# Corrections to the "PERFORMANCE OF MULTICLASS MARKOVIAN QUEUEING NETWORKS VIA PIECEWISE LINEAR LYAPUNOV FUNCTIONS", by Bertsimas, Gamarnik and Tsitsiklis, The Annals of Applied Probability 2001, Vol. 11, No. 4, 13841428

The aforementioned paper contains some technical errors concerning some of the lower bounds on the performance of multiclass Markovian queueing networks. The corrections are listed below.

## 1 Statement and proof of Proposition 2

The statement and the proof of Proposition 2 of the paper contain errors. The claimed statement with values of $p_{\min}$ and $\nu_{\min}$ as defined in the paper is not correct, and $p_{\min}$ and $\nu_{\min}$ have to be redefined. Specifically, the correct value of $p_{\min}$ should be as follows. For every station $\sigma_j$, let $I_j$ be the set of classes $i = 1, \ldots, I$ such that $(i, k) \in \sigma_j$ for some stage $k$. Namely, $I_j$ is the set of types that are eventually served by server $\sigma_j$. Instead of letting $p_{\min}$ to be $\sum_i \lambda_i$, as was done in the paper, we define

$$p_{\min} = \sum_{i \in I_j} \lambda_i.$$

Similarly, instead of defining $\nu_{\min} = \rho_{\sigma_j}/\lambda_{\max}$, we let

$$\nu_{\min} = \min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j +}}{\lambda_i}.$$

We claim that Proposition 2 is valid with these modified definitions of $p_{\min}$ and $\nu_{\min}$. The proof of the proposition is corrected as follows. The value of the Lyapunov function increases when an arrival into class $i \in I_j$ occurs, (as opposed to any arrival into the network, as was incorrectly stated in the proof of the proposition in the paper). In particular, an arrival into class $i$ for which no stages correspond to station $\sigma_j$ does not change the value of the Lyapunov function. An arrival into type $I$ occurs with probability $\lambda_i$ and therefore $p_{\min}$ is as stated. The derivation of the correct value of $\nu_{\min}$ is similar.

## 2 Statement and proof of Proposition 4

Similarly, the statement and the proof of Proposition 4 of the paper contain errors. In the statement, the correct value of $p_{\min}$ should be as follows. For every $K$-virtual station $V$, let $I_V$ be the set of classes $i = 1, \ldots, I$ such that $(i, k) \in V$ for some stage $k$. Then define

$$p_{\min} = \sum_{i \in V} \lambda_i.$$

Similarly, the definition of $\nu_{\min}$ is incorrect. The correct definition is

$$\nu_{\min} = \min_{i \in V} \frac{\rho_{i,1}^{V+}}{\lambda_i}.$$

The changes in the argument are similar to the ones for Proposition 2.

# 3 Implications for other statements

In light of these changes, the lower bounds appear in Theorem 2 should be corrected as follows.

$$\mathbb{P}\left(\sum_{i,j} \frac{\rho_{i,k}^{\sigma_j+}}{\lambda_i} Q_{i,k}(t) \geq \frac{1}{2}\left(\min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j+}}{\lambda_i}\right) m\right) \geq \left(\frac{(1/2)\left(\sum_{i \in I_j} \lambda_i\right)\left(\min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j+}}{\lambda_i}\right)}{(1/2)\left(\sum_{i \in I_j} \lambda_i\right)\left(\min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j+}}{\lambda_i}\right) + 1 - \rho_{\sigma_j}}\right)^m .$$

$$\mathbb{E}\left[\sum_{i,j} \frac{\rho_{i,k}^{\sigma_j+}}{\lambda_i} Q_{i,k}(t)\right] \geq \frac{\left(\sum_{i \in I_j} \lambda_i\right)\left(\min_{i \in I_j} \frac{\rho_{i,1}^{\sigma_j+}}{\lambda_i}\right)^2}{4(1 - \rho_{\sigma_j})}.$$

Similarly, the lower bounds appear in Theorem 3 should be corrected as follows.

$$\mathbb{P}\left(\sum_{i,j} \frac{\rho_{i,k}^{V+}}{\lambda_i} Q_{i,k}(t) \geq \frac{1}{2}\left(\min_{i \in V} \frac{\rho_{i,1}^{V+}}{\lambda_i}\right) m\right) \geq \left(\frac{(1/2)\left(\sum_{i \in V} \lambda_i\right)\left(\min_{i \in V} \frac{\rho_{i,1}^{V+}}{\lambda_i}\right)}{(1/2)\left(\sum_{i \in V} \lambda_i\right)\left(\min_{i \in V} \frac{\rho_{i,1}^{V+}}{\lambda_i}\right) + K - 1 - \rho(V)}\right)^m .$$

$$\mathbb{E}\left[\sum_{i,j} \frac{\rho_{i,k}^{V+}}{\lambda_i} Q_{i,k}(t)\right] \geq \frac{\left(\sum_{i \in V} \lambda_i\right)\left(\min_{i \in V} \frac{\rho_{i,1}^{V+}}{\lambda_i}\right)^2}{4(K - 1 - \rho(V))}.$$


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