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Continuity of Quark and Hadron Matter

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Abstract

We review, clarify, and extend the notion of color-flavor locking. We present evidence that for three degenerate flavors the qualitative features of the color-flavor locked state, reliably predicted for high density, match the expected features of hadronic matter at low density. This provides, in particular, a controlled, weak-coupling realization of confinement and chiral symmetry breaking in this (slight) idealization of QCD.

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In a recent study [1] of QCD with three degenerate flavors at high density, a new form of ordering was predicted, wherein the color and flavor degrees of freedom become rigidly correlated in the groundstate: color-flavor locking. This prediction is based on a weak coupling analysis using a four-fermion interaction with quantum numbers abstracted from one gluon exchange. One expects that such a weak coupling analysis is appropriate at high density, for the following reason [2, 3, 4]. Tentatively assuming that the quarks start out in a state close their free quark state, *i.e.* with large Fermi surfaces, one finds that the relevant interactions, which are scatterings the states near the Fermi surface, for the most part involve large momentum transfers. Thus, by asymptotic freedom, the effective coupling governing them is small, and the starting assumption is confirmed.

Of course, as one learns from the theory of superconductivity [5], even weak couplings near the Fermi surface can have dramatic qualitative effects, fundamentally because there are many low-energy states, and therefore one is inevitably doing highly degenerate perturbation theory. Indeed, the authors of [1] already pointed out that their color-flavor locked state, which is constructed by adapting the methods of superconductivity theory to the problem of high-density quark matter, displays a gap in all channels except for those associated with derivatively coupled spin zero excitations, *i.e.* Nambu-Goldstone modes. This is confinement. For massless quarks, they also demonstrated spontaneous chiral symmetry breaking.

In very recent work we [6], and others [7], have reinforced this circle of ideas by analyzing renormalization of the effective interactions as one integrates out modes far from the Fermi surface. A fully rigorous treatment will have to deal with the extremely near-forward scatterings, which are singular due to the absence of magnetic mass for the gluons, at least in straightforward perturbation theory. This problem, which is presumably technical, is any case ameliorated self-consistently for states of the color-flavor locking type, wherein all the gluons acquire mass through the Anderson-Higgs mechanism.

In the earlier work [1], several striking analogies between the calculated properties of the color-flavor locked state and the expected properties of hadronic matter at low or zero density, based on standard lore and observed phenomenology, were noted. In addition to confinement and chiral symmetry breaking, the authors observed that the dressed elementary excitations in the color-flavor locked state have the spin quantum numbers of low-lying hadron states and for the most part carry the expected flavor quantum numbers, including integral electric charge (in units of the electron charge). Thus, as we shall spell out immediately below, the gluons match the octet of vector mesons, the quark octet matches the baryon octet, and an octet of collective modes associated with chiral symmetry breaking matches the pseudoscalar octet. However there are also a few apparent discrepancies: there is an extra massless singlet scalar, associated with the spontaneous breaking of baryon number (superfluidity); there are eight rather than nine vector mesons (no singlet); and there are nine rather than eight baryons (extra singlet). We will argue that these "discrepancies" are superficial – or rather that they are features, not bugs.

Let us first briefly recall the fundamental concepts of color-flavor locking. The case of three massless flavors is the richest due to its chiral symmetry (and adding a common mass does not change anything essential) so we shall concentrate on it. The primary condensate, which one calculates using the methods of superconductivity theory near the Fermi surface, involves diquarks. It takes the form [1]

$$\langle q^{i\alpha}_{La} q^{j\beta}_{Lb} \epsilon_{ij} \rangle = -\langle q^{\alpha}_{R\dot{k}a} q^{\beta}_{R\dot{l}b} \epsilon^{\dot{k}\dot{l}} \rangle = \kappa_1 \delta^{\alpha}_{a} \delta^{\beta}_{b} + \kappa_2 \delta^{\alpha}_{b} \delta^{\beta}_{a} \tag{1}$$

Here L, R label the helicity, i, j, k, l are two-component spinor indices, a, b are flavor indices, and α, β are color indices. A common space-time argument is suppressed. κ_1, κ_2 are parameters (depending on chemical potential, coupling, ...) whose non-zero values emerge from a dynamical calculation.

This equation must be interpreted carefully. The value of any local quantity which is not gauge invariant, taken literally, is meaningless, since local gauge invariance parametrizes the redundant variables in the theory, and cannot be broken [8]. But as we know from the usual treatment of the electroweak sector in the Standard Model, it can be very convenient to use such quantities. The point is that we are allowed to fix a gauge during intermediate stages in the calculation of meaningful, gauge invariant quantities – indeed, in the context of weak coupling perturbation theory, we must do so. For our present purposes however it is important to extract non-perturbative results, especially symmetry breaking order parameters, that we can match to our expectations for the hadronic side. To do this, we can take suitable products of the members of (1) and their complex conjugates, and contract the color indices. In this way we can produce the square of the standard chiral symmetry breaking order parameter of type $\langle \bar{q_L} q_R \rangle$ and a baryon number violating order parameter of type $\langle (qqq)^2 \rangle$, both scalars and singlets under color and flavor. At this level only the square of the usual chiral order parameter appears, fundamentally because our condensates preserve left-handed quark number modulo two. This conservation law is violated by the six-quark vertex associated with instantons, and by convolving that vertex with our four-quark condensate we can obtain the usual two-quark chiral symmetry breaking order parameter [9].

By demanding invariance of the diquark condensate directly, we infer the symmetry breaking pattern $SU(3)^c \times SU(3)^L \times SU(3)^R \times U(1) \rightarrow SU(3)^\Delta$. Here among the initial microscopic symmetries $SU(3)^c$ is local color symmetry, while the remaining factors are chiral family and baryon number symmetries. The final residual unbroken symmetry is a global diagonal symmetry. Indeed, the Kronecker deltas in the final term of (1) are invariant only under simultaneous color and flavor rotations, so the color and flavor are "locked". This locking occurs separately for the left and right handed quarks, but since color symmetry itself is vectorial, the effect is also to lock left and right handed flavor rotations, breaking chiral symmetry. The global baryon number symmetry is, of course, manifestly broken, but quark number is conserved modulo two. Projecting onto the gauge invariant, color singlet, sector this implies that baryon number is violated only modulo two. The same conclusions would emerge from analysis of the gauge invariant symmetry generators only, upon consideration of the gauge invariant order parameters we constructed above.

Ordinary electromagnetic gauge invariance, like color symmetry, is violated by (1), but a linear combination of hypercharge (diagonal matrix -2/3, 1/3, 1/3) and electromagnetic charge (diagonal matrix 2/3, -1/3, -1/3) annihilates the combinations correlated by color-flavor locking, and generates a true symmetry. The physical result is that there is a massless gauge degree of freedom, representing the photon as modified by its interaction with the condensate. As seen by this modified photon, all the elementary excitations have appropriate charges to match the corresponding hadronic degrees of freedom. In particular, their charges are all integral multiples of the electron charge [1]. This is, of course, another classic aspect of confinement.

It was essential, in this construction, that the charges of the quarks add up to zero. If that were not so, we would not have been able to find a color generator capable of compensating the violation of naive electromagnetic gauge invariance. Yet it seems somewhat accidental that these charges do add up to zero, and one would be quite worried if any qualitative aspect of confinement depended on this accident. This worry touches the form rather than the substance of our argument. If the quark charges did not up to zero, it would not be valid to ignore Coulomb repulsion. One would have to add a compensating charge background as a mathematical device, or contemplate inhomogeneous states. Insofar as we want to use external gauge fields as a probe of pure QCD, we must restrict ourselves to those which preserve the overall neutrality of the QCD groundstate. Fortunately, in our slightly idealized version of QCD no awkwardness arises for the physically important gauge field, *i.e.* the physical photon.

The elementary excitations are of three types. The color gluons become massive vector mesons through the Anderson-Higgs mechanism. Due to color-flavor locking, they acquire flavor quantum numbers, which makes them an octet under the residual $SU(3)^{\Delta}$. The quark fields give single-particle spin 1/2 excitations whose stability is guaranteed by the residual Z_2 quark (or baryon) number symmetry. These excitations are massive, due to the color-flavor superconducting gap. They form an octet with the quantum numbers of the nucleon octet, plus a singlet. It might seem peculiar on first hearing that a single quark can behave as a baryon, but remember that there is a condensate of diquarks pervading this phase. In addition there are collective Nambu-Goldstone modes, associated with the spontaneously broken global symmetries. These are a massless pseudoscalar octet associated with chiral symmetry breaking, and a scalar singlet associated with baryon number violation. A common quark mass lifts the pseudoscalar octet, but not the singlet, because it spoils microscopic chiral symmetry but not microscopic baryon number.

Clearly, there are striking resemblances between the elementary excitations of color-flavor locked quark matter and the low-energy hadron spectrum. One is tempted to ask whether they might be identified. More precisely, one might ask whether strongly coupled hadronic matter at low density goes over into the calculable, weak-coupling form of quark matter just described without a phase transition. If so, then the confinement and chiral symmetry breaking calculated for the weak coupling phase not only resemble these central properties of low-density QCD, but are rigorously indistinguishable from them. This sort of possibility, that Higgs and confined phases are rigorously indistinguishable, has long been known to occur in simple abstract models [10].

As mentioned above, however, at first sight there appear to be several difficulties with this identification. We now debunk them in turn.

The most profound of the apparent difficulties is the existence of an extra scalar Nambu-Goldstone mode, and the related phenomenon that baryon number is spontaneously violated (indicating, as in liquid He^4, superfluidity). The answer to this comes through proper recognition of an important though somewhat exotic phenomenon for three degenerate flavors on the hadron side. Several years ago R. Jaffe discovered [11], in the context of the MIT bag model, that a particular dibaryon state, the H, a spin 0 SU(3) singlet with quark content (udsuds), is surprisingly light. This arose, in his calculations, because of a particularly favorable contribution from color magnetism. Roughly speaking, in the H configuration the color fields associated with the quark sources are minimized, together with the energy they would otherwise store, by arranging both the colors and spins to cancel pairwise to the greatest extent possible. It has been debated, for QCD with realistic quark masses, whether H might be only slightly above the nn or nΛ thresholds. Though at this level the outcome for realistic QCD is unclear, both theoretically [12] and experimentally [13], it has come to seem quite likely that in QCD with three degenerate quarks the H will be the particle with smallest energy per unit baryon number. Thus at any finite baryon number density, however small, at zero temperature one should expect, in this context, to find a Bose condensate of H dibaryons. This condensate gives us precisely – *i.e.* with the appropriate quantum numbers – the superfluid we were led to expect from our superficially very different considerations on the quark matter side.

If the H is above dibaryon threshold, one will have a narrow range of chemical potentials where baryon number is built up by single baryons. Based on the same calculations [11, 12], it is extremely plausible that in this case there will be attraction in the H channel at the Fermi surface, and hence superfluidity of the required type, now through a BCS mechanism.

This superfluidity, whatever its source, supplies us with the key to the riddle of the missing vector meson. For once there is a massless singlet scalar, the putative singlet vector becomes radically unstable, and should not appear in the effective theory. It might be objected here that the octet of vector mesons is also unstable – for massless quarks – against decay into massless scalar and pseudoscalar mesons. A quick answer is that this is not really an objection at all, because there is no harm in having redundant states (whose instability will appear immediately upon more accurate calculation). There is a much prettier and more satisfying answer, however. If we turn on non-zero masses for the quarks the pseudoscalar octet (but not the singlet) will become massive. Eventually the decay of the vector octet (but not the singlet) will be blocked, and then we will be grateful for the prescience of the theory in providing the appropriate degrees of freedom.

Finally, there is the question of the "extra" singlet baryon. This is the most straightforward. In the original calculations [1], it was found that the singlet gap is much larger than the octet gap. Thus the singlet baryon is predicted to be considerably heavier than the octet. This is not problematic: a particle of this sort is expected in the quark model, it could well exist in reality, and in any case it is radically unstable against decay into octet baryon and octet pseudoscalar, at least for massless or light quarks.

So all the objections have been answered. Continuity of quark and hadron matter, far from being paradoxical, now appears as the default option.

Clearly, superfluidity of quark/hadron matter has been essential for the argument. There is considerable evidence for pairing in nuclei [14]. Its full realization is limited by the finite size of nuclei, which in turn arises from the non-negligible strange quark mass and the Coulomb energy that arises in the most favorable (for QCD), symmetric arrangement of neutrons and protons. These limitations might be relieved to some extent in heavy ion collisions accompanied by creation of many strange-antistrange pairs, followed by charge segregation. An important signature for this, emphasized by the considerations above, is broadening of vector mesons, especially the singlet. This effect might be observable in the dimuon spectrum.

Our considerations here are clearly relevant to any attempt to model the deep interior of neutron stars, or conditions during supernova and hypernova explosions. To do justice to these questions, it will be very important to include the effects of unequal quark masses and of electromagnetism. That is an important task for the future.

In the remainder of this paper we shall consider a related but simpler problem, that of extending the analysis to larger numbers of degenerate quarks. An important foundational result, which emerges clearly from this analysis, is that the color-flavor locked state for three flavors, which was first guessed to be favorable because of its large residual symmetry and by analogy to the B phase of superfluid He^3, is in fact the global minimum for three flavors. It also reappears as a building block for larger numbers of flavors.

The renormalization group analysis in [6, 7] allows one to classify possible instabilities, and to assess their relative importance, for small but otherwise arbitrary couplings near the Fermi surface. It was found that the dominant instability corresponds to scalar diquark condensation. The analysis does not fix the color and flavor channel of this instability uniquely, independent of initial conditions for the couplings, since there are two equally enhanced marginal interactions. One gluon exchange, which dominates for weak coupling, is attractive in the color anti-symmetric $\bar{3}$ channel, and favors one of these interactions. During the evolution this interaction will grow, while the repulsive interaction in the color symmetric 6 channel is suppressed. Thus the instability is driven by a leading interaction of the form

$$\begin{aligned}\mathcal{L} &= K\left(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma}\right)\left(\delta^{ac}\delta^{bd}-\delta^{ad}\delta^{bc}\right)\\ &\qquad\left\{\left(\psi^{\alpha}_{a}C\gamma_5\psi^{\beta}_{b}\right)\left(\bar\psi^{\gamma}_{c}C\gamma_5\bar\psi^{\delta}_{d}\right)-\left(C\gamma_5\leftrightarrow C\right)\right\},\end{aligned} \qquad (2)$$

where as before $\alpha,\beta,\ldots$ are color indices and $a,b,\ldots$ are flavor indices. The Dirac structure of the interaction becomes more transparent when written in a chiral basis. We have

$$\epsilon_{ij}\epsilon_{kl}\psi^{i}_{L}\psi^{j}_{L}\bar\psi^{k}_{L}\bar\psi^{l}_{L}+(L\leftrightarrow R). \qquad (3)$$

The renormalization group analysis only provides the form of the dominant interaction, not the structure of the order parameter. In particular, it does not tell us whether color-flavor locking is the preferred state in three flavor QCD. In order to answer this question, we have to perform a variational analysis. Since the interaction is attractive in s-wave states, it seems clear that the dominant order parameter is an s-wave, too. We then only have to study the color-flavor structure of the primary condensate. For this purpose, we calculate the effective potential for the order parameter

$$\langle\psi^{i\alpha}_{L\,a}\psi^{j\beta}_{L\,b}\rangle=\epsilon^{ij}\Delta^{\alpha\beta}_{ab}. \qquad (4)$$

$\Delta^{\alpha\beta}_{ab}$ is a $N_f\times N_f$ matrix in flavor space and a $N_c\times N_c$ matrix in color space. Overall symmetry requires that $\Delta^{\alpha\beta}_{ab}$ is symmetric under the combined exchange $(a\alpha)\leftrightarrow(b\beta)$. Also, since the interaction only involves color and flavor anti-symmetric terms, the effective potential does not depend on color and flavor symmetric components of $\Delta^{\alpha\beta}_{ab}$. This means that the effective potential has at least $N_c(N_c+1)N_f(N_f+1)/4$ flat directions. These trivial flat directions will be lifted by subleading interactions not included in our analysis. We will comment on the importance of subleading terms below.

We calculate the effective potential in the mean field approximation. This approximation corresponds to resumming all "cactus" diagrams. These diagrams are expected to be dominant both in the limit of large chemical potential and in the large N_c, N_f limit. In the mean field approximation, the quadratic part of the action becomes

$$\mathcal{M}_{ab}^{\alpha\beta}\psi^a_{L\,\alpha}\psi^b_{L\,\beta} = K\left(\Delta^{\alpha\beta}_{ab} - \Delta^{\beta\alpha}_{ab} - \Delta^{\alpha\beta}_{ba} + \Delta^{\beta\alpha}_{ba}\right)\psi^a_{L\,\alpha}\psi^b_{L\,\beta}. \tag{5}$$

Integrating over the fermion fields we obtain the familiar tr log term in the effective potential. In order to evaluate the logarithm, we have to diagonalize the mass matrix $\mathcal{M}$. Let us denote the corresponding eigenvalues by δ_ρ ($\rho = 1,\ldots,N_cN_f$). These are the physical gaps for the N_fN_c fermion species. Adding the mean field part of the effective potential, we finally obtain

$$V_{eff}(\Delta) = -\sum_\rho \epsilon(\delta_\rho) + \mathcal{M}^{ab}_{\alpha\beta}\Delta^{\alpha\beta}_{ab}. \tag{6}$$

Here, $\epsilon(\delta)$ is the kinetic term in the effective action for one fermion species,

$$\epsilon(\delta) = \int \frac{d^3p}{(2\pi)^3}\left(\sqrt{(p-\mu)^2+\delta^2} + \sqrt{(p+\mu)^2+\delta^2}\right). \tag{7}$$

This integral has an ultra-violet divergence. This divergence can be removed by expressing δ in terms of the renormalized interaction [15]. In this work we are not really interested in the exact numerical value of the gap, but only in the symmetries of the order parameter. For simplicity, we therefore regularize the integral by introducing a sharp three-momentum cutoff Λ.

The effective potential (6) depends on $N_c(N_c-1)N_f(N_f-1)/4$ parameters. We minimize this function numerically. In order to make sure that the minimization routine does not become trapped in a local minimum we start the minimization from several different initial conditions. For the numerical analysis we also have to fix the value of the chemical potential μ, the coupling constant K, and the cutoff Λ. We have checked that the symmetry breaking pattern does not depend on the values of these parameters. We have used $\mu = 0.5$ GeV, $\Lambda = 0.6$ GeV and $K = 3.33/\Lambda^2$, similar to what was considered in [3, 4].

After we determine the matrix $\Delta^{\alpha\beta}_{ab}$ that minimizes the effective potential we study the corresponding symmetry breaking. Initially, there are $N_f^2 - 1$ global flavor symmetries for both left and right handed fermions, as well as $N_c^2 - 1$ local gauge symmetries. Superfluidity reduces the am-mount of symmetry. In order to find the unbroken generators we study the second variation of the order parameter $\delta^2\Delta/(\delta\theta_i\delta\theta_j)$, where θ_i $(i = 1, \ldots, N_f^2 + N_c^2 - 2)$ parameterizes the flavor and color transformations. Zero eigenvalues of this matrix correspond to unbroken color-flavor symmetries. The corresponding eigenvectors indicate whether the unbroken symmetry is a pure color, a pure flavor, or a coupled color-flavor symmetry.

Our results are summarized in Table 1. The two flavor case is special. In this case, the dominant order parameter does not break the color symmetry completely, and the flavor symmetry is completely unbroken. This is the scenario discussed in [3, 4]. Subdominant interactions can break the remaining color symmetry, either with or without [3] flavor symmetry breaking.

The main result is that, for three flavors, we verify that color-flavor locking is indeed the preferred order parameter. We find that all quark species acquire a mass gap, and both color and flavor symmetry are completely broken. There are eight combinations of color and flavor symmetries that generate unbroken global symmetries. These are the generators of the diagonal $SU(3)^{c+L+R}$. Also, the quark mass gaps fall into representations (8+1) of the unbroken symmetry. And, as mentioned above, the singlet state is twice heavier than the octet.

Note that in the present analysis, which only takes into account the leading interaction, the order parameter is completely anti-symmetric in both color and flavor. We find $\Delta^{\alpha\beta}_{ab} \sim \epsilon^{\alpha\beta I}\epsilon_{abI}$. If subleading interactions are taken into account, the order parameter will have the more general form $\Delta^{\alpha\beta}_{ab} = \kappa_1\delta^\alpha_a\delta^\beta_b + \kappa_2\delta^\alpha_b\delta^\beta_a$. This order parameter leaves the same residual symmetry.

The main qualitative results we found for three flavors extend to $N_f > 3$. Color symmetry is always completely broken, and all quarks acquire a mass gap. The only remaining symmetries are global coupled color-flavor symmetries. For massless quarks, chiral symmetry is spontaneously broken. For an odd number of flavors, there are subleading instanton operators that, after the dominant gap is formed, can give an expectation value to operators of the form $\bar\psi_L\psi_R$. For even numbers of flavors, generating a non zero $\langle\bar\psi_L\psi_R\rangle$ is more subtle. Instantons can only give an expectation value to operators

N_c	N_f	N_{par}	gaps (deg)	Δ	$-\epsilon/(N_cN_f)$	N_{sym}
3	2	3	Δ (4)	Δ_0	ϵ_0	3 (fl) + 3 (col)
3	3	9	Δ (8), 2Δ (1)	$0.80\Delta_0$	$1.27\epsilon_0$	8
3	4	18	Δ (8), 2Δ (4)	$0.63\Delta_0$	$1.21\epsilon_0$	6
3	5	30	Δ (5), 2Δ (7), 3Δ (3)	$0.43\Delta_0$	$1.18\epsilon_0$	3
3	6	45	Δ (16), 2Δ (2)	$0.80\Delta_0$	$1.27\epsilon_0$	9

Table 1: Groundstate properties of the s-wave superfluid state in QCD with $N_c = 3$ colors and N_f flavors. $N_{par} = N_c(N_c-1)N_f(N_f-1)/4$ is the number of totally anti-symmetric gap parameters. The column labeled "gaps (deg)" gives the relative magnitude of the gaps in the fermion spectrum, together with their degeneracy. The numerical values of the gap and the condensation energy per species are given in units of $\Delta_0 = 36\,\text{MeV}$ and $\epsilon_0 = 0.73\,\text{MeV/fm}^3$, respectively. N_{sym} is the number of unbroken color-flavor symmetries.

of the form $(\bar\psi_L\psi_R)^2$.

$N_f = N_c$ (or a multiple thereof) is the most favorable case, in the precise sense that in this case the condensation energy per species is maximal. If N_f is a multiple of N_c, the dominant gap corresponds to multiple embeddings of the $N_f = N_c$ order parameter. We have not studied the case $N_c \neq 3$ systematically, but since color and flavor are interchangeable in (2), the case $N_f = 3$ for various numbers of flavors is covered implicitly. Also, we have not studied the interesting case $N_f = N_c \to \infty$.

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