In the presence of a quantizing magnetic field, the lowest energy manifold of bilayer graphene (BLG) has spin, valley, and a two-fold orbital degeneracy. Experimentally, it can be tuned by applying a parallel magnetic field and by applying a perpendicular electric field, which tends to polarize the layers. At charge neutrality, which corresponds to $\nu=0$, the system can choose among a rich set of possible ground states based on the effective interactions and external tuning parameters. We will set up a simple model Hamiltonian containing symmetry-allowed terms and analyze it in the Hartree-Fock approximation to obtain several potential ground states, some of which have been obtained before (such as the canted antiferromagnet and the fully layer polarized phase) and one which has not. This new ground state breaks two different U(1) symmetries spontaneously, and could appear as an intermediate phase between the canted antiferromagnet and the fully layer polarized phases.