Locomotion via retrograde waves can be simply induced by out-of-plane waves generated on the underside of the foot. It is likely that, should marine snails crawl via the retrograde mechanism described below, it would be used in conjunction with an adhesive type of locomotion similar to that described in section II B. However, the retrograde mode proposed in this section can be used exclusively in developing mechanical crawlers, as described in section III A, and the analysis simplifies considerably if we consider the two mechanisms separately. The propulsive power from this type of wave is generated in a manner analogous to a peristaltic pump and the physical mechanism can be simply understood as follows. As the lowest point in the wave ([A] in figure 2), moves backwards, viscous fluid to the left of the wave is squeezed into a narrow gap resulting in an increase in pressure. Conversely, at the back of the wave, fluid flows unobstructed into a widening gap, resulting in a decrease in pressure. These two pressures, acting normal to the interface, result in a net force opposite to the direction of wave propagation.

To calculate the velocity of the snail, we first move into a reference frame moving with the wave such that \( \hat{h} = \hat{h}(\hat{x}) \), i.e. the shape of the foot is no longer time dependent (see figure 2). We consider a two-dimensional crawler that is periodic in \( \hat{x} \). \( \hat{V}_s \) is the snail velocity as measured in the lab frame and \( \hat{V}_w \) is the wave velocity as measured in the snail frame. Ultimately we would like to answer the question: Given a motor that can drive a deflection at speed \( \hat{V}_w \) in the mechanical foot, how fast will the “snail” go? Unlike the direct wave mechanism described in section II B, the simplest retrograde crawler can operate on a Newtonian fluid which we will assume in our model. If the average gap thickness, \( \hat{H} \), is much smaller than the wavelength, \( \lambda \), we can use a standard lubrication approximation [e.g. 9] to model the interstitial fluid. Conservation of momentum in the \( \hat{x} \) and \( \hat{y} \)
Specifically, which may be used to solve for the unknown constant $Q_u$ subject to the boundary conditions $u(x, y = 0) = 1 - V_s$, and $u(h) = 1$. Integrating twice we find

$$u(x, y) = \frac{1}{2} \frac{dp}{dx} y (y - h) + V_s \left( \frac{y}{h} - 1 \right) + 1. \quad (3)$$

At steady state, the volume flux per unit width through a plane normal to the substrate, $Q = \int_0^h u \, dy$, must be constant for all $x$. Integrating (3) from 0 to $h$ and solving for the pressure gradient we find

$$\frac{dp}{dx} = \frac{12}{h^3} + \left[ h \left( 1 - \frac{1}{2} V_s \right) - Q \right]. \quad (4)$$

Since $h(x)$ is periodic with wavelength 1, $p(1) = p(0)$, which may be used to solve for the unknown constant $Q$. Specifically,

$$\int_0^1 \frac{dp}{dx} \, dx = p(1) - p(0) = 0. \quad (5)$$

Integrating (4) over one wavelength, applying condition (5) and solving for $Q$, we find

$$Q = \left( 1 - \frac{1}{2} V_s \right) \frac{I_2}{I_3}, \quad \text{where} \quad I_3 = \int_0^1 \frac{dx}{h^3}. \quad (6)$$

Hence, given the wave shape, $h(x)$, we can compute the dimensionless flux, $Q$, which in turn gives us the pressure gradient, $dp/dx$ and the fluid velocity profile, $u(x)$.

Finally, to find the velocity of the snail we perform a force balance on the foot. The friction acting on the foot is $F = \hat{\sigma} \cdot \hat{n}$ where $\hat{\sigma}$ is the dimensional fluid stress tensor and $\hat{n}$ is the unit outward normal to the foot. Integrating over $x$ and applying force balance on the entire snail in the $x$ direction [17] to lowest order in $\hat{H}/\lambda$ yields (in dimensionless form)

$$\int_0^1 p \frac{dh}{dx} + \frac{\partial u}{\partial y} \bigg|_{y=h} \, dx = 0. \quad (7)$$

Recall that we have already assumed that the snail does not accelerate but travels at a steady velocity, $V_s$. The first term in the integral represents the pressure forces acting normal to the foot and the second represents viscous drag. Integrating the first term by parts and substituting (4) for $dp/dx$ we find

$$3QI_2 = (3 - 2V_s) I_1. \quad (8)$$

Finally, using the expression for $Q$ from (6) and defining a shape function,

$$A = \frac{I_2^2}{I_3 I_1}, \quad (9)$$

we find an expression for the velocity of the snail

$$V_s = \frac{6 (1 - A)}{4 - 3A}. \quad (10)$$

Thus, given the shape of the deformations in the foot, we can predict the velocity of the animal.

It is straightforward to modify the above calculation to include an inclined substrate and determine whether our mechanical snail is capable of climbing a near-vertical wall. Adding the gravitational body force into equation (7) we find

$$-W \sin \phi = \int_0^1 p \frac{dh}{dx} + \frac{\partial u}{\partial y} \bigg|_{y=h} \, dx, \quad (11)$$

where $\phi$ is the angle of inclination measured from the horizontal, $W = \frac{Hmg}{\mu V_s \lambda b}$ is a dimensionless weight parameter, $g$ is gravity, $m$ is the mass of the crawler and $b$ is the width of the crawler in the $z$ direction. Solving for the snail velocity we find

$$V_s = \frac{6 (1 - A)}{4 - 3A} - \frac{W \sin \phi}{I_1 (4 - 3A)}. \quad (12)$$

Thus the tangential force balance indicates that the snail can climb up any surface provided $6 (1 - A) > W/I_1$. However, it is important to note that the normal force balance limits our snail’s climbing capabilities to inclinations of $\phi < \pi/2$ i.e. the crawler cannot traverse an overhang nor can it climb up a truly vertical surface. Since there is no normal force adhering the crawler to the substrate, it cannot retain contact with a vertical wall, in contrast to the adhesive mechanism described in the next section.

### B. Direct Waves

Adhesive locomotion via direct waves was first correctly described by Denny [4, 6] in an article in which he answers the thought provoking question “How can an animal with only one foot walk on glue?”. The analogy of a kink propagating across a carpet, often used to describe the motion of dislocations, is equally apt here. To propel itself forward, the snail uses its muscles to initiate a
small region of compression at the tail (the “kink”). This compressed region then travels from the tail to the head along the foot (the “carpet”). When it reaches the head, the snail once more extends to its full length, shifting the entire animal forward [18].

Denny [4] correctly postulated that this mode of locomotion requires that the forces generated by the moving portion of the foot must be offset by the adhesive forces under the interwave portions stating “a gastropod will move forward if the reactive force beneath the stationary parts of the foot (the interwaves) is sufficient to offset the frictional force due to the movement of the waves and the rim.” That is if the adhesive forces do not exceed the shear forces, the entire snail will simply slip backwards as the wave propagates forward resulting in no net translation. Denny also correctly identified the source of this differential friction arguing that the muscles in a snail are not strong enough to lift the foot and generate a variable gap thickness, therefore the difference must result from a change in the properties of the pedal mucus in the wave and interwave regions. This change is reflected in the finite yield stress of the mucus. In regions where the foot is shearing the fluid sufficiently, namely the wave regions, the applied stress exceeds the critical yield stress and drives a flow; in the interwave regions, the fluid does not yield, effectively gluing the foot to the substrate.

To relate the externally applied stresses and the properties of the pedal mucus to locomotion, we first compute the velocities and pressures within the fluid. We consider a flat foot on top of a fluid layer of thickness, $H$, with periodically alternating wave and interwave regions (see figure 3). As with the retrograde crawler, we restrict our analysis to two dimensions with periodic variations in $\hat{x}$. In the interwave, the fluid is stationary. In the wave region, of length $L_w$, where the muscles are compressed, the foot drives the upper boundary of the fluid forward with surface velocity $\hat{V}_{\text{w}}(\hat{x}, \hat{t})$. The boundary between the wave and interwave moves forward with a velocity $\hat{V}_w$. Note, $\hat{V}_w$ and $\hat{V}_{\text{w}}(\hat{x}, \hat{t})$ need not be the same velocity and the compatibility condition relating the two to the motion of the snail as a whole can be constructed as follows.

First, we sketch the geometry of the contraction wave on the space-time plane (figure 4). The instantaneous velocity of the foot in the contraction wave is fixed in the frame moving with the wave: $\hat{V}_w = \hat{V}_s(\hat{x} - \hat{V}_w\hat{t}) = \hat{V}_s(\xi)$, where $\xi$ is the space coordinate in the moving frame. If $L_w$ and $L_I$ denote the lengths of the contraction wave and interwave, then the time taken for the wave and interwave to pass a fixed point of reference are $\hat{t}_w^\text{fixed} = L_w/\hat{V}_w$ and $\hat{t}_I = L_I/\hat{V}_w$, respectively. However, a particular material point in the foot prolongs the interval, $\hat{t}_w$, it spends in the contraction wave as a result of its own motion, i.e., $\hat{t}_w > \hat{t}_w^\text{fixed}$. Over this extended interval, the point on the foot jumps a total distance $\hat{X}_w$. Consequently, the snail speed must be given by

$$\hat{V}_\text{snail} = \frac{\hat{X}_w}{\hat{t}_w + \hat{t}_I}$$  \hspace{1cm} (13)

where $\hat{X}_w = \int_0^{\hat{t}_w} \hat{V}_s(\hat{x} - \hat{V}_w\hat{t}) d\hat{t}$. From figure 4 we see that the time interval spent by a point on the foot in the contraction wave is $\hat{t}_w = (L_w + \hat{X}_w)/\hat{V}_w$. Similarly, $\hat{t}_I = L_I/\hat{V}_w$. Hence

$$\hat{V}_\text{snail} = \frac{\hat{V}_w\hat{X}_w}{\hat{X}_w + L_w + L_I}.$$  \hspace{1cm} (14)

If $\hat{V}_s$ is a characteristic value of $\hat{V}_s(\xi)$, then $\hat{X}_s \sim \hat{V}_s \hat{t}_w$ and we estimate

$$\hat{V}_\text{snail} \sim \frac{\hat{V}_w \hat{V}_s}{\hat{V}_w(1 + R) - R\hat{V}_s},$$  \hspace{1cm} (15)
where $R = L_f/L_w$ is the ratio of the size of the inter-wave segment to the wave segment. As we shall see, this number cannot be arbitrarily changed as $L_f$ must be sufficient to prevent the snail from slipping backwards. As expected, the larger the characteristic footspeed, $V_s$, the greater the snailspeed. For typical land snails, surface velocities are approximately two times faster than the animal.

Returning to the stresses generated in the mucus layer, in the wave region, conservation of momentum of the fluid is given by

$$\nabla \hat{p} = \nabla \cdot \hat{\tau}$$

(16)

where $\hat{p}$ is the pressure and $\hat{\tau}$ is the deviatoric stress (again hats indicate dimensional variables). To model the interstitial fluid we again assume that $\tilde{H}/L_w \ll 1$ and, unlike the retrograde case, this thickness does not vary with $\hat{x}$. Any spatial variations in the flow field are driven by local variations in the surface velocity. We model the mucus as a Herschel-Bulkley fluid which captures the property of both a finite yield stress and a shear dependent effective viscosity. Such a fluid, in the lubrication limit, is governed by the following constitutive law, relating the $xy$-component of the stress tensor, $\hat{\tau}$, to the strain rate, $\dot{\gamma} = \partial u/\partial y$, the yield stress, $\tau_0$, the consistency, $\mu$, and the power law index, $n$,

$$\hat{\tau} = (\tau_0 + \mu |\dot{\gamma}|^n) \text{sgn}(\dot{\gamma})$$

(17)

where $|\dot{\gamma}| \geq \tau_0$. If $|\dot{\gamma}| < \tau_0$, there is no shear in the layer, namely $\partial u / \partial y = 0$. Rescaling by $\hat{u} = \frac{n+1}{n+1+1} \tilde{H} \left( \frac{2\tau_0}{\mu} \right)^{1/n} u$, $\hat{y} = \tilde{H} y$, $\hat{x} = L_w x$, $\hat{p} = \frac{L_p \tau_0 p}{\tilde{H}}$ and applying a lubrication approximation as before, we find that the pressure does not vary across the depth, i.e. $\hat{p} = \hat{p}(\hat{x}, t)$, and that the velocity is governed by one of two relations depending on whether the local stress exceeds the yield stress of the fluid:

$$|\dot{\gamma}| \geq \tau_0 : \quad \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left[ \left( 1 + \frac{2}{n+2} \right) \frac{n}{n+1} \frac{\partial u}{\partial y} \right] \text{sgn}(\dot{\gamma})$$

(18)

$$|\dot{\gamma}| < \tau_0 : \quad \frac{\partial u}{\partial y} = 0$$

(19)

where we have neglected terms of order $\tilde{H}/L_w$ and higher.

Since the fluid is incompressible and the thickness of the layer does not change, the net flux through any cross-section perpendicular to the foot must be the same at any position, $\hat{x}$. In the interwave regions, this flux is zero since the fluid is stationary. Therefore, the net flux in the wave region must also be zero. This is only possible if there exists a positive pressure gradient within each wave segment, that pushes fluid backwards somewhere in the layer as the foot drags fluid near the top forward. Thus the velocity profile can be divided into three parts as shown in figure 3: region A at the top ($Y_+ < \hat{y} \leq \tilde{Y}_+$), region B ($\tilde{Y}_- < \hat{y} \leq Y_+$) and region C ($0 \leq \hat{y} \leq Y_-$) again parabolic (for $n = 1$), connecting the no-slip boundary condition at the substrate to the backwards plug flow. As we shall see, region B is not a true plug flow as the velocity and the position of the the yield surfaces vary slowly with $\hat{x}$. Hence we refer to the region as a pseudo-plug rather than a true plug [10].

We now solve for the flow field in each region beginning with A. In this region, $\dot{\gamma} \geq 0$ and $|\dot{\gamma}| \geq \tau_0$. Solving (19) with boundary conditions $u = u_p$ and $\partial u / \partial y = 0$ at $y = Y_+$, we find the velocity, $u_A$ is given by

$$u_A = \left( \frac{1}{2} \frac{dp}{dx} \right)^{1/n} (y - Y_+)^{(n+1)/n} + u_p.$$

(20)

In region B, $|\dot{\gamma}| < \tau_0$, thus the velocity field is not a function of $y$. Hence

$$u_B = u_p.$$  

(21)

In region C, $\dot{\gamma} \leq 0$ and $|\dot{\gamma}| \geq \tau_0$. Applying the boundary conditions $u = u_p$ and $\partial u / \partial y = 0$ at $y = Y_-$ we obtain

$$u_C = \left( \frac{1}{2} \frac{dp}{dx} \right)^{1/n} (Y_- - y)^{(n+1)/n} + u_p.$$  

(22)

The four unknowns in the system are now $dp/dx$, $u_p$ (the velocity in the pseudo-plug region), and the dimensionless positions of the yield surfaces, $Y_+$ and $Y_-$. To solve for these unknowns we apply conservation of mass, boundary conditions at $y = 1$ and $y = 0$ (which have not yet been enforced), and a yield condition. Applying the boundary conditions $u(y = 0) = 0$ and $u(y = 1) = V_s$ we find our first two conditions

$$\left( \frac{1}{2} \frac{dp}{dx} \right)^{1/n} Y_-^{(n+1)/n} + u_p = 0.$$  

(23)

$$\left( \frac{1}{2} \frac{dp}{dx} \right)^{1/n} (1 - Y_+)^{(n+1)/n} + u_p = V_s.$$  

(24)

where $V_s$ is the dimensionless surface velocity. If the dimensional surface velocity is written as $V_s = V_0 f(\hat{x}, t)$
where \( V_0 \) is the maximum surface velocity in the wave region, then

\[
V_s = \frac{f^{n+1}}{n} \left( \frac{V_0^n \mu}{2 \tau_0 H^n} \right)^{1/n} \equiv \frac{n+1}{n} B_n^{-1/n} f \tag{25}
\]

where \( B_n \) is the Bingham number and represents a dimensionless yield stress.

By conservation of mass we know that the total flux \( Q \) through a surface perpendicular to the foot must be zero. In the wave, \( Q = Q_A + Q_B + Q_C = 0 \) where \( Q_i \) is the flux in region \( i \). Integrating equations (20–22) in \( y \) and rearranging, we find our third condition

\[
Y_-^{(n+1)/n} = \frac{n}{2n+1} \left( 1 - Y_+^{(2n+1)/n} + Y_-^{(2n+1)/n} \right) \tag{26}
\]

Finally, \( \tau \) is a linear function of height with slope \( dp/dx \). Considering the slope of the line in the stress plot in figure 3, we find our final yield condition \( dp/dx = 2 \tau_0 / \hat{h} \), where \( \hat{h} = \hat{Y}_+ - \hat{Y}_- \) is the dimensional thickness of the plug region, or in dimensionless form

\[
\frac{dp}{dx} (Y_+ - Y_-) = 2. \tag{27}
\]

Equations (23), (24), (26) and (27) can now be combined to solve for the four unknowns \( dp/dx, u_p, Y_+ \) and \( Y_- \). After some algebra, the system can be reduced to a single transcendental equation for \( Y_- \) with the dimensionless surface velocity (or equivalently, the Bingham number) and the power law exponent as the only remaining parameters:

\[
Y_-^{n+1} \left[ \left( \frac{2n+1}{nY_-} - 1 \right)^{n+1} + 1 \right] = V_s \left[ 1 - Y_- \left( \frac{2n+1}{nY_-} - 1 \right)^{n+1} - Y_- \right]^{1/n}. \tag{28}
\]

The real solutions to this equation as a function of \( V_s \) (or equivalently \( B_n \)) along with \( u_p = -\left( \frac{1}{2 \pi \tau_p} \right)^{1/n} \hat{Y}_+^{(n+1)/n} \), the surface stress, \( \tau_s \), and \( Y_+ = 1 - Y_- \left( \frac{2n+1}{nY_-} - 1 \right)^{n/(2n+1)} \) are all plotted in figure 5 for various \( n \). As expected, for large surface velocities (or small yield stress), the yield surfaces come together and the velocity profiles approach the Newtonian parabolic solution. As the surface velocity becomes small (or yield stress becomes large), the majority of the fluid resides in the pseudo-plug, moving slowly backwards with small shear layers near the top and bottom surfaces.

Now that we have expressions for pressures, yield surfaces and stresses in terms of the surface velocity, we can return to real space and construct the full solution in the gap. Switching to a frame following the wave, the surface velocity becomes a function of \( \xi \) only, i.e. \( V_s = \frac{n+1}{n} B_n^{-1/n} \hat{f}(\xi) \). So, within a contraction wave, the surface velocity (dictated by the local foot speed), starts at \( V_s = 0 \) at the one end of the wave and achieves a maximum velocity, \( V_s = \frac{n+1}{n} B_n^{-1/n} \) somewhere in the middle, then drops back down to \( V_s = 0 \). For example, if \( \hat{f}(\xi) = 16\xi^2(1-\xi)^2 \), we arrive at the yield surfaces shown in figure 6. Note that it is most expeditions to solve for \( \xi \) explicitly in terms of \( Y_- \), rather than implicitly solve for all the variables in terms of \( \xi \).

III. DESIGN AND IMPLEMENTATION OF MECHANICAL CRAWLERS

To test the efficacy of these mechanisms, two robotic snails, one retrograde (Robosnail I) and one direct (Robosnail II) were designed and constructed. Unlike previously developed mechanical snails [7, 8], both snails possess self-contained crawling mechanisms and do not require energy input from the substrate. As we shall see, the mechanics of retrograde versus direct crawling is quite different, hence each snail is uniquely suited to quite different environments.

A. Retrograde Crawler

Robosnail I, our prototype retrograde crawler, has a solid polycarbonate body as shown in figure 7. The motor, powered by an external DC power source, is capable of supplying 1.5, 3.0 and 4.5 volts, and is connected to a variable-speed gear box. A toothed pulley connects the gearbox to a shallow brass helix which passes through an array of aluminum sheets perforated with slots. Each of
the sheets is constrained to vertical motion as they ride in the equally spaced slots along the body. The bottom edges of the aluminum sheets are glued onto a flexible foam sheet. When the helix is spun by the motor and gearbox, it causes the plates to translate up and down inside their tracks in a traveling sinusoidal wave (evident when viewed from the side). As described in section II A, this sinusoidal wave generates regions of high pressure in front of the wave where the fluid is squeezed into a narrow gap, and regions of low pressure behind the wave where the fluid is allowed to expand. These pressures generate forces normal to the interface, driving the snail in the opposite direction of the wave.

A test track slightly larger than the width of the snail, constructed to minimize the leakage of fluid past the open sides of the foot, was filled with a 5 mm thick layer of glycerol and Robosnail I was activated on top of the layer. After the motion reached steady state, measurements of wave speed and snail speed were recorded. The velocities achieved by Robosnail I as a function of wave speed are shown in figure 11(b). As predicted the snail velocity scales linearly with the wave velocity. It is simple to verify that the locomotion is indeed driven by stresses generated within the fluid layer; when the crawler is activated on a dry surface it moves slowly in the direction of wave propagation opposite to what is predicted and observed when the fluid layer is present. This simple test underscores the importance of the thin fluid film in the propulsion mechanism.

Taking a best fit of the data in figure 11(b), and using equation (10) with a sinusoidal wave profile, we predict an average gap thickness of $H = (1.9 \pm 0.1)\bar{a}$ where $\bar{a} = 1$ mm. Interstitial heights were not directly measured and, although this estimate is quite reasonable, it is likely that the model slightly over-predicts the gap thicknesses achieved experimentally. Despite the narrow track, there was considerable leakage out the side of the foot as the crawler progressed reducing the effective pressures within the gap. Hence, we expect the mathematical model to over-estimate the speed of the mechanical crawler or equivalently, over-estimate the gap thickness.

B. Direct Wave Crawler

Robosnail II was built to mimic the direct wave, adhesive locomotion of land snails. Unlike real snails, which have one continuous foot of muscle, the foot of Robosnail II is made up of five discrete, sliding sections. Each of the five sections move forward along a track by a small fixed amount, $\Delta L$, relative to the body. After all five segments have advanced, the body slides forward, returning the segments to their original positions. These small motions occur in sequence as illustrated in figure 9.

The prototype consists of five Nitinol wires attached to nylon cords wrapped 180 degrees around a pulley and tied to five sheets of polycarbonate (see figure 8). Each of the sheets is attached to a pair of guides that allow only fore-aft translation. A leaf spring held by the main body returns each of the sheets to its original position when there is no other force present. The springs supply the restoring forces necessary to return the wires to their outstretched positions. The wires are crimped in loops at the ends; one end is tied to the cords controlling the foot, and the other end of each wire is looped around adjustable brass fasteners to allow proper tightening of the wire and cords. It was important for the wires to be correctly tensioned such that the springs succeed in re-stretching the wires, but not so tight as to limit the motion of the foot segments. The tightness adjustment is achieved by turning the screws through which the wire mounts are threaded.

Laponite [11, 12] was used as a lubricating fluid, chosen for its finite yield stress and ready availability. To test the wall climbing ability of Robosnail II, we mounted it on a tiltable platform covered in a 1.5 mm thick layer of Laponite. Since Laponite has a yield stress of 100 Pa, this gives us an approximate Bingham number of $B_n \sim O(1)$, which corresponds to a surface velocity of order $V_\infty \sim O(1)$. The tilt angle, $\phi$, was measured from...
the horizontal. $\phi = \pi/2$ corresponds to a vertical wall; angles greater than $\pi/2$ indicate that the snail crawls up an overhang in an inverted position; $\phi = \pi$ corresponds to a fully inverted traverse. Robosnail II was able to climb the Laponite-coated surface tilted at any angle, including both a vertical and fully inverted position.

Experimental data shows that the motion per cycle is in all cases slightly less than the translation of one foot, $\Delta L = \bar{X}_w$. Ideally, if there is no backwards slip, the crawler displacement per cycle, $D$, should be $\bar{X}_w$. However, data in figure 11(a) shows that there is some slippage at all inclinations with the maximum occurring around $\theta \approx 3\pi/4$ when gravity is working to oppose both the direction of motion and the normal force that contributes to adhesion. At high tilt angles, much of the adhesive force of the Laponite supports both the weight of the snail and the viscous resistance of the moving foot section, while at near-flat angles near 0 and $\pi$, the adhesive force is required primarily to resist the friction on the moving foot. Thus, we expect a minimum displacement somewhere between $\pi/2$ and $\pi$, as is observed in the experiments, when gravity works to decrease traction and to increase the resistive force of the weight.

The slipping of the stationary sections could be attributed to a number of phenomena. Neither the Laponite layer nor the foam rubber of the crawler’s foot sections is perfectly flat and imperfections in the thickness of the Laponite layer could perturb the local stress field driving a flow. Another likely cause for slippage is that the Laponite under the stationary sections may not have had enough time to resolidify. Once the Laponite has yielded, the microstructure reforms gradually, hence the effective yield stress after flowing is less than it was immediately before yielding. Only after a characteristic “healing” time does the yield stress return to its original value. This last problem can be remedied by increasing the time of each cycle, allowing the Laponite to resolidify at the expense of slowing down the translational speed of the snail. Finally, it was observed that there was delamination of the Laponite layer from the segments of the foot, decreasing the traction of the stationary foot segments.

IV. DISCUSSION

It is clear that the two different crawling mechanisms we have investigated are ideally suited to very different types of terrain. The retrograde crawler which uses peristaltic forces rather than adhesive forces cannot climb vertical walls. However, it has the advantage of being insensitive to the fluid properties and thus can easily maneuver over mud or other mushy surfaces. In contrast, the direct wave crawler can traverse extremely complex topologies including climbing vertical walls and traversing overhangs in an inverted position. The trade-off for this versatility is that the mechanical crawler has very specific requirements regarding the interstitial fluid properties. In particular, in order to perform these complex maneuvers, the yield stress of the fluid must be sufficient to support the weight of the crawler. The mathematical models discussed in sections II A and II B give us some insight as to how to design crawlers to maximize their versatility and minimize the drawbacks outlined above.

A. Exact Solutions and Optimal Wave Profiles for the Retrograde Crawler

The optimization questions that arise in designing retrograde crawlers are primarily geometric. Ideally we would like to know how to change the shape of the deformations in the foot to optimize various quantities such as speed and efficiency. We begin by investigating the limit of both small and large amplitude deformations.

For small wave amplitudes we consider $h(x) = 1 + a\delta(x)$ where $\delta = 0$ and the overbar indicates an $x$-average. Taylor expanding the integrands in (9) we find the shape function $A$ becomes

$$A = \frac{\left(1 + 3a^2 \delta^2\right)^2}{\left(1 + a^2 \delta^2\right) \left(1 + 6a^2 \delta^2\right)}.$$  

This gives, to lowest order, a snail velocity of

$$V_s = 6a^2 \delta^2.$$  

Thus, the snail speed is maximized for profiles that make the average of $\delta^2$ as large as possible (recall that the surface velocity, $V_s$ is the same as the snail velocity, $V_{snail}$, in the retrograde crawler). Square-wave-type profiles accomplish such a maximization while sharp jagged profiles do the worst job.

As the wave amplitude becomes large, the snail foot nearly touches down on the plane beneath and locally $h \approx h_s + (\partial^2 h_s/\partial x^2)(x-x_s)^2/2$. The $I_j$ integrals all then become dominated by local contributions near $x = x_s$ and reduce to derivatives of Lorentzians:

$$I_1 \sim \pi \sqrt{\frac{2}{h_s(\partial^2 h_s/\partial x^2)}}, \quad I_2 \sim \frac{1}{2h_s} I_1, \quad I_3 \sim \frac{3}{8h_s^2} I_1.$$  

Hence, $A \rightarrow 2/3$ and so $V \rightarrow 1$. In other words, for any profile with a generic parabolic minimum, when extended near the underlying plane, the foot acts like a caterpillar tread and drives the snail at the wavespeed.
FIG. 10: Dimensionless snail speeds for different wave profiles. The square-wave-like profile is given by $\delta = \tanh 12\cos(2\pi x)$, and the sawtooth-like profile by a Fourier series truncated at 16 terms. The spiky, sharp profile was built using the function $\exp(-6\sqrt{1 + \cos 2\pi x})$. The dotted lines show the initial parabolic behaviour; the dashed lines show the sawtooth and square wave snail velocity as a function of wave amplitude.

In addition to these important limits, there are several special cases for which the crawler velocity can be exactly computed. For a sinusoidal deformation, the integrations in $A$ can be performed analytically. If the gap thickness is given by $h(x) = 1 - a \cos 2\pi x$ in dimensionless form (or equivalently $h(\bar{x}) = \bar{H} - \bar{a} \cos \frac{2\pi \bar{x}}{\lambda}$), the integrals in (9) become $I_1 = (1 - a^2)^{-1/2}$, $I_2 = (1 - a^2)^{-3/2}$ and $I_3 = \frac{(2 + a^2)}{2(1 - a^2)^{5/2}}$ resulting in a simple expression for snail speed

$$V_s = \frac{3a^2}{1 + 2a^2}.$$  \hspace{1cm} (32)

Since the dimensional amplitude of the wave, $\bar{a}$, cannot exceed the average gap thickness, $\bar{H}$, $a \leq 1$. $V_s$ as a function of $a$ is plotted in figure 10. We find that, for a sinusoidal deformation, the snail velocity monotonically approaches the wave speed, i.e. $V_s \to V_w$, as $\bar{a} \to \bar{H}$.

Two other analytical examples (not necessarily realizable, though) are the square wave and sawtooth, for which

$$I_1 = \frac{1}{1 - a^2}, \quad I_2 = \frac{1 + a^2}{(1 - a^2)^2}, \quad I_3 = \frac{1 + 3a^2}{(1 - a^2)^3},$$ \hspace{1cm} (33)

and

$$I_1 = \frac{1}{2a} \log \left( \frac{1 + a}{1 - a} \right), \quad I_2 = \frac{1}{1 - a^2}, \quad I_3 = \frac{1}{(1 - a^2)^2},$$ \hspace{1cm} (34)

respectively. It is easy to see that the square wave does well near $a = 0$ since it has a large average of $\delta^2$, but does poorly as $a \to 1$, where the sawtooth is better. A comparison between the sinusoidal, square-wave-like and sawtooth-like profiles is shown in figure 10. Also shown is a spiky, sharp profile built using the function $\exp(-6\sqrt{1 + \cos 2\pi x})$. Clearly there is a trade-off between a preference for squarer profiles at low amplitude, to profiles with sharp minima at large amplitude. It is also possible for the snail to be driven faster than the wave for the latter type profiles. (Note that the smooth profiles swerve sharply near $a = 1$ to give the expected result for profiles with parabolic minima as $a \to 1$.) Obviously, this limit is a bit artificial, since the profile cannot be too sharp without violating the lubrication assumption. Analytic and approximate solutions are summarized in Table I.

We can now also determine if the crawler can climb a near-vertical slope. For a sinusoidal deformation, the snail can climb a wall if $6\left(1 - \frac{a^2}{(2 + a^2)}\right) > W\sqrt{1 - a^2}$. Note that the gravitational sliding can always be dominated by the forward motion if $a \to 1$.

It is also interesting to compare this solution to that obtained by Taylor [13] for a free swimming undulating sheet. Taylor’s sheet, which has fluid on both sides and does not interact with a bounding wall, displays the same scaling with amplitude as $\bar{a} \to 0$ namely $V_s \propto \bar{a}^{2}$. For a periodic wave profile, and a wall parallel to the crawler, we can solve the full biharmonic problem without making the lubrication assumption by decomposing the wave profile into a Fourier series (as in Taylor’s article). It is fairly straightforward to show that the first term of the asymptotic series in wave amplitude cannot contribute to driving the snail, and so $V \sim O(a^2)$, wide or narrow fluid layers alike.

### B. Optimized Design of Direct Crawlers

In the direct crawler, it is clear that the interwave segments play a critical role in adhesive locomotion. As we have seen, without the resistance provided by the interwave regions, the snail slips backward as the waves move forward resulting in no net translation. Hence the maximum allowed wave speed is directly related to the interwave area. We can now use the relations derived in section II B to design the appropriate ratio of wave and interwave areas in a mechanical crawler given a fluid with known properties (or conversely, given a snail foot,

<table>
<thead>
<tr>
<th>Profile</th>
<th>$A$</th>
<th>$V_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x) = 1 - a \delta(x)$</td>
<td>$\approx 1 - a^2 \delta^2$</td>
<td>$\approx 6a^2 \delta^2$</td>
</tr>
<tr>
<td>$h \approx h_\infty + h(</td>
<td>x</td>
<td>- x_\infty)/2$</td>
</tr>
<tr>
<td>$h(x) - 1 - a \cos 2\pi x$</td>
<td>$\frac{2}{1 + a^2}$</td>
<td>$\frac{3a^2}{1 + a^2}$</td>
</tr>
<tr>
<td>Square wave</td>
<td>$\frac{(1 + a^2)^2}{1 - 3a^2}$</td>
<td>$\frac{6a^2(1 - a^2)}{1 + 6a^2 - 6a^4}$</td>
</tr>
<tr>
<td>Sawtooth</td>
<td>$\frac{2a}{3\pi}$</td>
<td>$\frac{3(M - 2a)}{2M - 3a}$</td>
</tr>
</tbody>
</table>

**TABLE I:** Summary of snail velocities, $V_s$, and shape functions, $A$, for various wave profiles. The top two represent generic small and large amplitude waves respectively; the bottom three are exact solutions for a sinusoidal, square wave and sawtooth profile. In the sawtooth expressions, $M = \ln \left( \frac{1 + a}{1 - a} \right)$. 

In our mechanical crawler, the right-hand-side of equation (37) is particularly advantageous in this regard if the mucus is shear thinning (see figure 5).

Having built two prototypes and demonstrated the feasibility of propelling crawlers using direct and retrograde mechanisms, future studies will turn to the challenge of using the mathematical models to optimize the design of new crawlers. Again, the issues that arise in retrograde versus direct crawlers are radically different. As we have briefly discussed, for retrograde crawlers, an important design focus is likely to be a geometrical optimization problem, namely, to derive optimal designs for the shape of the foot and hence the shape of the traveling wave. In contrast, optimization of direct wave crawler relies heavily on optimizing the material properties of the interstitial fluid. An ideal fluid would have a large yield stress to support the weight of the snail and it should have a fast heal-time to maximize the velocity of the crawler. As a sample point of reference, we note that the healing time scale for Laponite is on the order of minutes [12] compared to the heal-time for pedal mucus which has evolved to be as fast as 0.1 s [6]. Clearly there is potential to vastly increase crawling capabilities through design and optimization of synthetic fluids. So far, we have not yet explored the effect of rheology on retrograde crawlers which is likely to play a role in addition to the geometrical considerations outlined above. Both material and geometric optimization questions are topics of current investigation.

Acknowledgments

This research was partially supported by NSF grant number CCF-0323672 and Schlumberger-Doll Research (SDR).
It is interesting to note that a similar force balance in the $y$ direction results in no net lift for a wave that is symmetric about the trough, despite experimental evidence to the contrary. This puzzle and various resolutions are discussed in more detail in [14] and [15].

Hancock proposed a similar mechanism for fully immersed filaments, swimming in a Newtonian fluid, in which a longitudinal compression propagates along the body propelling in the organism forward [16].