

Selected Publications of Frank Wilczek, with Brief Commentary

The full bibliography has grown by accretion. It contains quite a bit of secondary material (that is, conference proceedings, reviews, and so forth) and lacks cohesive organization. The following version is less complete, but better organized. It also contains considerable explanatory material. For most purposes, it should be more useful.

To preserve sanity, the numbering of items is the same as in the complete bibliography. Quite a few papers straddle the boundary between categories, but I always chose a unique classification.

QCD: Foundational Papers

In the first three papers listed below, Gross and I did several things. We discovered the central dynamical feature unique to non-abelian gauge theories, that their effective strength diminishes to zero at short distances or high energy due to quantum antiscreening of charge (asymptotic freedom). We explored many examples, including some with scalar fields and some with non-trivial weak-coupling infrared fixed points. We proposed one such gauge theory, now known as QCD, as the fundamental theory of the strong interaction. And we proposed specific experimental tests of our ideas. In the fourth paper some technical objections to the theory were cleared up, and in the fifth and sixth papers further experimental consequences, regarding the pointwise evolution of structure functions, were derived. The most dramatic of these, that protons viewed at ever higher resolution would appear more and more as field energy (soft glue), was only clearly verified at HERA twenty years later. Finally I have included an early review, which was among the first to discuss QCD as an established theory, and my lecture on receipt of the Dirac Medal.

1. Ultraviolet Behavior of Non-Abelian Gauge Theories (with D. Gross), *Phys. Rev. Lett.* **30**, 1343 (1973).
2. Asymptotically Free Gauge Theories, I (with D. Gross), *Phys. Rev.* **D8**, 3633 (1973).
3. Asymptotically Free Gauge Theories, II (with D. Gross), *Phys. Rev.* **D9**, 980 (1974).
4. Gauge Dependence of Renormalization Group Parameters (with W. Caswell), *Phys. Lett.* **B49**, 291 (1974).
5. Possible Non-Regge Behavior of Electroproduction Structure Functions (with A. DeRujula, S.L. Glashow, H.D. Politzer, S.B. Treiman and A. Zee), *Phys. Rev.* **D10**, 1649 (1974).
6. Scaling Deviations for Neutrino Reactions in Asymptotically Free Field Theories (with S. Treiman and A. Zee) *Phys. Rev.* **D10**, 2881 (1974).
68. QCD - The Modern Theory of Strong Interactions, *Ann. Rev. Nucl. Sci.* **V32**, 177 (1982).
235. Asymptotic Freedom, Lecture on receipt of the Dirac Medal, October 1994, published by ICTP, Trieste, Italy. hep-th/9609099 [96-95]

QCD: High Temperature

Item 91 is foundational. In it, Pisarski and I applied the renormalization group to the QCD chiral transition, and argued that for three massless flavors it could not be a second-order transition, while for two massless flavors it might be second-order in the $O(4)$ universality class. This seems to be borne out by numerical simulation work. In Items 181 and 186 the full machinery of the renormalization group was brought to bear on this problem, and a host of universal predictions are made, including the existence and properties of a tricritical point as both temperature and strange quark mass are varied. Some of these have been crudely tested already. I anticipate that in future work many more will be. In Item 189 Rajagopal and I demonstrated a mechanism whereby the return to equilibrium after a quench following chiral symmetry restoration can be accompanied by dramatic non-thermal radiation. This suggestion has stimulated a lot of activity, both theoretical and experimental.

91. Remarks on the Chiral Phase Transition in Chromodynamics (with R. Pisarski), *Phys. Rev.* **D29**, 338 (1984).
181. Application of the Renormalization Group to a Second Order QCD Phase Transition, *Int. Jour. Mod. Phys.* **A7**, 3911 (1992). [91/65]
186. Static and Dynamic Critical Phenomena at a Second Order QCD Phase Transition (with Krishna Rajagopal) *Nucl. Phys.* **B399** (1993) 395. hep-ph/9210253 [92/60]
189. Emergence of Coherent Long Wavelength Oscillations After a Quench: Application to QCD, (with Krishna Rajagopal), *Nucl. Phys.* **B404** (1993) 577. hep-ph/9303281 [93/16]

QCD: High Density

This is a cluster of very recent papers wherein the methods of superconductivity theory are applied to QCD at high density. Item 261 is foundational. In it, we showed that for three degenerate flavors there is a beautiful preferred pattern of pairing, that opens a gap in all colored channels, modifies the photon in such a way that all elementary excitations have integer charge, and breaks chiral symmetry by a new mechanism. All this happens at weak coupling, in a tractable, controlled theoretical framework. In Item 271 we made the case that there need be no phase transition between this “color-flavor locked” phase and 3-flavor nuclear matter.

248. QCD at Finite Baryon Density: Nucleon Droplets and Color Superconductivity (with M. Alford and K. Rajagopal), *Phys. Lett.* **B 422**, 247-256, (1998). hep-ph/9711395 [97/119]
261. Color-Flavor Locking and Chiral Symmetry Breaking in High Density QCD, (with M. Alford and K. Rajagopal), *Nucl. Phys.* **B537**, 443-458, (1999). hep-ph/9804403 [98-29]
266. Imaginary Chemical Potential and Finite Fermion Density on the Lattice (with M. Alford and A. Kapustin), *Phys. Rev.* **D59**, 054502, (1999). hep-lat/9807039 [98-67]
270. High Density Quark Matter and the Renormalization Group in QCD with two and three flavors (with T. Schäfer), *Phys. Lett.* **B450**, 325-331, (1999). hep-ph/9810509 [98-90]
271. Continuity of Quark and Hadron Matter, (with Thomas Schäfer), accepted in *Phys. Rev. Lett.* hep-ph/9811473 [98-100]

276. Quark Description of Hadronic Phases (with T. Schäfer), submitted to *Phys. Rev. D*. hep-ph/9903503. [99-32]

Heavy Quarks

These papers were all inspired, in one way or another, by the experimental discovery of the J/ψ family of particles, which were beautifully adapted to serve as a demonstration system for the newly emerging Standard Model. Items 36 and 37 contain especially original uses of QCD. They have been both developed and “rediscovered” in subsequent literature.

11. Remarks on the New Resonances at 3.1GeV and 3.7GeV (with C.G. Callan, R.L. Kingsley, S.B. Treiman and A. Zee), *Phys. Rev. Lett.* **34**, 52 (1975).
12. Weak Decays of Charmed Hadrons (with R.L. Kingsley, S.B. Treiman and A. Zee), *Phys. Rev.* **D11**, 1919 (1975).
13. Weak Decays of Charmed Hadrons, II: Soft Meson Theorems (with R.L. Kingsley, S. Treiman and A. Zee), *Phys. Rev.* **D12**, 106 (1975).
32. Instantons and Spin Forces Between Massive Quarks (with A. Zee), *Phys. Rev. Lett.* **40**, 83 (1977).
36. Low Energy Manifestations of Heavy Particles: Application to Neutral Current (with J. Collins and A. Zee), *Phys. Rev.* **D18**, 242 (1978).
37. Effect of Instantons on the Heavy Quark Potential (with C. Callan, R. Dashen, D. Gross and A. Zee). *Phys. Rev.* **D18**, 4686 (1978).
43. $SU(3)$ Predictions for Charmed Meson Decays (with S. Treiman), *Phys. Rev. Lett.* **43**, 816 (1979).
44. Interference Effects in Charmed Meson Decays (with S. Treiman), *Phys. Rev. Lett.* **43**, 1059 (1979).

Standard Model Phenomenology

Item 25 contains, I believe, the first discussion of kinetic neutrino mixing, and shows how it can arise in simple gauge theory models. Item 30 introduced the Higgs-gluon-gluon coupling, which plays a central role in the anticipated phenomenology of that particle.

25. Rare Muon Decays, Heavy Leptons and CP Violations (with S. Treiman and A. Zee), *Phys. Rev.* **D16**, 152 (1977).
29. Sum Rules for Spin-Dependent Electroproduction - Test of Relativistic Constituent Quarks (with S. Wandzura), *Phys. Lett.* **B72**, 195 (1977).
30. Decay of Heavy Vector Mesons into Higgs Particles, *Phys. Rev. Lett.* **39**, 1304 (1977).

Axions

In Item 33 I introduced and named the axion. It also contains a discussion of the theoretical background and alternatives that holds up well today. Item 78 demonstrates the non-thermal production mechanism for axions during the Big Bang. This mechanism established axions as a plausible dark matter candidate. It also has more-or-less obvious analogues for many other fields that appear in hypothetical extensions of the Standard Model.

33. Problems of Strong P and T Invariance in the Presence of Instantons, *Phys. Rev. Lett.* **40**, 279 (1977).
78. Cosmology of Invisible Axions (with J. Preskill and M. Wise), *Phys. Lett.* **B120**, 127 (1983).
79. Axions and Family Symmetry Breaking, *Phys. Rev. Lett.* **49**, 1549 (1982).
88. Formation of Structure in an Axion-Dominated Universe (with M. Turner and A. Zee), *Phys. Lett.* **B125**, 35, 519(E) (1983).
93. The $U(1)$ Problem: Instanton, Axions, and Familons, in *How Far Are We from the Gauge Forces*, ed. A. Zichichi (Plenum, 1985).
110. Calculations for Cosmic Axion Detection (with L. Krauss, J. Moody and D. Morris), *Phys. Rev. Lett.* **55**, 1797 (1985).

Supersymmetry

Each of the first three papers in this cluster presents an idea that plays a continuing major part in assessing the possible role of supersymmetry in the description of Nature. In Item 56 we discussed how allowing for low-energy supersymmetry changes the analysis of coupling constant unification, raising the scale without grossly modifying the structure. In Item 58 we showed how supersymmetric models based on the group $SO(10)$ (or, of course, a larger group) contain a very natural mechanism for lifting the troublesome Higgs color triplet to very high mass, leaving the doublet behind. This is now known as the “Dimopoulos-Wilczek mechanism”. Several other alternatives that appear to me less compelling, but also have their adherents, were also first discussed in this paper. In Item 66 we introduced a discrete matter symmetry sufficient to banish catastrophic proton decay in supersymmetric models. It still appears as if this symmetry, or some closely related variant, is also necessary. Also in this paper we provided the foundational analysis of branching modes. Items 253 and 269 are recent contributions to this subject, provoked by the SuperK observation of neutrino mass, which greatly encourages this circle of ideas. We point out additional contributions to proton decay amplitudes which arise more-or-less directly from the interactions that generate neutrino mass, and try our best to give serious quantitative estimates of the expected rates. Finally Item 172 is an attempt to portray the big picture, for a wide physics audience.

56. Supersymmetry and the Scale of Unification (with S. Dimopoulos and S. Raby), *Phys. Rev.* **D24**, 1681 (1981).
58. Supersymmetric Unified Models (with S. Dimopoulos), in *The Unity of the Fundamental Interactions*, ed. A. Zichichi (Plenum, New York, 1983).
66. Proton Decay in Supersymmetry Theories (with S. Dimopoulos and S. Raby), *Phys. Lett.* **B112**, 133 (1982).
172. Unification of Couplings, (with S. Dimopoulos and S. Raby), *Physics Today* **44**, October 1991, p.25. [91/63]
253. Suggested New Modes in Supersymmetric Proton Decay, (with K.S. Babu and J. Pati), *Phys. Lett.* **B 423**, 337-347, (1998). hep-ph/9712307 [97/136]
269. Fermion masses, neutrino oscillations, and proton decay in the light of SuperKamiokande (with K.S. Babu and J. Pati), hep-ph/9812538 [98-80].

Other Beyond the Standard Model Phenomenology

Item 45 was an early model for how to use effective field theory ideas to analyze possible deviations from the Standard Model in a systematic way. Item 55 was an early use of higher-dimensional spinor representations, when this was considered quite adventurous. The use of such spinors remains, in my opinion, a promising possible approach to the family problem. Item 92 was a pioneering discussion of the possibility of deviations from Newton's law at short distances, both from an experimental and from a theoretical point of view. Especially original is the discussion of monopole-dipole forces, which I believe remains an important subject for experimental investigation. Items 106 and 109 present ideas for neutrino and dark matter detection, that have influenced experimental work.

- 45. Operator Analysis of Nucleon Decay (with A. Zee), *Phys. Rev. Lett.* **43**, 1571 (1979).
- 49. Symmetry Relations in Nucleon Decay (with Anya Hurlbert), *Phys. Lett.* **B92**, 95 (1980).
- 55. Families from Spinors (with A. Zee), *Phys. Rev.* **D25**, 553 (1982).
- 92. New Macroscopic Forces? (with J.E. Moody), *Phys. Rev.* **D30**, 130 (1984).
- 106. Bolometric Detection of Neutrinos (with B. Cabrera and L. Krauss), *Phys. Rev. Lett.* **55**, 25 (1985).
- 107. Solar Neutrino Oscillations (with L. Krauss), *Phys. Rev. Lett.* **55**, 122 (1985).
- 109. Solar System Constraints and Signatures for Dark Matter Candidates (with L. Krauss and M. Srednicki), *Phys. Rev.* **D33**, 2079 (1986).

Baryogenesis

These papers present pioneering discussions of how the phenomenon of baryon number violation, natural within unified gauge models, can lead an initially symmetric Universe to evolve an asymmetry between matter and antimatter, as observed. We discovered the main phenomena for ourselves, as presented in Item 39, but we were scooped by Yoshimura as we were developing them. Later we learned that they had been anticipated in a general way by Sakharov. Item 40 is a clean discussion of the minimal requirements. Item 50 was the first to address the problem that later interactions tend to restore equilibrium in a serious, quantitative fashion.

- 39. Matter-Antimatter Accounting, Thermodynamics, and Black Hole Radiation (with D. Toussaint, S. Treiman and A. Zee), *Phys. Rev.* **D19**, 1036 (1979).
- 40. Elementary Examples of Baryon Number Generation (with D. Toussaint), *Phys. Lett.* **B81**, 238 (1979).
- 50. Thermalization of Baryon Asymmetry (with S. Treiman), *Phys. Lett.* **B95**, 222 (1980).
- 52. Cosmic Asymmetry Between Matter and Antimatter, *Scientific American*, December 1980, p. 82.

Other Cosmology

The titles of most of the papers in this section give a pretty clear idea of their subject matter. Items 80-82 were talks at the famous Nuffield conference.

71. Reheating an Inflationary Universe (with A. Aubrecht, P. Steinhart and M. Turner) *Phys. Rev. Lett.* **48**, 1437 (1982).
72. Might Our Vacuum be Metastable? (with M. Turner), *Nature* **298**, 633 (1982).
74. Positivity Conditions on $SU(5)$ Potentials (with R. MacKenzie), *Phys. Rev.* **D26**, 3679 (1982).
80. Particle Physics and Cosmology: Foundation and Working Pictures, in *The Very Early Universe*, eds. Gibbons, Hawking, Siklos (Cambridge University Press, 1983).
81. Fun with Monopoles and Axions, in *The Very Early Universe*, eds. Gibbons, Hawking, Siklos (Cambridge University Press, 1983).
82. Conference Summary, in *The Very Early Universe*, eds. Gibbons, Hawking, Siklos (Cambridge University Press, 1983).
157. Positron Line Radiation as a Signature of Particle Dark Matter in the Halo (with M.S. Turner), *Phys. Rev.* **D42**, 1001 (1990).
159. Inflationary Axion Cosmology (with M.S. Turner), *Phys. Rev. Lett.* **66**, 5 (1991).
161. Relic Gravitational Waves and Extended Inflation (with M.S. Turner), *Phys. Rev. Lett.* **65**, 3080 (1990).
163. Cosmological Implications of Axinos (with K. Rajagopal and M.S. Turner), *Nucl. Phys.* **B358**, 447 (1991). [90/79]
165. Cosmology and Broken Discrete Symmetry (with S. Trivedi, J. Preskill and M.B. Wise), *Nucl. Phys.* **B363**, 207 (1991). [91/11]

Quantum Properties of Solitons

In Item 57 we generalized the “halving” of quantum numbers discovered abstractly by Jackiw and Rebbi, and in the context of polyacetylene by Su, Schrieffer, and Heeger, to a much wider class of field theories, featuring other fractions. The method used, tracing the flow of quantum numbers as one turns on some external field or control parameter, has been fruitfully applied to many other problems since. Several examples are presented in Items 97, 98, and 104. Item 102 is of some methodological interest; it identifies a limit in which heavy particles can be shown to spontaneously form semi-classical bags, substantially deforming the vacuum around themselves. Several of the other papers here are devoted to analysis of exotic dynamics around non-abelian strings. Especially interesting I think is Item 147, where the concept of (delocalized) Cheshire charge was first introduced.

57. Fractional Charge on Solitons (with J. Goldstone), *Phys. Rev. Lett.* **47**, 986 (1981).
97. Illustrations of Vacuum Polarization by Solitons (with R. MacKenzie), *Phys. Rev.* **D30**, 2194 (1984).
98. Examples of Vacuum Polarization by Solitons (with R. MacKenzie), *Phys. Rev.* **D30**, 2260 (1984).
102. Possible Form of Vacuum Deformation by Heavy Particles (with R. MacKenzie and A. Zee), *Phys. Rev. Lett.* **53**, 2203 (1983).

104. Adiabatic Methods in Field Theory, in *TASI Lectures in Elementary Particle Physics*, ed. Williams (TASI Publications, Ann Arbor, MI, 1984).
135. Aharonov-Bohm Interaction of Cosmic Strings with Matter (with M. G. Alford), *Phys. Rev. Lett.* **62**, 1071 (1989).
141. Enhanced Baryon Number Violation Around Cosmic Strings (with M. Alford and J. March-Russell), *Nucl. Phys.* **B328**, 140 (1989).
147. The Interactions and Excitations of Nonabelian Vortices (with M. Alford, K. Benson, S. Coleman and J. March-Russell), *Phys. Rev. Lett.* **64**, 1632 (1990). [89/69]
150. Zero Modes of Non-Abelian Vortices (with M. Alford K. Benson, S. Coleman, and J. March-Russell), *Nucl. Phys.* **B349**, 414 (1991).
178. Internal Frame Dragging and a Global Analogue of the Aharonov-Bohm Effect (with J. March-Russell and J. Preskill), *Phys. Rev. Lett.* **68**, 2567 (1992). hep-th/9112054 [91/92]

Statistical Transmutation, Anyons, and Chern-Simons Theory

In this cluster of papers the possibility of fractional statistics for two-dimensional particles was introduced and extensively analyzed. Some earlier work on this subject had been done by Leinaas and Myrheim, and roughly contemporary independent work on the foundations was done by Goldin, Menikoff, and Sharp. I discovered the possibility of fractional statistics for myself, and put it in the context of gauge theories, in the first three items listed below. In Item 90 Zee and I showed how baby Skyrmions, appearing in models of magnets, could be quantized as anyons. The Chern-Simons construction of statistical transmutation is implicit in that paper, made more explicit (and used!) in Item 94, and spelled out so even the blind can see it in Item 129. Item 94 also contains a calculation of the virial coefficient for the ideal anyon gas; this was perhaps the first “physical consequence” of anyon statistics to be cleanly identified. Item 131 contains a pioneering use of concepts that later became extremely popular, both in condensed matter and high energy theory, using duality transformations to identify theories with apparently quite different coupling constants. After an injection of ideas from Laughlin, the analysis of many-body problems using statistical transmutation became quite sophisticated; this is reflected in the papers Items 142, 143, 154, and the book Item 155. (See also the following group of papers.) Extremely interesting possibilities, as yet insufficiently explored, arise in theories with several Chern-Simons fields, particularly when several distinct two-dimensional planes are involved. Item 241 introduces an essentially new phenomenon in this context, providing limpid models that exhibit confinement of discrete quantum numbers.

64. Magnetic Flux, Angular Momentum, and Statistics, *Phys. Rev. Lett.* **48**, 1144 (1982).
65. Remarks on Dyons, *Phys. Rev. Lett.* **48**, 1146 (1982).
76. Quantum Mechanics of Fractional Spin Particles, *Phys. Rev. Lett.* **49**, 957 (1982).
90. Linking Numbers, Spin, and Statistics of Solitons (with A. Zee), *Phys. Rev. Lett.* **51**, 2250 (1983).
94. Statistical Mechanics of Anyons (with D. Arovas, J.R. Schrieffer and A. Zee), *Nucl. Phys.* **B251**, [FS13], 917 (1985).
129. Peculiar Spin and Statistics in 2+1 Dimensions (with R. MacKenzie), *Int. J. Mod. Phys.* **A3**, 2827 (1988).

- 131. Self-Dual Models with θ Terms (with A. Shapere), *Nucl. Phys.* **B320**, 669 (1989).
- 132. Field Corrections to Induced Statistics (with A. Goldhaber, R. MacKenzie), *Mod. Phys. Lett.* **A4**, 21 (1989).
- 142. On Anyon Superconductivity, (with Y-H. Chen, E. Witten and B. Halperin), *Int'l. Jour. Mod. Phys.* **B3**, 1001 (1989).
- 149. Space-Time Approach to Holonomy Scattering (with Y.-S. Wu), *Phys. Rev. Lett.* **65**, 13 (1990). [90/2]
- 154. States of Anyon Matter, *Int'l. Jour. of Mod. Phys.* **B5**, 1273 (1991). [90/29]
- 155. Fractional Statistics and Anyon Superconductivity, a monograph and reprint collection, World Scientific (September, 1990).
- 176. Disassembling Anyons, *Phys. Rev. Lett.* **69**, 132 (1992). [91/70]
- 241. Cross-Confinement in Multi-Chern-Simons Theories (with Lorenzo Cornalba), *Phys. Rev. Lett.* **78**, 4679, (1997). hep-th/9703131 [97/22]

Quantum Hall States

In this group of papers I applied the ideas of Chern-Simons theory and statistical transmutation more specifically to the quantum Hall effect. Item 100 started the whole subject. Items 153, 171, and 218 are applications of the general philosophy mentioned in the previous Section to the quantum Hall effect context. In Items 229 and 241 partially elucidate the new, nonabelian statistics that arises for the paired Hall (or Pfaffian) state. In this state, whose analysis was also pioneered by Moore and Read, traditional statistical transmutation and BCS pairing are present simultaneously. I am told it is the leading candidate to describe the observed incompressible state at $\nu = 5/2$.

- 100. Fractional Statistics and the Quantum Hall Effect (with D. Arovas and J.R. Schrieffer), *Phys. Rev. Lett.* **53**, 722 (1984).
- 153. Heuristic Principle for Quantized Hall States (with M. Greiter), *Mod. Phys. Lett.* **B4**, 1063 (1990). [90/35]
- 167. Paired Hall State at Half Filling (with M. Greiter and X.G. Wen), *Phys. Rev. Lett.* **66**, 3205 (1991) [91/18].
- 171. Exact solutions and the adiabatic heuristic for quantum Hall states (with M. Greiter), *Nucl. Phys.* **B370**, 577 (1992). [91/45]
- 174. Paired Hall States (with M. Greiter and X.G. Wen), *Nucl. Phys.* **B374**, 567 (1992). [91/66]
- 180. Paired Hall States in Double Layer Electron Systems (with M. Greiter and X.G. Wen), *Phys. Rev.* **B46**, 9586, (1992). [92/1],
- 197. Non-Fermi Liquid Fixed Point in 2+1 Dimensions, (with Chetan Nayak), *Nucl. Phys.* **B417**, (1994) 359. cond-mat/9312086 [93/89]
- 199. Exclusion Statistics: Low Temperature Properties, Fluctuations, Duality, Applications, (with C. Nayak), *Phys. Rev. Lett.* **73**, (1994) 2740. cond-mat/9405017 [94/25]
- 210. Quantum Hall States of High Symmetry (with C. Nayak), *Nucl. Phys.* **B 450**, 558, (1995). cond-mat/9501052 [94/109]
- 218. Spin-Singlet to Spin Polarized Phase Transition at $\nu = 2/3$: Flux-Trading in Action (with C. Nayak), *Nucl. Phys.* **B 455**, 493, (1995). cond-mat/9507016 [95/59]

- 222. Quantum Numbers of Textured Hall Effect Quasiparticles (with C. Nayak), *Phys. Rev. Lett.* **77**, 4418, (1996). cond-mat/9512061 [95/104]
- 229. $2n$ Quasihole States Realize 2^{n-1} -Dimensional Spinor Braiding Statistics in Paired Quantum Hall States (with Chetan Nayak), *Nucl. Phys. B* **479**, 529, (1996). cond-mat/9605145 [96/52]
- 249. A Chern-Simons Effective Field Theory for the Pfaffian Quantum Hall State (with E. Fradkin, C. Nayak, and A. Tsvetlik), *Nucl. Phys. B* **516**, 704-718 (1998). cond-mat/9711087 [97-120]

Models of Condensed Matter

This somewhat disparate group of papers contains my additional work in condensed matter theory, that does not fall comfortably into the earlier categories. I am especially proud of Item 95, which anticipates the concept of domain wall fermions, which has since become quite popular in high energy theory, in a concrete condensed matter context. Item 136 describes a possible subtle new form of spin ordering, that exhibits a gap but no easily accessible order parameter, and supports exotic quasiparticle excitations. I am told it is a leading candidate to describe solid helium 3 films. Items 225 and 233 are among the earliest theoretical papers on narrow stripes in cuprates (later than Zaanen, but roughly contemporary with Emery and Kivelson), which have since become a very hot topic.

- 95. Solitons in Superfluid $^3\text{He-A}$: Bound States on Domain Walls (with J.L. Ho, J.R. Fulco and J.R. Schrieffer), *Phys. Rev. Lett.* **52**, 1524 (1984).
- 136. Chiral Spin States and Superconductivity (with X.G. Wen, A. Zee), *Phys. Rev.* **B39**, 11413 (1989).
- 143. Hydrodynamic Relations in Superconductivity, (with M. Greiter and E. Witten), *Mod. Phys. Letts.* **B3**, 903 (1989).
- 209. Space-Time Aspects of Quasiparticle Propagation (with R. Levien and C. Nayak) *Int. J. of Mod. Phys.* **B 9**, 3189, (1995). cond-mat/9501050 [94/108]
- 225. Possible Electronic Structure of Domain Walls in Mott Insulators (with C. Nayak), *Int. J. Mod. Phys.* **B 10**, 2125, (1996). cond-mat/9602112 [95/111]
- 233. Populated Domain Walls (with C. Nayak), *Phys. Rev. Lett.*, **78** , 2465, (1997). cond-mat/9609094 [96/93]

Mathematics of Gauge Theory

On a few occasions I have published exploratory mathematical ideas that seemed to me especially pretty or naturally connected to physics, without any immediate sense of how they might be applied. Item 26 presented an ansatz for instanton solutions, which in a generalized form became quite influential. Item 137 is foundational. Item 254 is a heterodox but I think extremely intriguing approach to construction of gravity as a gauge theory. One aspect of this, which requires less radical departures than the full program, is the connection between the cosmological term and the question whether one should require invariance under volume-changing transformations.

18. Non-Uniqueness of Gauge Field Potentials (with S. Deser), *Phys. Lett.* **65B**, 391 (1976).
19. Inequivalent Embeddings of $SU(2)$ and Instanton Interactions, *Phys. Lett.* **65B**, 160 (1976).
26. Geometry and Interaction of Instantons, *Quark Confinement and Field Theory: Proceedings of a Conference at the University of Rochester, Rochester, NY, June 14-18, 1976*, Stump and Weingarten, eds. (Wiley-Interscience, NY, 1977), pp. 211-219.
137. Discrete Gauge Symmetry in Continuum Theories (with L.M. Krauss), *Phys. Rev. Lett.* **62**, 1221 (1989).
151. Infrared Behavior at Negative Curvature (with C. Callan), *Nucl. Phys.* **B340**, 366 (1990). [90/4]
244. Mass Splittings from Symmetry Obstruction (with L. Cornalba), *Phys. Lett.* **B 411**, 112-116, (1997). hep-th/9706014 [97-48]
245. Some Examples in the Realization of Symmetry, *Nucl. Phys.* **B 68** (Proc. Suppl.), 367, (1998). hep-th/9710135 [97/116]
254. Riemann-Einstein Structure from Volume and Gauge Symmetry, *Phys. Rev. Lett.*, **80**, 4851, (1998). hep-th/9801184 [97-142]

Geometric Phase and Applications

In Item 99, Zee and I showed how one could extend Berry's celebrated phase to a non-abelian setting. Item 112 presents a simple, natural example. Items 120, 123, 124, and 128 demonstrate the use of gauge theory concepts to describe two problems in the mechanics of deformable bodies: self-orientation (say of divers or falling cats) at zero angular momentum, and self-propulsion (say of bacteria) at low Reynolds number. Item 130 has become a standard reference on the subject.

99. Appearance of Gauge Structures in Simple Dynamical Systems (with A. Zee), *Phys. Rev. Lett.* **52**, 2111 (1984).
112. Simple Realizations of Magnetic Monopole Gauge Fields: Diatoms and Spin Precession (with J. Moody and A. Shapere), *Phys. Rev. Lett.* **56**, 893 (1986).
120. Geometry of Self Propulsion at Low Reynolds Number (with A. Shapere), *Jour. Fluid Mech.* **198**, 557 (1989).
122. Two Applications of Axion Electrodynamics, *Phys. Rev. Lett.* **58**, 1799 (1987).
124. Self-Propulsion at Low Reynolds Number (with A. Shapere), *Phys. Rev. Lett.* **58**, 2051 (1987).
128. Gauge Kinematics of Deformable Bodies (with A. Shapere), *Am. J. Phys.* **57**, 514 (1989).
130. Geometric Phases in Physics (a text and reprint volume, edited with A. Shapere) (World Scientific, 1989).

Black Holes

This is a cluster of papers concerned with the quantum theory of black holes. Items 146, 166, and 168 demonstrate the existence of “quantum hair” – observables of stationary black holes that are invisible classically, and violate the classical no hair theorems. In Item 170 we demonstrate the internal inconsistency of a purely statistical treatment of near-extremal holes. In Item 177 Holzhey and I discussed a class of dilaton black holes that behave much like elementary particles; it later developed, that in certain cases these solutions in fact do correspond to elementary objects in string theory. In Items 203 and 205 we show, by tough concrete calculations, how the worst aspects of the inconsistency can be relieved by enforcing full energy conservation (including gravitational self-energy). A line element that is regular through the horizon and particular well adapted to such calculations, and has other features of interest, was introduced in Item 202 (the mathematics, though not the quantum physics, of this line element has a prehistory).

- 146. Discrete Quantum Hair on Black Holes and the Nonabelian Aharonov-Bohm Effect, (with M. Alford and J. March-Russell), *Nucl. Phys.* **B337**, 695 (1990).
- 166. Dynamical Effect of Quantum Hair (with S. Coleman and J. Preskill), *Int’l. Jour. Mod. Phys. Lett.* **A6**, 1631 (1991). [91/17]
- 168. Growing Hair on Black Holes (with J. Preskill and S. Coleman), *Phys. Rev. Lett.* **67**, 1975 (1991). [91/32]
- 170. Limitations on the Statistical Description of Black Holes (with P. Schwarz, A. Shapere and S. Trivedi), *Mod. Phys. Lett.* **A6**, 2353 (1991). [91/34]
- 173. Quantum Hair on Black Holes (with S. Coleman and J. Preskill), *Nucl. Phys.* **B378**, 175 (1992). hep-th/9201059 [91/64]
- 177. Black Holes as Elementary Particles (with C.F.E. Holzhey), *Nucl. Phys.* **B380**, 447 (1992). hep-th/9202014 [91/71]
- 198. On Geometric Entropy, (with Curtis Callan), *Phys. Lett.* **B333**, (1994) 55-61. hep-th/9401072 [93/87]
- 202. Some Applications of a Simple Stationary Line Element for the Schwarzschild Geometry, (with P. Kraus), *Mod. Phys. Lett.* **A 9**, 3713, (1994). gr-qc/9406042 [94/46]
- 203. Self-Interaction Correction to Black Hole Radiance, (with P. Kraus), *Nucl. Phys.* **B 433**, (1995) 403. gr-qc/9408003 [94/61]
- 205. Effect of Self-Interaction on Charged Black Hole Radiance (P. Kraus), *Nucl. Phys.* **B437** (1995) 231. hep-th/9411219 [94/101]
- 265. Global Structure of Evaporating Black Holes (with M. Parikh), *Phys. Lett.* **B449**, 24-29, (1999). gr-qc/9807031 [98-57]

String Theory

Item 119 is one of the earliest papers on T-duality. In Item 169 we studied black holes in supergravity, and demonstrated their dual spectrum. In Item 220 we pointed out the non-zero horizon area of extremal black holes with several charges, showed that this area is independent of various moduli (consistent with its interpretation as an entropy) and argued from duality that something very close to the Bekenstein-Hawking formula must be valid. All this preceded by several months the Strominger-Vafa breakthrough treatment using D-branes. In Item 236 Larsen and I made a first attempt to apply the improved understanding

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of string black holes to cosmology. Although the model presented there is far from complete or satisfactory, I'm convinced cosmology is the most promising direction for string theory, and am determined to pursue it further.

- 119. Compactification of the Twisted Heterotic String (with V. Nair, A. Shapere and A. Strominger), *Nucl. Phys.* **B287**, 402 (1987).
- 169. Dual Dilaton Dyons (with A. Shapere and S. Trivedi), *Mod. Phys. Lett.* **A6**, 2677 (1991). [91/33]
- 220. Internal Structure of Black Holes (with F. Larsen), *Phys. Lett* **B 375**, 37, 1996. hep-th/9511064 [95/92]
- 236. Resolution of Cosmological Singularities (with Finn Larsen), *Phys. Rev.* **D 55**, 4591, (1997). hep-th/9610252 [96-108]

Popular and Semi-Popular Pieces

I have long been interested in communicating results from the frontiers of physics to the scientific (and, ideally, the general) public. In recent years I have been asked to do this on a regular basis for both Nature and Physics Today. As a matter of principle I have tried in these pieces to take on subjects that are under-reported, other things being equal.

- 115. *Longing for the Harmonies*, [a book] W.W. Norton (January, 1988).
- 116. "Virtual Particles" [a sonnet] Norton Anthology of Light Verse, ed. Baker, 1986.
- 164. Anyons, *Scientific American* **264**, #5, p. 58 (May, 1991).
- 193. 10^{12} Degrees in the Shade (preprinted as "Hot Stuff: The High Temperature Frontier"), *The Sciences*, January/February 1994, 22. [93/50]
- 226. A crack in the Standard Model?, *Nature* **Vol. 380**, 19-20, (7 March 1996).
- 246. Panning for Gold at the K Stream, *Nature* **389**, 671 (16 Oct. 1997).
- 250. Colour Takes The Field, *Nature* **390**, 659 (18/25 Dec. 1998).
- 251. Neutrino Deficit Challenges Conservation Laws, *Nature* **391**, 123 (8 Jan. 1998).
- 252. Why are there Analogies between Condensed Matter and Particle Theory?, *Physics Today*, 11 (Jan. 1998).
- 255. Liberating Quarks and Gluons, *Nature*, **391**, 330-331, (22 Jan. 1998).
- 256. Back to Basics at High Temperature, *Physics Today*, 11 (April 1998).
- 264. Particle Physics: The Standard Model Transcended, *Nature* **394**, 13-15, (2 July 1998).
- 267. Nuclear and Subnuclear Boiling, *Nature* **395**, 220-221 (17 September 1998).
- 272. The Persistence of Ether, *Physics Today*, **52**, 11-13, (January 1999).
- 277. Cosmic Molasses for Particle Masses, *New Scientist*, (April 1999).

Miscellaneous

These are three papers that don't fit naturally anywhere else. Item 89 arose out of discussions of the annihilation of magnetic monopoles and antimonopoles in the early Universe, but quickly took on a different flavor. We discussed how a proper treatment of spatial fluctuations can invalidate the usual treatment of chemical kinetics, which implicitly assumes homogeneity. Later authors including Cardy developed a very pretty renormalization group analysis of the phenomena we found here, and generalizations. Item 214 is the only observable tip of a much larger iceberg representing my interest in quantum computing. In itself it

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is a clever paper, I think, but it was rapidly overtaken by the explosive development of the field. Finally in Item 278 we discuss how some extremely simple, symmetric potentials can be shown both analytically and numerically to develop an exponential set of inequivalent local minima.

- 89. Particle-Antiparticle Annihilation in Diffuse Motion (with D. Toussaint), *Jour. Chem. Phys.* **78**, 2642 (1983).
- 214. Realization of the Fredkin Gate Using A Series of One- And Two-body Operators, (with H.F. Chau), *Phys. Rev. Lett* **75** 748 (1995). quant-ph/9503005 [95/15]
- 278. Minimal Potentials with Very Many Minima (with Marin Soljačić). cond-mat/9904190 [99-39]

Special

These are four papers I prepared for special occasions. I put a lot of thought into them, and I am pleased with how they came out. Implicitly, and in part explicitly, they present my vision of physics. Item 238 was prepared as part of the celebration of the 250th anniversary of Princeton University. Item 259 was prepared as part of the American Physical Society Centennial volume. Item 274 was prepared as a special News and Views Feature for Nature. Finally, Item 279 was prepared for a special supplement issue of Nature, also designed to celebrate the century in physics. It is brief, but of very broad scope.

- 238. The Future of Particle Physics as a Natural Science, Published in “Critical Problems in Physics” in celebration of the 250th Anniversary of Princeton University, November 1996, eds. Fitch, Marlow, and Dementi, Princeton University Press; also in *Int. Jour. Mod. Phys. A* **13**, 863, (1998); also in *Magazine of Physics, Science & Ideas* Vol. 1 No. 2) 12-25, (Dec. 1996). hep-ph/9702371 [97-11]
- 259. Quantum Field Theory, in the American Physical Society Centenary issue of *Rev. Mod. Phys.* **71**, S85-S95, (1999); More Things in Heaven and Earth— A celebration of Physics at the Millennium, ed. B. Bederson, (Springer-Verlag, New York), (1999). hep-th/9803075 [98-20]
- 274. Getting Its from Bits *Nature* **397**, 303-306 (28 Jan. 1999).
- 279. Reaching Bottom, Laying Foundations, *Nature*, special issue “A Celebration of Physics,” for American Physical Society 100th anniversary, 4-5, (April 1999).

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