The Blume-Emery-Griffiths Spin Glass and Inverted Tricritical Points

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The Blume-Emery-Griffiths (BEG) model\textsuperscript{[1,2]} is the simplest system for the study of the various meetings of first- and second-order phase boundaries between ordered and disordered phases, in a plethora of phase diagram topologies\textsuperscript{[3]}. In these diagrams, the second-order phase transitions are dominated by thermal fluctuations and occur at high temperatures. The first-order phase transitions evolve to finite temperatures, from zero-temperature ground-state energy crossings and occur at low temperatures. In a well-known phase diagram topology, a tricritical point separates the high-temperature second-order boundary and the low-temperature first-order boundary. In the present work, we find that a temperature sequence of transitions that is reverse to the above can occur with the inclusion of quenched randomness. Thus, an inverted tricritical point is obtained, separating a high-temperature first-order boundary and a low-temperature second-order boundary. Interest is further compounded with spin-glass type of quenched randomness\textsuperscript{[3]}, as the spin-glass phase appears within the Blume-Emery-Griffiths global phase diagram (Fig.1). Thus, a new spin-glass phase diagram topology is found, in which disconnected spin-glass regions occur close to the ferromagnetic and antiferromagnetic phases, but are separated by a paramagnetic gap.

We have studied, in spatial dimension \(d = 3\), the model with Hamiltonian

\[-\beta \mathcal{H} = \sum_{<ij>} [J_{ij} s_i s_j + K s_i^2 s_j^2 - \Delta(s_i^2 + s_j^2)],\]  

(1)

where \(s_i = 0, \pm 1\) at each site \(i\) of the lattice and \(<ij>\) indicates summation over nearest-neighbor pairs of sites. The spin-glass type of quenched randomness is given by each local \(J_{ij}\) being ferromagnetic with the value \(+J\) with probability \(1-p\) and antiferromagnetic with the value \(-J\) with probability \(p\). Under the scale change induced by renormalization-group transformation, all renormalized interactions become quenched random and the more general Hamiltonian

\[-\beta \mathcal{H} = \sum_{<ij>} [J_{ij} s_i s_j + K_{ij} s_i^2 s_j^2 - \Delta_{ij}(s_i^2 + s_j^2) - \Delta_{ij}^\dagger(s_i^2 - s_j^2)],\]  

(2)

has to be considered. The renormalization-group flows are in terms of the joint quenched probability distribution \(P(J_{ij}, K_{ij}, \Delta_{ij}, \Delta_{ij}^\dagger)\), which is renormalized through the convolution\textsuperscript{[3]}

\[ P'((K'_{ij}, J'_{ij})) = \int \prod_{ij} dK_{ij} P(K_{ij}) \delta(K'_{ij} - \mathbf{R}(\{K_{ij}\})), \]  

(3)

where primes refer to the renormalized system, \(K_{ij} \equiv (J_{ij}, K_{ij}, \Delta_{ij}, \Delta_{ij}^\dagger)\), and \(\mathbf{R}(K_{ij})\) is the local recursion re-
lution through which 108 unrenormalized local interactions in \{K_{ij}\} determine 4 renormalized local interactions in \(K'_{ij'}\). The local recursion relation \(R(K_{ij})\) is effected by a mixed Migdal-Kadanoff procedure [6] with \(d = 3\) and length rescaling factor \(b = 3\) necessary for the equal \(a \text{ priori}\) treatment of ferromagnetism and antiferromagnetism. Thus, our treatment is approximate for the cubic lattice and exact for the hierarchical lattice [6, 7, 8, 9, 10, 11, 12, 13, 14, 15] shown in Fig. 2. This hierarchical lattice is known to give very accurate results for the critical temperatures of the \(d = 3\) isotropic and anisotropic Ising models [6]. The probability distribution \(P(J_{ij}, K_{ij}, \Delta_{ij}, \Delta'_{ij})\) is represented by histograms lodged on a four-dimensional interaction space \((J_{ij}, K_{ij}, \Delta_{ij}, \Delta'_{ij})\). Eq (3) is effected by 8 pairwise convolutions, which are either bond-moving or decimation in the appropriate sequence, between intermediate distributions. The number of histograms rapidly grows from the starting two described after Eq (1). Thus, for calculational purposes, before each pairwise convolution, the histograms are combined by using a binning procedure, so that our results are obtained by the renormalization-group flows of 22,500 histograms.

Tricritical phase diagram cross-sections of the purely ferromagnetic system for constant chemical potential \(\Delta/J\) of the non-magnetic state are in Fig. 4. The outermost cross-section has \(\Delta/J = -\infty\), meaning no \(s_i = 0\) states, and therefore is the phase diagram of the spin-1/2 Ising spin glass, showing as temperature is lowered the paramagnet-ferromagnet spin-glass reentrance [16, 17]. The annealed vacancies, namely the nonmagnetic states \(s_i = 0\), are introduced in cross-sections with successively higher values of \(\Delta/J\). For \(\Delta/J\) greater than the non-random tricritical value of \(\Delta/J = 0.192\), first-order transitions between the ferromagnetic and paramagnetic phases are introduced from the low randomness side, but are converted to the strong-coupling second-order transition at a threshold value of randomness \(\sigma\). This constitutes an \emph{inverted tricritical point}, since the phase boundary is converted from first order to second order as temperature is lowered, contrary to the ordinary tricritical points (as seen for example in Fig. 3). The above results are consistent with the general prediction that, in \(d = 3\), quenched randomness gradually converts first-order boundaries into second order [18] (In \(d = 2\), this conversion is predicted to happen with infinitesimal quenched randomness [18, 19]).

As the annealed vacancies \(s_i = 0\) are increased, at \(\Delta/J \geq 0.34\), of the second-order transitions between the ferromagnetic and paramagnetic phases, only the strong-coupling transition remains. At \(\Delta/J \geq 0.42\), the strong-coupling second-order transition also disappears, leaving only first-order transitions between the ferromagnetic
and paramagnetic phases. Also as the annealed vacancies are increased, all ordered phases recede. In this process, first the spin-glass phase disappears, at $\Delta/J = 0.37$, which is understandable, since it is tenously ordered due to frustration. The new, disconnected spin-glass phase diagram topology is obtained in this neighborhood, e.g., for $\Delta/J = 0.35$ as shown in Fig. 4, in which the spin-glass phase occurs close to the ferromagnetic (and, symmetrically, antiferromagnetic, not shown here) phase, but yields to the paramagnetic phase as $p$ is increased towards 0.5. The spin-glass phase disappears at $\Delta/J = 0.37$.

The paramagnetic-ferromagnetic-spin-glass reentrances, as temperature is lowered, of the Blume-Emery-Griffiths spin-glass cross-sections fall on the same reentrant second-order boundary, as seen in Fig. 4. As seen for $\Delta/J = 0.45$ and 0.48 in this figure, before disappearing at $\Delta/J = 0.5$, the ferromagnetic phase exhibits paramagnetic-ferromagnetic-paramagnetic reentrance as temperature is lowered.

Constant $p$ cross-sections of the global phase diagram in Fig. 1 are shown in Fig. 5. The outermost curve corresponds to the pure Blume-Emery-Griffiths model with no quenched randomness ($p = 0$). As spin-glass quenched randomness is introduced with increasing values of $p$, ordered phases and first-order phase transitions recede.

FIG. 4: (Color on-line) Blume-Emery-Griffiths spin-glass phase diagrams: Constant $\Delta/J$ cross-sections of the global phase diagram in Fig. 1. The outermost cross-section has $\Delta/J = -\infty$, meaning no $s_i = 0$ states. The annealed vacancies $s_i = 0$ are introduced in cross-sections with successively higher values of $\Delta/J$, making all ordered phases recede. The dotted and full lines are respectively first- and second-order phase boundaries. The dashed lines are strong-coupling second-order phase boundaries induced by quenched randomness. The inverted tricritical topology is seen between the second-order phase boundaries induced by quenched randomness. The outermost curve corresponds to the pure Blume-Emery-Griffiths model with no quenched randomness ($p = 0$). As spin-glass quenched randomness is introduced with increasing values of $p$, ordered phases and first-order phase transitions recede.

FIG. 5: (Color on-line) Spin-glass Blume-Emery-Griffiths phase diagrams: Constant $p$ cross-sections of the global phase diagram in Fig. 1. The dotted and full lines are respectively first- and second-order phase boundaries. The dashed lines are strong-coupling second-order phase boundaries induced by quenched randomness. The outermost curve corresponds to the pure Blume-Emery-Griffiths model with no quenched randomness ($p = 0$). As spin-glass quenched randomness is introduced with increasing values of $p$, ordered phases and first-order phase transitions recede.
butions of (a) the quenched randomness-induced second-order transitions between the ferromagnetic and paramagnetic phases, (b) the first-order transitions between the ferromagnetic and paramagnetic phases, (c) the second-order transitions between the ferromagnetic and spin-glass phases, (d) the second-order phase transitions between the spin-glass and paramagnetic phases, and (e) the sink fixed distribution for the spin-glass phase. Note that (a),(b),(c),(d) are runaways, in the sense that the couplings renormalize to infinity while the distribution retains its shape shown here. In the second-order phase transitions between the spin-glass and paramagnetic phases (d), $\Delta$ is a runaway (to minus infinity), while the other interactions remain finite. The fixed distributions in this figure are singly unstable, except for the sink (e), which is totally stable.

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