Tensor networks provide a useful class of variational wave functions suitable for numerical studies of ground states of quantum Hamiltonian as well as certain classical problems. In spite of an extensive body of work on the subject, so far no examples have been found where tensor networks describe exactly ground states of gapless systems. Here, I will describe the first such example: a construction of the ground states of the deformed Motzkin and Fredkin models. These models are spin chains that pose a novel quantum phase transition between low and high entanglement, and, as I will show, their ground state can be written as a new type of tensor network, describing tiling models in which the physical degrees of freedom live on the edge. I will also point out an interesting application to a well-known problem in non-equilibrium statistical mechanics.