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DOCTORAL GENERAL EXAMINATION

PART 1

SEPTEMBER 7, 2000

FIVE HOURS

1. This examination is divided into five sections, each consisting of four problems. Answer all the problems. Each problem is worth 5 points, thus the maximum score for the exam is 100.
2. Use a separate fold of paper for each problem. Write your name and the problem number (IV-3 for example) on each fold. A diagram or sketch as part of the answer is often useful, particularly when a problem asks for a quantitative response.
3. Read the problem carefully and do not do more work than is necessary. For example "give" and "sketch" do not mean "derive".
4. Calculators may be used but are not necessary.
5. No books, notes or reference materials may be used.

GROUP I

I-1. Heat Capacity of a Trapped Gas

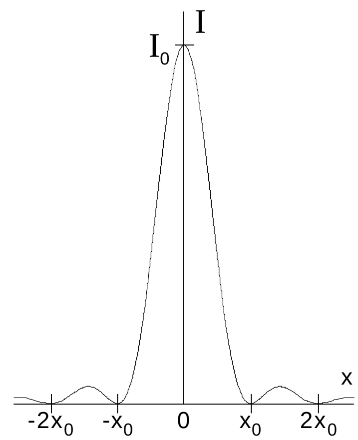
A gas of N atoms of mass m at temperature T is confined in a potential given by

$$\begin{aligned} V(x,y,z) &= ax^2 + by^2 + mgz & \quad z \geq 0 \\ &= \infty & \quad z \leq 0 \end{aligned}$$

Find the heat capacity of the gas.

I-2. Diffraction

A long thin silt is illuminated from behind by a monochromatic source of light. The figure shows the resulting intensity pattern on a screen in the far field region. The pattern changes when the right half of the width of the slit is covered by an absorptive filter which decreases the transmitted electric field in that region by a factor of two. Find the new intensity at the three locations $x = 0$, $x = x_0$, and $x = 2x_0$ in terms of I_0, the maximum intensity in the original pattern. [Hint: use a graphical method of adding complex E fields. Fourier techniques are not necessary.]



I-3. Spin-1 Particles

Two identical spin-1 particles obeying Bose-Einstein statistics are placed in an isotropic harmonic potential in three dimensions.

a) If the particles are non-interacting, give the energies and degeneracies of the lowest two energy levels of the two-particle system.

b) Now assume that the particles have a magnetic moment and interact through a term in the Hamiltonian of the form $A\,\vec{S}_1 \cdot \vec{S}_2$. How are the energies and degeneracies of the states in a) changed by this interaction?

I-4. Muon Decay

A muon (mass m_μ) at rest decays to an electron (mass m_e) and two neutrinos (both massless) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. What direction do the neutrinos travel when the electron has the maximum energy? What is the maximum total energy and momentum the electron may have in terms of the particle masses and fundamental constants?

GROUP II

II-1. Waveguide Propagation

A transverse electric wave propagates in a rectangular waveguide of height a and width b, with $b > a$.

a) Find the lowest frequency that can propagate, ω_o.

b) Find the frequency ω at which the phase velocity is twice the group velocity.

II-2. Relativistic Energy Shift

An ^{55}Fe x-ray source which emits 5.9 keV x-rays is located at sea-level. An absorber is located 30 m directly above the source.

a) Find the fractional energy shift of the x-ray photon when it reaches the absorber. Make any reasonable approximation.

b) The absorber has a very narrow resonance. How fast must the absorber move in order to bring the photon back into resonance?

II-3. Central Force Motion

A particle of mass m moves under an attractive central force with magnitude A/r^3. Find the condition for which it moves with constant radial speed and, for this special case, find the orbital equation: $r(\theta)$, where r and θ are polar co-ordinates.

II-4. Scattering from a Delta Function Potential

A 1-d stream of particles with momentum $\hbar k$ is incident from the $-\hat{x}$ direction on a 1-d delta function potential.

$$U(x) = K\delta(x)$$

Find the fraction of particles transmitted and the fraction that is reflected.

GROUP III

III-1. Light in a Moving Medium

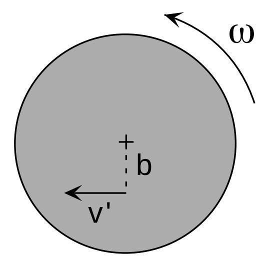
Light travels in a liquid with refractive index n. The liquid is moving with speed V with respect to the laboratory, in the direction of the light. Find the speed of light as measured in the laboratory, to lowest order in V.

III-2. Motion in Crossed Fields

A particle of mass m and charge Q is released from rest in orthogonal uniform electric and magnetic fields: $\vec{E}(\vec{r}) = E_0\hat{x}$ and $\vec{B}(\vec{r}) = B_0\hat{z}$ where in CGS units $E_0 < B_0$. Find the subsequent motion. (Hint: the problem is simplest in a moving coordinate system.)

III-3. Motion in a Rotating Reference Frame

A cylindrical space station of radius R rotates with angular velocity ω as is shown in the figure. Primed quantities indicate quantities measured in the space station frame, unprimed quantities are measured in the inertial frame. An observer in the space station sees a ball is moving at radius b with velocity v' tangentially as indicated in the figure. There are no applied forces.



a) Give the expression for the apparent forces on the ball, $\vec{F}'$ measured in the rotating frame.

b) Find the apparent force vector acting on the ball if the velocity measured in the rotating frame is

$$\text{i) } v' = 0\,; \quad \text{ii) } v' = \omega b\,; \quad \text{iii) } v' = \omega b/2\,.$$

c) In cases i-iii above, describe the ball's initial trajectory as seen by an observer in the non-rotating frame.

III-4. Specific Heat of Diatomic Molecules

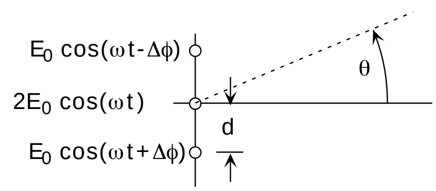
a) Show with a clear sketch how the molar specific heat of an ideal gas of diatomic molecules varies with temperature and indicate any characteristic temperatures.

b) Estimate the characteristic temperatures for a molecule of mass m with moment of inertia I and vibrational frequency ω_o.

GROUP IV

IV-1. Antennas Driven Unequally

Three identical radio antennas, radiating isotropically in the plane, are arranged in a line separated from each other by a distance d. The center antenna is driven with an amplitude **twice** that of its neighbors. The outside antennas are driven with phases $-\Delta\phi$ and $\Delta\phi$ with respect to the center one.



a) Find an expression for the time averaged intensity $< I >$ of the radio signal at a large distance from the array in terms of d, $\Delta\phi$, θ, the wavelength λ, and the intensity that would result from one outside antenna alone $< I_0 >$.

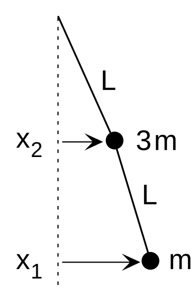
b) Sketch $< I > / < I_0 >$ for $\theta = 0$ as $\Delta\phi$ varies from 0 to 2π.

IV-2. Stark Effect

The $2S$ and $2P$ states of hydrogen are split by energy Δ (the Lamb shift). Find the change in energy of these states when an electric field $\vec{E}$ is applied along the z-axis, as a function of its magnitude E. Make a sketch of the results. Neglect all other states and the effects of spin. Do not evaluate any integrals but designate them by some symbol.

IV-3. Compound Pendulum

A pendulum is composed of two masses, $3m$ and m, and two strings of equal length L as shown. At $t = 0$ the system is released from rest with the upper (heavier) mass undisplaced and the lower mass displaced to the right: $x_2(0) = 0$, $x_1(0) = a \ll L$. Find an expression for the subsequent motion of the lower mass, $x_1(t)$.



IV-4. Blue Sky

Propose a model to explain why the sky is blue and justify your reasoning with a simple analytic calculation. [Note: the atmosphere is not made up of free electrons!]

GROUP V

V-1. Absorbing Sheet

An electro-magnetic plane wave propagates in free space with angular frequency ω and wave number k. The wave is incident normally on an absorbing sheet. The resistance of the sheet is adjusted until there is no reflection. Find an expression for the resistance of a square of the sheet of side l, and estimate the numerical value.

V-2. Pivoted Plank

A plank of length l and mass M hangs from a pivot on one side and a wire on the other. The wire is cut.

a) Find the force on the pivot immediately after the wire is cut.

b) Find the force on the pivot when the plank swings through vertical.

V-3. Hydrostatic System

The internal energy of a certain hydrostatic system is given by $U = AP^2V$ where the constant A has the units of (pressure)$^{-1}$. Find the slope, dP/dV, of an adiabatic path ($\not{d}Q = 0$) in the P-V plane in terms of A, P, and V.

V-4. Bead on Loop

An uncharged spin 1/2 bead is confined to move on a wire loop of radius a which lies in the $x-y$ plane. The bead's position may be specified by its distance x along the wire from a reference point, $0 \leq x \leq 2\pi a$. The bead has magnetic moment μ. ($\vec{\mu} = 2\mu\vec{S}$.)

a) The lowest energy state(s) of the system have energy E_o. The next highest energy state(s) have energy E_1. Find E_o and E_1. Discuss the degeneracy of each. Give a complete set.

b) A magnetic field $\vec{B} = B_o\hat{z}$ is applied. The magnetic field may lift the degeneracy of some of the state(s) in a). Find the new energies of all the states in a) with the magnetic field applied.