Hysteresis loops are obtained in the Ising spin-glass phase in $d=3$, using frustration-conserving hard-spin mean-field theory. The system is driven by a time-dependent random magnetic field $H_Q$ that is conjugate to the spin-glass order $Q$, yielding a field-driven first-order phase transition through the spin-glass phase. The hysteresis loop area $A$ of the $Q-H_Q$ curve scales with respect to the sweep rate $h$ of magnetic field as $A - A_0 \sim h^b$. In the spin-glass and random-bond ferromagnetic phases, the sweep-rate scaling exponent $b$ changes with temperature $T$, but appears not to change with antiferromagnetic bond concentration $p$. By contrast, in the pure ferromagnetic phase, $b$ does not depend on $T$ and has a sharply different value than in the two other phases.

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Frustration and non-equilibrium effects induce complicated ordering behaviors that challenge the methods of statistical physics. Perhaps the most ubiquitous non-equilibrium effect, hysteresis is the current topic of intense fundamental and applied studies.\[1, 2, 3, 4, 5\] In the present study, hard-spin mean-field theory, developed specifically to respect frustration\[6, 7\], is used to study the non-equilibrium behavior of the field-driven first-order phase transition that is implicit, but to-date unstudied, in spin-glass ordering. For the Ising spin-glass on a cubic lattice, the phase diagram is obtained and the temperature- and concentration-dependent ordering of the spin-glass phase is microscopically determined. The random magnetic field that is conjugate to this microscopic order is then identified and used to induce a first-order transition and hysteresis loops. We find qualitatively and quantitatively contrasting scaling behaviors in spin-glass, quenched random-bond ferromagnetic, and pure ferromagnetic phases of the system.

The model is defined by the Hamiltonian

$$-\beta H = \sum_{<ij>} J_{ij} s_i s_j + \sum_i H_i(t) s_i, \quad (1)$$

where $s_i = \pm 1$ at each site $i$ of a cubic lattice and $<ij>$ denotes summation over nearest-neighbor pairs. The
bond strengths \( J_{ij} \) are equal to \(-J\) with quenched probability \( p \) and \(+J\) with probability \(1-p\), respectively corresponding to antiferromagnetic and ferromagnetic coupling. \( H_i(t) \) is a linearly swept quenched random magnetic field, itself determined, as explained below, by the spin-glass local order of this system.

For our calculations we use the hard-spin mean-field theory \([6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]\), a method which is nearly as simply implemented as the conventional mean-field theory \([6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]\), but which conserves frustration by incorporating the effect of the full magnitude of each spin. The self-consistent equation for local magnetizations \( m_i \) in hard-spin mean-field theory is

\[
m_i = \sum_j \left[ \prod_j P(m_j, s_j) \right] \tanh \left( \sum_j J_{ij} s_j + H_i(t) \right),
\]

where the sum \{ \( s_j \) \} is over all interacting neighbor configurations and the sum and the product over \( j \) are over all sites that are coupled to site \( i \) by interaction \( J_{ij} \). The single-site probability distribution \( P(m_j, s_j) \) is \((1 + m_j s_j)/2\). The hard-spin mean-field theory has been used in time-dependent systems, in the study of field-cooled and zero-field cooled magnetizations in spin glasses.\([14]\)

**Equilibrium Phase Diagram** - The equilibrium local magnetizations \( m_i^{(0)} \) are determined by simultaneously solving \( N \) coupled Eqs. (2) for all \( N \) sites \( i \) of the system, at zero external magnetic field, \( H = 0 \). For \( 0 < p < 1 \), the system is degenerate, and many local magnetization solutions exist and are reached by hard-spin mean-field theory. The phase diagram (Fig.1) is obtained from temperature \( T = J^{-1} \) and concentration \( p \) scans of the equilibrium spin-glass order parameter \( Q^{(0)} = \frac{1}{N} \sum_i m_i^{2} \) and magnetization \( M^{(0)} = \frac{1}{N} \sum_i m_i \), illustrated in Fig.2 obtained by averaging over 20 realizations for a \( N = 20^3 \) spin system. The results do not change if a larger system is used. In the resulting phase diagram shown in Fig.1 the transition temperatures are gauged by comparing \( T_C \) at \( p = 0 \): The precise value \([21]\) is 4.51, the ordinary mean-field value is 6, the value obtained here is 5.06. Thus, the transition temperatures are exaggerated as expected from a mean-field theory, but considerably improved over ordinary mean-field theory. Our obtained
transition concentrations between the ferromagnetic and spin-glass phases are $p = 0.22$, in excellent agreement with the precise value of $p = 0.23$\cite{22}.

Fig.4 shows the zero-temperature spin-glass order parameter $Q^{(0)}$ as a function of antiferromagnetic bond concentration $p$. It seen that, as soon as frustration is introduced via the antiferromagnetic bonds, order does not saturate at zero temperature, both in the ferromagnetic and spin-glass phases, the latter of course showing more unsaturation. Moreover, the left column of Fig.4 shows the equilibrium local magnetizations $m_i$ in a cross-section of the system, in the ferromagnetic and spin-glass phases. These magnetization cross-sections are remarkably similar to the renormalization-group results\cite{22} and are consistent with the chaotic rescaling picture of the spin-glass phase\cite{24}.

**Spin-Glass Hysteresis Loops** - The quenched random magnetic field that is conjugate to the microscopic order is $H_i(t) = \Delta H(t)m_i^{(0)}$ in Eq.[1], where the $m_i^{(0)}$ are the equilibrium local magnetizations obtained with Eq.[2] for a given $T, p$. Hysteresis loops in the spin-glass order $Q(t) = \frac{1}{N}\Sigma_i m_i(t)m_i^{(0)}$ are obtained in the ordered phases, spin-glass or ferromagnetic, by cycling $H_Q(t)$ at constant $T, p$, via a step of magnitude $h$ for each time unit. Thus, at time $t = 0$, $Q(t = 0) = Q^{(0)}$, the equilibrium spin-glass order parameter. A time unit is $N$ updating of Eq.[2] at randomly selected sites. Thus, $h$ is the sweep rate of the linearly driven random magnetic field. The resulting hysteresis curves are illustrated in Figs.5. After one cycling, the subsequent hysteresis loops for a given sweep rate coincide, and are shown in Figs.5 and used in the scaling analysis further below.

**Cycling Effect of a Uniform Magnetic Field on Spin-Glass Order** - As a contrast to the hysteretic effect of the conjugate quenched random magnetic field $H_Q(t)$ introduced above, Fig.6 shows the effect on the spin-glass phase of turning on and then off a uniform magnetic field $H(t)$ at a sweep rate $h$. As expected, the spin-glass order $Q(t)$ starts at a finite value and returns to zero, while the uniform magnetization $M(t) = \frac{1}{N}\Sigma_i m_i(t)$ starts at zero and returns to a finite value.

**Spin-Glass Hysteresis Area Scaling** - The energy dissipation of a first-order phase transition is obtained from the

\[ A = \int_{-\infty}^{\infty} (Q_{\text{sat}} - Q)^2 \, dt \]

where $Q_{\text{sat}}$ is the saturated value of $Q(t)$ and $Q$ is the value of $Q(t)$ at a given sweep rate $h$. The hysteresis area $A$ is plotted as a function of sweep rate $h$ in Fig.7 for different values of $p$. It is seen that the hysteresis area $A$ decreases as $p$ increases, and that the scaling behavior of $A$ with $h$ is non-linear, consistent with the chaotic rescaling picture of the spin-glass phase.
hysteresis area $A$ of the $Q - H_Q$ curve: $A = \int Q dH_Q$. At fixed $T, p$, the loop area $A$ decreases with decreasing sweep rate $h$ and finally reaches a value of $A_0$. The area can be scaled as $A = A_0 + f(T)h^b$. The $(A - A_0)$ versus sweep rate $h$ scaling curves are shown in Figs. 7 for the pure ferromagnetic, quenched random-bond ferromagnetic, and spin-glass phases for various temperatures, where $A_0$ is fitted. The resulting sweep-rate exponents $b$ are given in Fig. 8 and Table I. From these results, we deduce that in the pure ferromagnetic phase, $p = 0$, the exponent $b$ is independent of temperature, as found previously. However, the value of $b = 0.64$ that we find here, under hard-spin mean-field dynamics, is distinctly different from that of $b = 2/3$ found in Ref. [5] under ordinary mean-field dynamics, thereby constituting a different dynamic universality class. By contrast, in the quenched random-bond ferromagnetic phase and in the spin-glass phase, the value of $b$ is distinctly smaller than that in the pure ferromagnetic phase, and dependent on temperature. Across both of these two phases, there appears to be no dependence of $b$ on concentration.

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![FIG. 8: The sweep-rate scaling exponent $b$ versus concentration $p$ for $T = 1.0, 1.5, 2.0$. These results are obtained by averaging over 10 realizations, with the standard deviation being used as the error bar.](image)

<table>
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<th>$T$</th>
<th>$p = 0$</th>
<th>$p = 0.1$</th>
<th>$p = 0.2$</th>
<th>$p = 0.3$</th>
<th>$p = 0.4$</th>
<th>$p = 0.5$</th>
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<td>0.64 ± 0.04</td>
<td>0.74 ± 0.04</td>
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<td>0.89 ± 0.04</td>
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</tr>
<tr>
<td>1.5</td>
<td>0.64 ± 0.04</td>
<td>0.84 ± 0.04</td>
<td>0.92 ± 0.04</td>
<td>0.95 ± 0.04</td>
<td>0.98 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>0.64 ± 0.04</td>
<td>0.94 ± 0.04</td>
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<td>0.99 ± 0.04</td>
<td>1.00 ± 0.04</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I: The sweep-rate scaling exponents $b$ at different temperatures and concentrations in the ferromagnetic and spin-glass phases.