Topological crystalline insulators (TCIs) are a new class of topological materials which harbor massless Dirac surface states (SS). Theory postulates that these SS are protected by crystalline symmetries, and that SS electrons can acquire a mass if these symmetries are broken. Moreover, this unique crystalline protection has led to a series of intriguing predictions of strain-generated phenomena, such as the appearance of pseudo-magnetic fields and topological phase transitions. In this talk, I will present our recent scanning tunneling microscopy (STM) investigations of two TCI systems: single crystals of Pb$_{1-x}$Sn$_x$Se and strained thin films of SnTe. Simultaneous imaging of the atomic and electronic structures in TCI single crystals reveals that a fraction of the Dirac electrons acquire mass due to a surface distortion that breaks a crystalline mirror symmetry. Furthermore, we discover that even in the absence of any symmetry breaking, nanoscale strain in TCI heteroepitaxial thin films can induce spatially dependent changes in the SS dispersion associated with the momentum-space shift of the Dirac nodes. Our experiments provide the first direct visualization of the effects of strain on the SS band structure in any topological material and suggest a novel pathway for manipulation of Dirac electrons via structural deformations.