Recent studies of out-of-time ordered thermal correlation functions (OTOC) in holographic systems and in solvable models such as the Sachdev-Ye-Kitaev (SYK) model have yielded new insights into manifestations of many-body chaos. So far the chaotic behavior has been obtained through explicit calculations in specific models. We propose a unified description of the exponential growth and ballistic butterfly spreading of OTOCs across different systems using a newly formulated "quantum hydrodynamics," which is valid at finite $\hbar$ and to all orders in derivatives. The scrambling of a generic few-body operator in a chaotic system is described as building up a "hydrodynamic cloud," and the exponential growth of the cloud arises from a shift symmetry of the hydrodynamic action. The shift symmetry also shields correlation functions of the energy density and flux, and time ordered correlation functions of generic operators from exponential growth, while leads to chaotic behavior in OTOCs. The theory also predicts a phenomenon of the skipping of a pole at special values of complex frequency and momentum in response functions of energy density and flux. This pole-skipping phenomenon may be considered as a "smoking gun" for the hydrodynamic origin of the chaotic mode.