“The two dimensional Hubbard model and beyond: what we know, what we think we can know, and what is open”

The model introduced by John Hubbard in 1963 is perhaps the simplest model that encapsulates all of the difficulty of many-fermion quantum mechanics. An essentially complete solution of the one dimensional Hubbard model was formulated in the mid 1960s and was understood over the next two decades; the infinite dimensional version was solved in the mid 1990s, but the behavior at intermediate dimensions, especially the two dimensional repulsive-interaction version believed to be relevant for copper-oxide superconductivity, has remained a challenge to theoretical physics. In this talk I will review recent progress, due in part to remarkable recent developments of some numerical algorithms and improvements of others, which has provided definitive information about some aspects of the phase diagram and excitation spectrum, reasonable indications of other aspects, and has sharpened the remaining theoretical questions. After 55 years, the solution of this basic model of electron correlations seems now to be within reach.