Stochastic Modeling of Electric Power Prices in a Multi-market Environment

Petter Skantze, Massachusetts Institute of Technology, Cambridge, MA
Marija Ilic, Massachusetts Institute of Technology, Cambridge, MA
Jeffrey Chapman, ABB Systems Control, Santa Clara, CA

Abstract

Over the past few years, a number of competitive electric power markets have emerged in the United States. While market structure differs by region, the common denominator has been the high level of price volatility experienced in these markets. As power suppliers, marketers and consumers seek to manage their positions in this volatile environment, understanding the locational spreads in power prices is becoming increasingly important. The paper develops a dynamic model describing the interplay of electricity prices in a multi-market environment. In contrast to most existing price models, it is based on the fundamental interaction of demand and supply processes. We illustrate how delays in the information flow to market participants causes price differentials to occur between markets with unconstrained transmission interfaces. Furthermore we examine how correlation between load processes translates into a correlation between the price processes in a dual-market environment. These results are illustrated in a simulated example.

In the context of the new model we examine current state of the art algorithms for valuing locational spread options. The work presented suggests that a single correlation factor may not be sufficient to describe the interplay of prices in a dual-market environment. The model illustrates how the price correlation shifts between two states based on the state of congestion on the system. Possibilities for extensions of current option valuation schemes are discussed.

I. Introduction

This paper addresses the question of how one should model the dynamics of electricity prices in the presence of multiple interconnected power markets. This question is becoming increasingly relevant for utilities and power marketers in the United States with the emergence of numerous power pools and power exchanges throughout the country. In practice this problem takes on different shapes. On the east coast we have the interplay between power pools in New England, New York and PJM, where transmission capacity is relatively scarce and prices can diverge significantly. California on the other hand has experienced the emergence of competing power exchanges (the California PX and the APX) covering the same geographic region. We attempt to formulate a general modeling framework, which can incorporate both the constrained and unconstrained interaction of power markets. Such modeling is crucial to the analysis of power markets from several perspectives.

1. Bidding Strategies: If a generating unit is positioned so that it is able to bid its output into two or more markets, it needs a good understanding of how the various prices evolve in relation to one another in order to optimally divide its capacity between the markets.
2. Asset Valuation: What additional value does one assign to a generator that can potentially bid into multiple markets.
3. Hedging Strategies: If the correlation between prices on various power markets can be accurately modeled, then this information can be used to generate effective risk management strategies for market participants.
4. Transmission valuation: In the case were markets are separated by a constrained transmission interface, modeling the evolution of the price differential between the markets can be used to estimate the value of holding transmission rights on that interface.
5. Market Structure: Understanding how price interactions change as a function of market structure allows regulatory agencies to address issues such as whether to allow physical bilateral contracts or competing markets inside a control area.

II. The Modeling of Electricity Prices

Before we can address the issue of how electricity prices in neighboring markets interact, we have to ask what is reasonable model for electricity prices in a single market environment. A significant amount of literature has emerged lately on modeling electricity prices (see [1], [3], [4], [7]). The models proposed are generally adaptations from financial markets (stocks and interest rates), or from mature commodities markets (oil and gas). Such adaptations of existing models often run into trouble because of the unique characteristics of power markets:

1. Extreme price volatility coupled with non-Gaussian characteristics of the implied probability distribution on spot and forward prices.
2. Strong seasonal effects on yearly, weekly and daily cycles.
3. Inability to store electric power.
4. Scarcity of transportation network, coupled with the noncompetitive, tariff based, pricing of inter-area transmission.

The models proposed to deal with these challenges are generally a combination of the following, well-studied, financial models for the evolution of the spot price (S). For an in depth discussion of these and other pricing models see Hull [2].

0-7803-5935-6/00/$10.00 (c) 2000 IEEE

Geometric Brownian motion:

$$\frac{dS}{S} = \mu dt + \sigma dz$$

This process, where z is a Wiener process, produces a lognormal distribution on S(t). This is the underlying process for the Black-Sholes equation.

Mean reverting Brownian motion (Hull and White):

$$dS = \alpha(\mu - x)dt + \sigma_x dz_x$$

or in log-normal form,

$$\frac{dS}{S} = \alpha(\mu - \ln x)dt + \sigma_x dz_x$$

Jump Diffusion model:

$$\frac{dS}{S} = \alpha(\mu - \lambda k)dt + \sigma_x dz_x + dq$$

where dq is a Poisson random incidence process.

Challenges for existing models

The usefulness of any pricing model depends on the ability of the user to accurately calibrate the model parameters such as, mean-reversion rate, volatility, jump size, and jump frequency. This process is further complicated by the fact that each of the parameters exhibits seasonality. Calibrating such a model would therefore require many years worth of consistent price data. In reality however, many of the markets have only existed for one or two years. Even in the case where historical prices are available for a longer period changes in the market rules make historical prices all but useless in the prediction of future volatility.

Modeling the price differential between two markets

As discussed in the introduction, modeling the price differential, or spread, between two markets, can be a very useful hedging and valuation tool. To create such a model we need to go through three basic steps:

1. Select a stochastic process describing the evolution of price in each market, assuming no interaction.
2. Postulate additional constraints for how the variables in the two models interact.
3. Solve for the joint stochastic process of the two markets.

Deng Johnson and Sogomonian [1] show how a locational spread option can be valued by assuming that the futures price process of electricity in each market ($F_{e,1}$, $F_{e,2}$) follow mean-reverting processes:

$$dF_{e,1} = \kappa_1(\mu_{e,1} - \ln F_{e,1})F_{e,1}dt + \sigma_{e,1}(t)F_{e,1}dB^1$$

$$dF_{e,2} = \kappa_2(\mu_{e,2} - \ln F_{e,2})F_{e,2}dt + \sigma_{e,2}(t)F_{e,2}dB^2$$

The interaction between the markets is modeled by assuming that the two Wiener processes B^1 and B^2 have instantaneous correlation ρ. Based on this assumption, the authors proceed to provide a solution to the valuation of locational spread options.

III. Bottom up approach to modeling electricity price dynamics

The model described above has many attractive features. Similar approaches have been used in other markets, and the solutions are well understood. Why then is there any need for further research in this area? The question we try to answer in this paper is whether the joint price dynamics in electric power markets can be accurately described by a simple correlation. To do this we develop a bottom up model of spot market prices, building on the fundamentals of supply and demand.

Defining 'Interactions Between Power Markets'

Up to this point we have used the term 'interaction between power markets' rather freely. The sense has been that the amount of interaction between the markets is somehow related to the degree of correlation between the prices. We will now provide a more rigorous definition of what we mean by markets interacting.

To better illustrate our definitions, consider the following scenario. Two power pools are connected by a single transmission line, with total transmission capacity T. Each pool operates under the following rules. On a day-ahead basis, generators submit marginal bid curves to the pool. Loads are assumed to be fully inelastic, and the pool operator estimates their demand. For each hour the pool operator stacks all supply bids to generate a total supply bid curve (SBC) for the pool. The market-clearing price for the pool is set at the intersection of the SBC and the anticipated demand. Generators are able to submit bids into either pool as long as sufficient transmission capacity is available.

Given this scenario we now attempt to postulate a joint process for the evolution of electricity prices on the two markets. First we recognize that at any given time t, the price (P^i) for a given pool (i) is fully defined by the current load (L) and current SBC for that pool.

$$P^i(t) = f(L^i(t), SBC^i(t))$$

Rather than modeling price directly as a stochastic process, we attempt to characterize the underlying supply and demand processes. The load and supply curve in each market is modeled as a separate stochastic process.

Load model:

$$dL^1 = \alpha_1(\mu_{L1} - L_1)dt + \sigma_{L1}dz_{L1}$$

$$dL^2 = \alpha_2(\mu_{L2} - L_2)dt + \sigma_{L2}dz_{L2}$$

The load in each pool is modeled as a mean reverting process. z_{L1} and z_{L2} are Wiener processes with instantaneous correlation ρ. The means (μ_{L1} and μ_{L2}) are time varying (seasonal) but assumed to be known. Since load is fundamentally weather driven (at least in the short and medium term) we would expect the process to mean revert at the same rate as we lose information in the weather forecast.

Supply Bid Curve Model

Our model for the SBC is based on two main assumptions:

1. At any given point in time, the bid curve (the function translating load to price) can be approximated as an exponential function.
2. All variations in electricity price can be explained by either changes in load, shifts in the bid curve, or a combination of both.

Given these assumptions, our expression for the spot price of electricity takes on the following form:

$$P_i(t) = e^{a_i L_i(t) + b_i(t)}$$

Where L(t) is the load process described above, b(t) represents a shift in the bid curve, and a is a constant.

Why an exponential bid curve?

The exponential shape of the bid curve was chosen because it fit two important criteria. Primarily it is a well-behaved function which in the end greatly simplifies any attempts to value electricity derivatives. For example, a normal distribution on the load produces a lognormal distribution on price, creating a nice link to traditional models. In addition the exponential captures an important characteristics observed in the marginal cost stack of all major markets, and reflected in the actual bids of these units. As load crosses a critical level, the slope of the stack/bid curve increases drastically. Furthermore the point at which this change in slope occurs varies over time. In our model, the change of slope is modeled as a smooth function of load (approximated as an exponential), and the uncertainty in the location of the breaking point is modeled as a stochastic shift in the bid curve.

Figure (1) shows actual day ahead market clearing prices and volumes from the California Power Exchange for July 1999. The scatter plot shows how the slope of the bid curve tends to increase at higher load levels. We approximate this as an exponential bid curve.

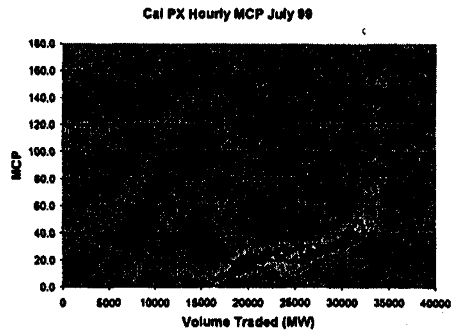


Figure 1

The notion of native bid curves

In our two-market scenario, we assume each generator is physically located inside the boundaries of one of the pools. A generator located in pool 1's territory is considered *native* to region 1. The supply bids into each pool can thus be decomposed into a subset of bids originating from native generators, and bids submitted by external generators. Note that since no demand side bidding is allowed, the load in each pool is composed entirely of native demand. We adopt the following notation for characterizing supply bid functions:

SBC_i^j = the aggregation of all supply bids into pool j, from generators native to region i.

Using this notation we can define the aggregate supply bid curve into a given pool as,

$$SBC^j = \sum_i SBC_i^j$$

We now introduce the notion of a native supply bid curve (SBC_i), defined as

$$SBC_i = \sum_j SBC_i^j$$

In contrast to the aggregate SBC into a pool, the native SBC cannot be directly observed in the market. It is a measure of how much power the native generators are willing to provide at a given price. From this point on, we will assume that the native SBCs for each region remain constant over time.

$$SBC_i(t) = SBC_i$$

This is a strong assumption to make. Market realities such as unit outages and supply side gaming will cause the native SBCs to shift (see [5]). We consider these events to be intra-market effects, in that they can be modeled for individual power markets. In order to isolate the dynamics associated with the interconnection of markets, we assume the intra-market effects to be negligible.

Consolidating the SBC constraints with the exponential price model

In a previous section we postulated that changes in the pool price could be modeled as changes in the regional load and shifts in an exponential bid curve.

$$P^j(t) = e^{a_j L_j(t) + b_j(t)}$$

Now assume that the SBC's take on this exponential form. We rewrite the equation to show the amount of power generators are willing to supply into pool j as a function of market clearing price P.

$$SBC^j(P^j, t) = \frac{1}{a_j}(\ln P^j(t) - b_j(t))$$

Setting supply equal to load at all times yields:

$$L_j(t) = \frac{1}{a_j}(\ln P^j(t) - b_j(t))$$

which is equivalent to the exponential price model above.

We now incorporate the constraints on the native SBCs into the model. Recall that we assumed the native supply bid curves to be stationary over time. This is equivalent to assuming that the quantity supplied is a constant function of price. Clearly this constraint does not hold for the pool bid curves, which have a time varying shift in the exponential curve, characterized by b(t). For the native SBC we therefore assume this term to be constant (b(t) = b).

$$SBC_i(P) = \frac{1}{a_i}(\ln P(t) - \bar{b}_i)$$

If the two markets were fully disconnected, that is the transmission capacity between them was zero, then all native generation would have to be bid in the local pool. As a result, the pool supply bid curve would have to equal to the native supply bid curve. As a result, price in each pool would be defined by,

$$P^1(t) = e^{a_1 L_1(t) + \bar{b}_1}$$

$$P^2(t) = e^{a_2 L_2(t) + \bar{b}_2}$$

As we introduce transmission capacity between the regions, we allow bid curves to shift. These shifts are characterized by deviations in the b parameter. Since bids are not lost or created, but merely moved from one market to another, every positive shift in the bid curve for the first market must be accompanied by a negative shift in the second market. Formally we derive the relationship between these shifts as follows.

Starting with the conservation of bids:

$$\sum_i SBC_i(P) = \sum_j SBC^j(P)$$

and combining it with the definition of the SBC:

$$\sum_i \frac{1}{a_i}(\ln P - \bar{b}_i) = \sum_j \frac{1}{a_j}(\ln P - b_j(t))$$

For our two-market example this reduces to:

$$\frac{b_1(t)}{a_1} + \frac{b_2(t)}{a_2} = \frac{\bar{b}_1}{a_1} + \frac{\bar{b}_2}{a_2} = \text{constant}$$

which shows how shifts in the supply curves are directly related,

$$\frac{\Delta b_1(t)}{a_1} = -\frac{\Delta b_2(t)}{a_2}$$

Closed loop model for supply curve shifts

The derivation in the previous section describes the relation between supply curve shifts in the two markets. We now address what is the driving force behind these shifts. We will assume that whenever a price differential exists between our markets, suppliers will shift bids from the low price region to the high price region to increase their profits. Because price information is not available until after the auction is closed however, there is a delay between the price signal and the redistribution of the bids. We capture this delay by modeling supply shifts for a given day as proportional to the previous days price differential.

$$b_1[k+1] = b_1[k] - G(P^1[k] - P^2[k])$$

Applying the constraint on supply curve shifts between markets:

$$b_2[k+1] = b_2[k] - G\left(\frac{a_2}{a_1}\right)(P^2[k] - P^1[k])$$

Summary of load driven price model

Price is a time varying exponential function of load.

$$P^1[k] = e^{a_1 L_1[k] + b_1[k]}$$

$$P^2[k] = e^{a_2 L_2[k] + b_2[k]}$$

Load is modeled as a mean reverting process.

$$L^1[k+1] - L^1[k] = \alpha_1(\mu_{L1} - L_1[k]) + \sigma_{L1} z_{L1}[k]$$

$$L^2[k+1] - L^2[k] = \alpha_2(\mu_{L2} - L_2[k]) + \sigma_{L2} z_{L2}[k]$$

Supply curve shifts are proportional to price differences between markets.

$$b_1[k+1] = b_1[k] - G(P^1[k] - P^2[k])$$

$$b_2[k+1] = b_2[k] - G\left(\frac{a_2}{a_1}\right)(P^2[k] - P^1[k])$$

IV. Incorporating transmission constraints

We now consider the case where there is a constraint on the total transmission capacity available between the regions. The notation for these constraints is given by,

T_i^j = total transmission capacity from market i to market j

Using the notation developed in the previous section for native supply, we can define imports (imp) into a market as,

Imp = Native Load – Native Supply

Substituting the expression for native supply,

$$SBC_i(P) = \frac{1}{a_i}(\ln P(t) - \overline{b_i})$$

and for native load,

$$L_i(t) = \frac{1}{a_i}(\ln P^i(t) - b_i(t))$$

we arrive at,

$$imp = \frac{1}{a_i}(\ln P^i(t) - b_i(t)) - \frac{1}{a_i}(\ln P(t) - \overline{b_i})$$

which can be simplified as,

$$imp = \left(\frac{\overline{b_i}}{a_i}\right) - \left(\frac{b_i(t)}{a_i}\right)$$

For our two-market system we can rewrite the transmission constraint as:

Imports into market 1 < T_2^1

Imports into market 2 < T_1^2

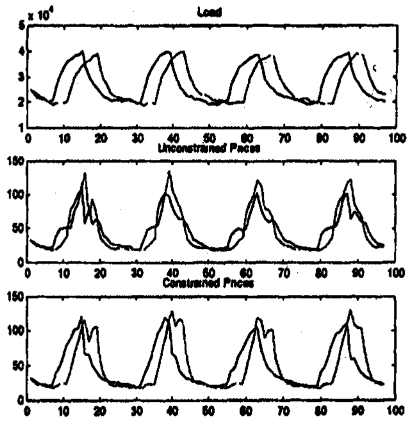
This allows us to derive upper and lower limits on the supply curve shifts:

$$-a_1T_2^1 + \overline{b_1} < b_1 < a_1T_1^2 + \overline{b_1}$$
$$-a_2T_1^2 + \overline{b_2} < b_2 < a_2T_2^1 + \overline{b_2}$$

V. Simulation

For the simulation, we assume that the two markets have identical native supply curves, and the same average on and off peak loads. Loads are modeled by the mean reverting process described in the previous sections. The mean of the process is time varying and cyclical over a 24 hour period to reflect on and off peak demand. To make the results more interesting we have chosen a case where the two markets have non-coincidental peaks. That is, the maximum load during the day occurs at different times in the markets.

In all parts of the simulation, a solid line is used for load/price in market one, and a dashed line is used for market two. The first part of the simulation shows the evolution of load in two markets. The second plot shows how prices evolve when unlimited transmission is available. The third plot shows price evolution in the presence of transmission congestion. Two aspects of the simulated price plots are especially worth noting.



In the unconstrained case, we clearly see the effects of the delay in price information to bidders. In a system with no delays we would expect prices in the two regions to track each other perfectly. As it stands we see, at times, a significant divergence in prices. If the suppliers' response to the price differential increases, corresponding to an increase in the G parameter in our model, prices will tend to converge faster. However, if the price feedback becomes too strong, the price dynamics will go unstable.

In the constrained case, prices closely reflect the time shift in the underlying load processes. As the transmission line becomes congested, the supply curves in the two markets can no longer shift and thus become static. The two price processes are then decoupled, and driven purely by the native load in each region.

VI. Relevance of results in valuing spread options

The modeling and simulation presented above illustrates some fundamental characteristics of electricity prices in multi-market environments.

If the markets are connected by an uncongested transmission line, prices are highly correlated, but sills exhibit some divergence. The extent to which a price differential exists, depends on the inertia of the bidding behavior of the participants.

If markets are connected by a congested interface, the price processes are decoupled and driven by native load. When the transmission interface is constrained, the b(t) state in the price process becomes a constant. We can then write the log of price in each market as:

$$\ln(P^j(t)) = a_j L_j(t) + b_j$$

Since $L_j(t)$ is a normal mean reverting function, $P^j(t)$ then becomes a lognormal mean revering function. In our case:

$$\frac{dP^i}{P^i} = \alpha_i(\mu_{P_i} - \ln(P^i)) + a_i\sigma_{Li}dz_{Li}$$

Note that the Wiener process z_{Li} is the same as for the loads. Therefore the degree of correlation between prices is governed by the correlation between loads.

These result indicate that the correlation between prices in dual-market environments will take on one of two possible values depending on the current state of congestion on the transmission interface. There are several possibilities for incorporating this effect into an option valuation scheme. One can stay with the single correlation model, where the correlation coefficient represents some form of weighted average between the congested and uncongested correlations. An alternative is to allow the correlation to behave as a stochastic variable with two possible states. The probability of being in a given state would then reflect the estimated probability of the transmission interface being congested.

VII. Conclusion

In this paper we have attempted to provide some insight into the interplay between electricity prices in multi-market environment. Rather than attempting to characterize price changes directly as a stochastic process, we used the basics of demand and supply to arrive at the closed loop price dynamics. In doing so we focused on the accuracy of the model in reflecting actual characteristics of the market, rather than finding a convenient mathematical structure for the valuation of derivatives. The modeling shows that the correlation between prices can take on a finite number of values, depending on the number of markets modeled. This challenges the effectiveness of the single-correlation models in accurately predicting the value of locational spread options for electricity. Further research is needed to translate this understanding into an effective algorithm for the valuation of such options. Yet even as it stands, the work provides some framework by which to evaluate models currently in use in the industry.

VIII. Acknowledgments

The authors would like to thank Ralph Masiello at ABB Energy Information Systems for insightful discussions on the future of power system restructuring.

We also greatly appreciate the interaction with the Strategies Group at Constellation Power Source. Specifically we would like to thank Scott Weinstein, Alan Yamamura and Jack Yang for their input on modeling electricity derivatives.

The financial support provided by the MIT Consortium for "New Concepts and Software for Competitive Power Systems" has made this research possible and is greatly appreciated.

IX. References

[1] Deng,S., B., Johnson, and A., Sogommonian, 1998, *Exotic Electricity Options and the Valuation of Electricity Generation and Transmission Assets.*

[2] Hull, J.C., 2000, *Options, Futures and other Derivatives*, (4'th edition), Prentice Hall.

[3] B., Johnson, G , Barz, 1999, *Selecting Stochastic Processes for Modelling Electricity Prices*, Risk Publications

[4] J , Corvin, 1999, *Pricing, Modeling and Managing Physical Power Derivatives*, Risk Publications

[5] P., Skantze, J., Chapman, 1999, *Price Dynamics in the Deregulated California Electricity Market*, Proceedings of the IEEE PES Winter Meeting, New York, NY.

[6] E., Allen, M., Ilic, 1998, *Price-Based Commitment Decisions in the Electricity Market*, Springer-Verlag London Series in Advances in Industrial Control.

[7] S , Deng, 1998, *Stochastic Models of Energy Commodity Prices and Their Applications: Mean-reversion with Jumps and Spikes.*

[8] R., Ethier, T., Mount, 1998, *Estimating the Volatility of Spot Prices in Restructured Electricity Markets and the Implications for Option Values.*

[9] The California Power Exchange website found at http://www.calpx.com.

X. Biographies

Petter Skantze received a B.S. and a MEng. In Electrical Engineering and Computer Science from MIT in 1997. He has been a research assistant with MIT's Energy Laboratory since September 1997, where he is currently pursuing a Ph.D. in EECS at MIT. His research interests are in the areas of power systems restructuring, market dynamics, and risk management in deregulated electricity markets.

Marija Ilic is a Senior Research Scientist in the Department of Electrical Engineering and Computer Science at MIT, where she teaches several graduate courses in the area of electric power systems and heads research in the same area She has twenty years of experience in research and teaching in this area. Prior to coming to MIT in 1987, she was an Assistant Professor at Cornell University, and tenured Associate Professor at the University of Illinois at Urbana-Champaign. Her main interest is in the systems aspects of operations, planning and economics of the electric power industry. She is an IEEE Fellow.

Jeffrey Chapman received his B.S. from the University of California at Santa Barbara, and his M.S. and Ph.D degrees from MIT in 1990, 1992 and 1996 respectively, all in Electrical Engineering. He is currently the West Coast Manager for Whole Sale Trading Applications with ABB Energy Information Systems. His interests are in dynamics of electric power systems and the electricity markets