Fermion Masses, Neutrino Oscillations, and Proton Decay in the Light of SuperKamiokande

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Abstract

Within the framework of unified gauge models, interactions responsible for neutrino masses can also provide mechanisms for nucleon instability. We discuss their implications concretely in the light of recent results on neutrino oscillation from the SuperKamiokande collaboration. We construct a predictive $SO(10)$-based framework that describes the masses and mixing of all quarks and leptons. An overconstrained global fit is obtained, that makes five successful predictions for quarks and charged leptons. The same description provides agreement with the SuperK results on atmospheric neutrinos and supports a small-angle MSW mechanism. We find that current limits on nucleon stability put significant stress on the framework. Thus improved searches in the decay channels predicted to dominate, $p \to \overline{\nu} K^+$ and $p \to \mu^+ K^0$, will either turn up events, or force us outside this circle of ideas.

I. INTRODUCTION

Recent SuperKamiokande observations on atmospheric neutrinos [1] establish the oscillation of ν_μ to ν_τ with a mass splitting $\delta m^2 \sim (10^{-2}$ to $10^{-3})$ eV^2 and an oscillation angle $\sin^2 2\theta_{\mu\tau}^{\mathrm{osc}} = (0.82 - 1.0)$. To be more precise, the observations do not directly exclude oscillation into to some other ν_{X}, as long as it is *not* to ν_e, but Occam's razor and the framework adopted in this paper suggest $X = \tau$, and we shall assume that in what follows, without further comment. These observations clearly require new physics beyond what is usually contemplated in the Standard Model.

As we shall presently discuss, a tau neutrino mass consistent with the observed oscillations fits extremely naturally into the framework supplied by unified gauge theories of the strong, weak, and electromagnetic interactions [2,3] that includes the symmetry structure $G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_C$, the minimal such symmetry being $SO(10)$ [4]. In this

framework, the low-energy degrees of freedom need be no larger than the Standard Model. Neutrino masses appear as effective nonrenormalizable (dimension 5) operators.

Unified gauge theories were already very impressive on other grounds. They combine the scattered multiplets of the Standard Model (five per family) into a significantly smaller number (two for $SU(5)$, one for $SO(10)$). They rationalize the otherwise bizarre-looking hypercharge assignments in the Standard Model [2,3]. Finally, especially in their supersymmetric version [5], they account quantitatively for the relative values of the strong, weak, and electromagnetic couplings [6,7].

This last feat is accomplished by renormalizing the separate couplings down from a single common value at a unification scale, taking into account the effects of vacuum polarization due to virtual particles, down to the much lower mass scales at which they are observed experimentally. A by-product of this overconstrained, and singularly successful, calculation, is to identify the mass scale at which the unified symmetry is broken, to be $M_{\mathrm{U}} \sim 2 \times 10^{16}$ GeV [7].

This value is interesting in several respects. First, from data and concepts purely internal to gauge theories of particle interactions, it brings us to the threshold of the fundamental scale of quantum gravity, namely the Planck mass 2×10^{18} GeV (in rational units). Reading it the other way, by demanding unification, allowing for both the classical power-law running of the gravitational coupling and the quantum logarithmic running of gauge couplings, we obtain a roughly accurate explanation of the observed strength of gravity.

Second, it sets the scale for phenomena directly associated with unification but forbidden in the Standard Model, notably nucleon decay and neutrino masses. Prior to the SuperKamiokande observations, the main phenomenological virtues of the large value of the unification mass scale were its negative implications. It explained why nucleon decay is rare, and neutrino masses are small, although both are almost inevitable consequences of unification. Now the scale can also be *positively* identified, at least semi-quantitatively.

Indeed, any unification based on G_{224} [2] requires the existence of right-handed neutrinos ν^R. When G_{224} is embedded in $SO(10)$, ν^R fills out, together with the 15 left-handed quark and lepton fields in each Standard Model family, the 16 dimensional spinor representation of $SO(10)$. The ν^R are Standard Model singlets, so that they can, and generically will, acquire large Majorana masses at the scale where unified $SO(10)$ symmetry breaks to the Standard Model $SU(3) \times SU(2) \times U(1)$. The ordinary, left-handed neutrinos couple to these ν^R much as ordinary quarks and leptons couple to their right-handed partners, through $SU(2) \times U(1)$ non-singlet Higgs fields. For the quarks and leptons, condensation of those Higgs fields transforms such interactions directly into mass terms. For neutrinos the effect of this condensation is slightly more involved. As mentioned, the ν^R have an independent, and much larger, source of mass. As a result, through the "see-saw" mechanism [8], the effective masses for the left-handed neutrinos, acquired through their virtual transitions into ν^R and back, are predictably tiny.

In this paper we do two things. First, we flesh out with quantitative detail the rough picture just sketched. Its most straightforward embodiment leads to the hierarchical pattern $m_{\nu_e} << m_{\nu_\mu} << m_{\nu_\tau}$, and to a value of m_{ν_τ} very consistent with the SuperK observations, interpreted as $\nu_\mu - \nu_\tau$ oscillations. We demonstrate that the large mixing angle observed does not force us to swerve from this straightforward direction. Indeed, it can arise rather plausibly in the context of ideas which have been applied successfully to understanding

quark masses and mixings. Motivated by the success of the circle of ideas mentioned above, we insist that the pattern of neutrino masses and mixings should be discussed together with those of the quarks and charged leptons, and not in isolation. A simple and predictive $SO(10)$–based mass structure that describes the observed masses and mixings of all fermions including those of the neutrinos will be presented and analyzed. We thus demonstrate by example how the large $\nu_\mu - \nu_\tau$ oscillation angle can be obtained quite naturally along with a large hierarchy in the $\nu_\mu - \nu_\tau$ masses. (This is in contrast to several recent attempts [9,10] where such a large oscillation angle is explained as a consequence of the near degeneracy of the $\nu_\mu - \nu_\tau$ system.) In this scheme, $\nu_e - \nu_\mu$ oscillation drives the small angle MSW explanation [11] of the solar neutrino puzzle.

Second, we revisit a previously noted link between neutrino masses and nucleon decay, in the framework of supersymmetric $SO(10)$ unified models [12]. Previously, motivated in part by possible cosmological indications for a hot dark matter component, we used numerical estimates for $m_{\nu_\tau} \sim 1$ eV, considerably larger than are now favored by the SuperK result ($\sim 1/10$ eV). This amounts to an increase in the Majorana mass of ν_τ^R compared to previous work, and correspondingly an increase in the strength of the neutrino mass related $d = 5$ proton decay rate. Another important change is caused by the large $\nu_\mu - \nu_\tau$ oscillation angle suggested by the SuperK result. With hierarchical neutrino masses, we will argue, their result strongly suggests substantial mixing in the charged lepton ($\mu - \tau$) sector. That, in turn, affects the strength and the flavor structure not only of the neutrino related, but also of the standard $d = 5$ proton decay operators induced by the exchange of color triplet partners of the electroweak Higgs doublets [13]. These adjustments in our expectations for proton decay turn out to be quite significant quantitatively. They considerably heighten the tension around nucleon decay: either it is accessible, or the framework fails.

This paper is organized as follows. In Sec. II, we discuss the scale of new physics implied by the SuperK observations. In Sec. III we describe a caricature model that accommodates large neutrino oscillation angle as suggested by SuperK without assuming neutrino mass degeneracy. Sec. IV is devoted to a more ambitious $SO(10)$ model that accounts for the masses of second and third generation quarks and leptons including the large neutrino oscillation angle. In Sec. V we suggest, by way of example, a predictive way to incorporate the first family fermions into the $SO(10)$ scheme. In Sec. VI we discuss the issue of proton decay in the context of neutrino masses. Three Appendices (A,B and C) contain relevant technical details of our proton decay calculations. Finally, a summary of our results and some concluding remarks are given in Sec. VII[1].

II. M_{ν_τ} AND THE UNIFICATION SCALE

Using the degrees of freedom of the Standard Model, small Majorana masses for neutrinos arise from dimension-5 operators in the form [15]

$$\mathcal{L} = \lambda_{ij} \frac{L_i L_j \phi \phi}{M} + \text{h.c.} \tag{1}$$

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[1]Preliminary results of this investigation were announced at summer conferences [14].

where $L_i = (\nu_i, \ell_i)^T$ denote the lepton doublets and $\phi = (\phi^+, \phi^0)^T$ the Higgs doublet of the Standard Model. Interpreting the SuperK result as a measure of $m_{\nu_\tau} = (1/30 - 1/10)$ eV, momentarily ignoring mixing, and using $\langle \phi^0 \rangle = 246$ GeV, we find $M/\lambda_{33} = (6 - 18) \times 10^{14}$ GeV.

According to the seesaw mechanism, and again putting off the question of mixing, the tau neutrino mass is given as

$$m_{\nu_\tau} = \frac{(m_{\nu_\tau}^D)^2}{M_\tau^R} \tag{2}$$

where m^D is the Dirac mass that the neutrino would acquire in the absence of the large Majorana mass of the right-handed neutrino, and M^R is the value of this Majorana mass.

If one assumes, within $SO(10)$, that the Dirac masses of the third family are dominated by a contribution from a fundamental Higgs $\mathbf{10_H}$ condensate then one obtains the relation $m_\tau(M_U) = m_b(M_U)$ between the masses of the tau lepton and bottom quark at the unification scale, which is known to be successful [16]. This suggests that the third family fermions get their masses primarily from a $\mathbf{10_H}$ condensate through a Yukawa coupling $\mathbf{16_3 16_3 10_H}$. ($\mathbf{16}_i$, $i = 1 - 3$ denotes the three generations of fermions.) This hypothesis entails the relation

$$m_{\nu_\tau}^D(M_U) = m_t(M_U) \sim (100 - 120) \text{ GeV} . \tag{3}$$

(This numerical estimate is valid for most values of the MSSM parameters, so long as $\tan \beta$ is neither too large (≥ 30) nor too small (≤ 2).) Combining this with the seesaw formula and the SuperK measurement, one obtains $M_\tau^R \approx (1 - 3) \times 10^{14}$ GeV.

In $SO(10)$, the Majorana masses of right-handed neutrinos can be generated using either the five-index self-dual antisymmetric tensor $\overline{\mathbf{126}}_\mathbf{H}$, or a bilinear product of the spinorial Higgs $\overline{\mathbf{16}}_\mathbf{H}$. The relevant interactions are the renormalizable interaction $\tilde{f}_{ij} \mathbf{16_i 16_j \overline{126}_H}$ or the effective nonrenormalizable interaction $f_{ij} \mathbf{16_i 16_j \overline{16}_H \overline{16}_H}/M$, respectively. In terms of the $SU(2)_L \times SU(2)_R \times SU(4)_C$ subgroup the relevant multiplets transform as $(1,3,10)_H$ or $(1,2,4)_H$ respectively.

If the $\overline{\mathbf{126}}_\mathbf{H}$ is used to induce the Majorana mass of ν_τ^R, then with $\langle \overline{\mathbf{126}}_\mathbf{H} \rangle \approx M_U$ we require the Yukawa coupling $f_{33} \sim 10^{-2}$. Considering that it was constructed by balancing quantities of vastly different magnitudes, the nearness of this coupling to the 'natural' value unity is encouraging.

Still more interesting is the situation that arises if we employ the $\overline{\mathbf{16}}_\mathbf{H}$. In this case, as mentioned just above, we require an effective nonrenormalizable interaction. Such an interaction could well arise through the exchange of superheavy states associated with quantum gravity. Then using $\langle \overline{\mathbf{16}}_\mathbf{H} \rangle \sim M_U \approx 2 \times 10^{16}$ GeV and $M \approx M_{\text{Planck}} \approx 2 \times 10^{18}$ GeV we find

$$M_\tau^R \approx f_{33} \frac{\langle \overline{\mathbf{16}}_\mathbf{H} \rangle^2}{M} \approx (f_{33}) \times 2 \times 10^{14} \text{ GeV} . \tag{4}$$

This is nicely consistent with the required value of M_τ^R, for f_{33} close to unity.

Many of the considerations that follow do not depend on which of these alternatives, $\overline{\mathbf{16}}_\mathbf{H}$ or $\overline{\mathbf{126}}_H$, is chosen. Motivated partly by the foregoing numerology, and partly by some suggestions from higher symmetry schemes and string theory [17], we will mainly discuss

models in which the pair ($\mathbf{16_H}, \overline{\mathbf{16_H}}$) is used to break ($B - L$). Let us note, however, that the $\overline{\mathbf{126}}_H$ does have some advantages. Specifically, its couplings $\tilde{f}_{ij}$ are renormalizable, and its vacuum expectation value violates ($B - L$) by two units, and thus conserves an R parity automatically [18]. The latter property is important for eliminating catastrophic – dimension 4 – sources of proton decay. With the $SO(10)$ spinor condensates, we must postulate a suitable Z_2 symmetry for this purpose separately.

III. LARGE ($\nu_\mu - \nu_\tau$) OSCILLATION ANGLE WITH HIERARCHICAL MASSES

Based on its measurements of atmospheric cosmic ray neutrino oscillations, the SuperK group estimates a large oscillation angle $\sin^2 2\theta^{\rm osc}_{\mu\tau} = (0.82 - 1)$ [1]. If we compare this to the analogous angle for quarks, $\sin^2 2\theta_{cb} \approx 4|V_{cb}|^2 \approx 6 \times 10^{-3}$, a challenge arises. How are we to understand the enormous difference between these two mixings, in a framework where quarks and leptons are unified?

One widely considered possibility is to propose that the two relevant neutrino flavor eigenstates are nearly degenerate [9,10]. Then a small perturbation that lifts this degeneracy will induce maximal mixing. Such behavior is familiar from the $K^0 - \overline{K^0}$ system. But while there is a well–established fundamental symmetry (CPT) which guarantees exact degeneracy of K^0 and $\overline{K^0}$, no such symmetry is known to operate for the different flavors in the quark–lepton system. In the context of the seesaw mechanism, such degeneracy is awkward to accommodate, at best. Furthermore, if ν_μ and ν_τ are nearly mass degenerate with a common mass of $\geq 1/30$ eV, then there is no simple way to address the solar neutrino puzzle through $\nu_e - \nu_\mu$ or $\nu_e - \nu_\tau$ oscillation. The possibilities appear bizarre – either ν_e is nearly degenerate with ν_μ and ν_τ, or there is a fourth "sterile" neutrino with the right mass and mixing parameters. In light of all this, it seems reasonable to consider alternatives sympathetically.

We will argue now that large $\nu_\mu - \nu_\tau$ oscillation angle can in fact arise, without requiring the neutrinos to be nearly degenerate, along the lines of some not entirely unsuccessful attempts to relate mixing angles and hierarchical masses in the quark sector.

In the two sections following this one we shall analyze a complete model for the quark and lepton masses that is both overconstrained and phenomenologically successful (though, to be sure, its construction involves considerable guesswork). Since that analysis becomes rather complicated and perhaps intimidating, we shall first, in this section, highlight some salient features in a caricature model.

A leading idea in many attempts to understand large mixing in the quark sector is to utilize the properties of special matrices, whose form might be constrained by simple symmetry requirements (e.g., requiring symmetry or antisymmetry) and selection rules. Such constraints can easily arise, as we shall see, from the group theory of unification, given specific choices for the Higgs fields whose condensation generates the masses.

The first attempts along these lines, which remain quite intriguing and instructive, utilized a 2×2 system of the form [19]

$$M_f = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}_f , \quad f = (u, d) \tag{5}$$

with $a_f \ll b_f$. Symmetric matrices arise, for example, in $SO(10)$ (or left–right symmetric models [20]) where the matter fields are $\mathbf{16}$s if the relevant Higgs fields are $\mathbf{10}$s. The vanishing

of the (1,1) entry can be ensured by a suitable flavor symmetry that distinguishes the two relevant families. The eigenvalues of this matrix are $(m_1, m_2)_f \simeq (a^2/b, b)_f$. Thus the off–diagonal element a_f is nearly the geometrical mean of the two eigenvalues. For convenience in discussion, we will refer to mass matrices of this form as type A. For type A mass matrices, the mixing angles in each sector, up and down, are given by

$$\tan\theta_f = \sqrt{\left|\frac{m_1}{m_2}\right|_f} \ . \tag{6}$$

To obtain the observable mixing angle one must of course combine the mixing angles of the up and the down sectors. In a world with two flavors, this leads to the well–known formula for the Cabibbo angle [19]:

$$\theta_C \simeq \left|\sqrt{\frac{m_d}{m_s}} - e^{i\phi}\sqrt{\frac{m_u}{m_c}}\right| \ . \tag{7}$$

Using $\sqrt{m_d/m_s} \simeq 0.22$ and $\sqrt{m_u/m_c} \simeq 0.06$, we see that Eq. (7) works within 30% for any value of the phase ϕ, and perfectly for a value of the phase parameter ϕ around $\pi/2$.

A notable feature of the type A pattern, which remains valid even if the requirement of symmetry is relaxed, is that it can support a strong hierarchy of eigenvalues by means of a much weaker hierarchy of matrix elements. For example, with $(a/b)_f = 1/10$, one obtains a large hierarchy $(m_1/m_2)_f \sim 1/100$.

Now let us consider the implications of adopting the type A pattern for the $\mu - \tau$ sector, including the 2×2 Dirac mass matrices of the charged leptons $(\mu - \tau)$ and the neutrinos $(\nu_\mu - \nu_\tau)$, and the Majorana mass matrix of the right–handed neutrinos $(\nu_\mu^R - \nu_\tau^R)$. Including the first family generally will not much affect the discussion $\mu - \tau$ sector, as we shall see.

The three matrices in the leptonic sector have the form

$$M_{\ell,\nu}^D = \begin{pmatrix} 0 & d_{\mu\tau} \\ d_{\mu\tau} & d_{\tau\tau} \end{pmatrix}^{\ell,\nu} \ ; \ \ M_\nu^M = \begin{pmatrix} 0 & y \\ y & 1 \end{pmatrix} M_R \tag{8}$$

with the understanding that in each case the $(\tau\tau)$ entry dominates. One finds easily that the physical mass matrix for the light left-handed neutrinos, $M_\nu^{\text{light}} = -M_\nu^D (M_\nu^M)^{-1} (M_\nu^D)^T$, takes a similar form, viz.

$$M_\nu^{\text{light}} = \frac{1}{M_R y^2} \begin{pmatrix} 0 & -d_{\mu\tau}^2 y \\ -d_{\mu\tau}^2 y & \{d_{\mu\tau}^2 - 2d_{\tau\tau} d_{\mu\tau} y\} \end{pmatrix}^\nu \ . \tag{9}$$

Denoting the two eigenvalues as m_{ν_2} and m_{ν_3}, this yields:

$$\frac{m_{\nu_2}}{m_{\nu_3}} \simeq \frac{-y^2}{(1 - 2y\frac{d_{\tau\tau}^\nu}{d_{\mu\tau}^\nu})^2}, \ \ \tan\theta_{\mu\tau}^\nu = \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}, \ \ \tan\theta_{\mu\tau}^\ell = \sqrt{\frac{m_\mu}{m_\tau}} \ . \tag{10}$$

Observe that the square–root formula holds for the mixing angles in all sectors, including the light neutrinos.

To illustrate the possibilities, let us consider the phenomenologically interesting hierarchy ratio $m_{\nu_\mu}/m_{\nu_\tau} \simeq 1/10$. With this ratio, one can accommodate $m_{\nu_\tau} \simeq .03$ eV as suggested by

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atmospheric neutrino oscillations and a value of m_{ν_μ} consistent with the small-angle MSW solution (with very small m_{ν_e}) [21]. This ratio is achieved for $(d^\nu_{\mu\tau}/d^\nu_{\tau\tau}, \ y) = (1/5, \ 1/13)$. Combining the contributions from the charged lepton sector and from the neutrino sector, the physical oscillation angle following from Eq. (10) is

$$\theta^{\rm osc}_{\mu\tau} \simeq \left| \sqrt{\frac{m_\mu}{m_\tau}} - e^{i\eta} \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_\tau}}} \right| \ . \tag{11}$$

Although the mixing angle is not large in either sector ($\theta^\ell_{\mu\tau} \simeq \sqrt{m_\mu/m_\tau} \simeq 0.25 \approx 14^o$ and $\theta^\nu_{\mu\tau} \simeq \sqrt{m_{\nu_\mu}/m_{\nu_\tau}} \simeq \sqrt{1/10} \simeq 0.31 \approx 18^o$), if $\eta \approx \pi$ one obtains a near-maximal value for the physical mixing parameter. Actually the small angle approximation used in Eq. (11) is too crude; the precise expression is given by

$$\sin^2 2\theta_{\mu\tau} = \frac{4 \left(\sqrt{\frac{m_\mu}{m_\tau}} \pm \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_\tau}}} \right)^2 \left(1 \mp \sqrt{\frac{m_\mu}{m_\tau}} \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_\tau}}} \right)^2}{\left(1 + |\frac{m_\mu}{m_\tau}| \right)^2 \left(1 + |\frac{m_{\nu_\mu}}{m_{\nu_\tau}}| \right)^2} \ , \tag{12}$$

where the $\pm$ corresponds to $\eta = (\pi, 0)$. For $\eta = \pi$ and $m_{\nu_\mu}/m_{\nu_\tau} = 1/10$, $\sin^2 2\theta^{\rm osc}_{\mu\tau} \simeq 0.79$. This example shows that large oscillation angles can arise in a simple hierarchical model, without any extreme adjustment of parameters.

IV. A MORE AMBITIOUS $SO(10)$ MODEL

The model discussed in the previous section is not adequate for a unified treatment of the masses and mixings of the second and the third family fermions. Indeed, the square-root formula for V_{cb} reads

$$|V_{cb}| \simeq \left| \sqrt{\frac{m_c}{m_t}} - e^{i\chi} \sqrt{\frac{m_s}{m_b}} \right| \ , \tag{13}$$

so that with $\sqrt{m_s/m_b} \simeq 0.17$ and $\sqrt{m_c/m_t} \simeq 0.06$, one cannot obtain observed value $V_{cb} \simeq 0.04 \pm 0.003$ for any value of χ. Thus the simplest symmetrical type A mass matrices (Eq. (5)) cannot adequately describe the hierarchical masses and the mixings of the quarks.

We now propose to study in detail a concrete proposal for asymmetric type A mass matrices that can be obtained within $SO(10)$, and predicts correlations among the quark–lepton and up–down mass matrices that are phenomenologically acceptable. Before plunging into the analysis, let us briefly summarize its main conclusions. In this section, we shall temporarily ignore the first family.

The four Dirac mass matrices (U, D, L, and N) will be generated via four Yukawa couplings. The Majorana matrix of the right-handed neutrinos involves two additional Yukawa couplings. Thus there are seven parameters (six Yukawa couplings plus one ratio of vacuum expectation values) to describe 10 observables (eight masses – $(m_c, m_t, m_s, m_b, m_\mu, m_\tau, m_{\nu_2}, m_{\nu_3})$ – and two mixing angles – $\theta^{\rm osc}_{\mu\tau}$ and V_{cb}). The system is overconstrained, and predicts three relations among observables. Two of these concern the charged fermion sector, and they are reasonably well satisfied. The third prediction

concerns the neutrino sector: the neutrino oscillation angle will be predicted as a function of the mass ratio m_{ν_2}/m_{ν_3}. For $m_{\nu_2}/m_{\nu_3} = (1/10 - 1/20)$, consistent with mass determinations from the SuperK atmospheric neutrino data and the small angle MSW solution of the solar neutrino puzzle, a large $(\nu_\mu - \nu_\tau)$ oscillation angle is obtained, as indicated by the SuperK atmospheric data.

In forming hypotheses for the form of the couplings responsible for the mass matrices, we are guided by several considerations. We assume an underlying $SO(10)$ unification symmetry, and that unified symmetry breaking occurs through a minimal system of low dimensional Higgs multiplets – specifically, $\langle \mathbf{45_H} \rangle$, one pair of $\langle \mathbf{16_H} \rangle$ and $\langle \overline{\mathbf{16_H}} \rangle$. A single $\mathbf{10_H}$ is employed for electroweak symmetry breaking. The hierarchical masses will be assumed to arise from type A mass matrices, as in Eq. (5). In addition, we must of course satisfy the broad phenomenological constraints $m_b(M_U) \simeq m_\tau(M_U)$, $m_s(M_U) \neq m_\mu(M_U)$, and $V_{CKM} \neq 1$. Combining these considerations with a restriction to low dimensional operators ($d \leq 5$), we are led to suggest the following set of Yukawa couplings [22]:

$$\mathcal{L}_{\text{Yukawa}} = h_{33} \mathbf{16_3} \mathbf{16_3} \mathbf{10_H} + \frac{a_{23}}{M} \mathbf{16_2} \mathbf{16_3} \mathbf{10_H} \mathbf{45_H} +$$
$$\frac{g_{23}}{M} \mathbf{16_2} \mathbf{16_3} \mathbf{16_H} \mathbf{16_H} + h_{23} \mathbf{16_2} \mathbf{16_3} \mathbf{10_H} \ . \tag{14}$$

Here M, a scale associated with the effective non–renormalizable interactions, could plausibly lie somewhere between the unification scale M_U and M_{Planck}. For example, the a_{23} term might result from integrating out a $\mathbf{16} + \overline{\mathbf{16}}$ superfield pair with mass of order M; or M might be identified as M_{Planck} itself if the nonrenormalizable interactions are associated with gravity. A mass matrix of type A results if the first term, $h_{33} \langle \mathbf{10_H} \rangle$, is dominant. This ensures $m_b(M_U) \simeq m_\tau(M_U)$ and $m_t(M_U) \simeq m_{\nu_\tau}^D(M_U)$. The remaining terms, responsible for off-diagonal mixings, must be smaller by about one order of magnitude.

In more detail, our rationale for favoring the particular off–diagonal couplings a_{23} and g_{23} is the following. If the fermions acquired mass only through $\mathbf{10}$s, one would obtain too much symmetry between down quark and charged lepton masses, and in particular the bad relation $m_s(M_U) = m_\mu(M_U)$. To avoid this, while eschewing proliferation of extraneous Higgs multiplets, the simplest possibility is to bring in the vacuum expectation value of the (1,1,15) component (under G_{224}) in the $\mathbf{45_H}$ that is proportional to $(B - L)$. A vacuum expectation value of this form figures prominently in the Dimpoulos-Wilczek mechanism for doublet-triplet splitting in $SO(10)$ [23], as we shall discuss at length in connection with proton decay. Now if one restricts to $d = 4$ and $d = 5$ operators, the only relevant $SO(10)$ invariant effective coupling that involves the $\mathbf{45_H}$ is our $a_{ij} \mathbf{16_i} \mathbf{16_j} \mathbf{10_H} \mathbf{45_H}/M$. In this term, only the $\mathbf{120}$ in the decomposition $\mathbf{10_H} \times \mathbf{45_H}$ ($= \mathbf{10} + \mathbf{120} + \mathbf{320}$) can contribute to fermion mass matrix, and its contribution is antisymmetric in (i,j). These two couplings still do not distinguish up and down quark mass patterns, and so if they were the whole story one would have a trivial, identity, Cabibbo-Kobayashi-Maskawa (CKM) matrix. The effective couplings $g_{ij} \mathbf{16_i} \mathbf{16_j} \mathbf{16_H} \mathbf{16_H}/M$ remedy this problem, without requiring addition to our small set of Higgs multiplets. When a vacuum expectation value of order the unification scale for the $SU(3) \times SU(2) \times U(1)$ singlet component of $\mathbf{16_H}$ and a vacuum expectation value of order the electroweak scale for the $SU(2) \times U(1)$ breaking component of $\mathbf{16_H}$ are inserted, this term contributes to quark and lepton mass matrices. To be more precise, since the electroweak doublet contained in $\mathbf{16_H}$ has the quantum numbers of a down–type Higgs

doublet, it contributes to the down quark and charged lepton mass matrices, but not to the up sector. The resulting up–down asymmetry generates non–zero CKM mixing angles.

With these four effective Yukawa couplings, the Dirac mass matrices of quarks and leptons of the second and the third families take the form:

$$U = \begin{pmatrix} 0 & \epsilon + \sigma \\ -\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon + \eta \\ -\epsilon + \eta & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon + \sigma \\ 3\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad L = \begin{pmatrix} 0 & -3\epsilon + \eta \\ 3\epsilon + \eta & 1 \end{pmatrix} m_D . \tag{15}$$

Here the matrices are multiplied by left–handed fermion fields from the left and by anti–fermion fields from the right. (U, D) stand for the mass matrices of up and down quarks, while (N, L) are the Dirac mass matrices of the neutrinos and the charged leptons.

The entries $(1, \epsilon, \sigma)$ arise respectively from the h_{33}, a_{23} and h_{23} terms in Eq. (14), while η entering into D and L receives contributions from both g_{23} and h_{23}; thus $\eta \neq \sigma$. Note the quark–lepton correlations between (U, N) as well as (D, L), and the up–down correlation between (U, D) as well as (N, L). These correlations arise because of the symmetry structure of $SO(10)$. The relative factor of -3 between quarks and leptons involving the ϵ entry reflects the fact that $\langle \mathbf{45_H} \rangle \propto (B - L)$, while the antisymmetry in this entry arises from the $SO(10)$ structure as explained above.

Assuming $\epsilon, \eta, \sigma \ll 1$, we obtain

$$\left| \frac{m_c}{m_t} \right| \simeq |\epsilon^2 - \sigma^2|, \quad |\frac{m_s}{m_b}| \simeq |\epsilon^2 - \eta^2| ,$$

$$\left| \frac{m_\mu}{m_\tau} \right| \simeq |9\epsilon^2 - \eta^2|, \quad |m_b| \simeq |m_\tau||1 - 8\epsilon^2| ,$$

$$|V_{cb}| \simeq |\sigma - \eta| . \tag{16}$$

Here all the mass and mixing angle parameters are to be identified with those at the unification scale. One can evaluate σ, η, ϵ in terms of the observed masses and mixing (extrapolated to the unification scale):

$$\sigma \simeq [V_{cb}^2 + m_s/m_b - m_c/m_t]/(2V_{cb}) ,$$

$$\eta \simeq [-V_{cb}^2 + m_s/m_b - m_c/m_t]/(2V_{cb}) ,$$

$$\epsilon^2 \simeq \{\sigma^2 + m_c/m_t\} \simeq (\eta^2 + m_\mu/m_\tau)/9 . \tag{17}$$

This leads to the sum rule:

$$\frac{m_s}{m_b} \simeq \frac{m_c}{m_t} - \frac{5}{4} V_{cb}^2 \pm V_{cb} \left[\frac{9}{16} V_{cb}^2 + \frac{1}{2} \frac{m_\mu}{m_\tau} - \frac{9}{2} \frac{m_c}{m_t} \right]^{1/2} . \tag{18}$$

A word of explanation is needed here. The parameters σ, η are in general complex. In Eqs. (16)-(18) therefore, one should interpret the mass ratios and V_{cb} to be also complex. Let us denote $m_c/m_t = \eta_{ct}|m_c/m_t|, m_s/m_b = \eta_{sb}|m_s/m_b|, V_{cb} = \eta_{cb}|V_{cb}|$, with $|\eta_{ij}| = 1$. Noting that each term on the RHS of Eq. (18) is small compared to $m_s/m_b \simeq (1/30 - 1/50)$ (e.g: $m_c/m_t \approx 1/300$, $|V_{cb}||m_\mu/(2m_\tau)|^{1/2} \approx 1/140$), we see that essentially only one choice of the phase factors can possibly allow the sum rule to work so that the RHS is maximized – i.e.,

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$\eta_{ct} = -1, \eta_{\mu\tau} = +1$ and $\eta_{sb} = -1$ with $\{\pm V_{cb}[\ \ ]^{1/2}\}$ having a negative sign. Thus, all phases are constrained to be near the CP conserving limit (0 or π). For simplicity, we will take the η_{ij}'s to be real.

The relatively simple pattern shown in Eq. (15) provides a reasonable fit to all the masses and the mixing of the quarks and the charged leptons in the second and the third families. For example, if we take as input $m_t^{\text{physical}} = 174$ GeV, $m_c(m_c) = 1.37$ GeV, $V_{cb} = 0.045$, in agreement with the values advocated in Ref. [24,25], and the known μ and τ lepton masses, then we obtain the predictions

$$m_b(m_b) \simeq 4.9 \text{ GeV} ,$$
$$m_s(1 \text{ GeV}) \simeq 116 \text{ MeV} . \tag{19}$$

In quoting the numbers in Eq. (19), we have extrapolated the GUT scale values down to low energies using the beta functions of the minimal supersymmetric extension of the Standard Model (MSSM), assuming $\alpha_s(M_Z) = 0.118$, an effective SUSY threshold of 500 GeV and $\tan\beta = 5$. Our results depend only weakly on these input choices, so long as $\tan\beta$ is neither too large (≥ 30) nor too small (≤ 2).

The mass of the strange quark is somewhat low compared to the central value advocated in Ref. [24], but agrees well with recent lattice determinations [26]. The b–quark mass prediction is also in reasonable agreement with determination from Υ spectroscopy. In this regard, it should be mentioned that exact $b-\tau$ unification in supersymmetric unified models, without taking finite threshold effects due to the gluino into account, would yield a b–quark mass which is about 10-20% above the experimental value for a wide range of the parameter $\tan\beta$ [27]. In our case, b and τ masses are not exactly equal at M_U, m_b is about 8% lower than m_τ (see Eq. (16)). This difference, which has its origin in the $(B - L)$ generator associated with the off–diagonal entry ϵ in Eq. (15), leads to better agreement with the experimental value of $m_b(m_b)$.

In addition, the Dirac masses of the neutrinos are determined to be

$$m_{\nu_\tau}^D(M_U) \simeq m_t(M_U) \simeq 100 - 120 \text{ GeV} ,$$
$$m_{\nu_\mu}^D(M_U) \simeq (9\epsilon^2 - \sigma^2)m_U \simeq 8 \text{ GeV} . \tag{20}$$

Note that the Dirac mass of ν_μ is quite different from $m_c(M_U) \simeq 300$ MeV, which is often anticipated in $SO(10)$. This difference arises because of the type A pattern of the mass matrices and the factor of -3 associated with the $(B - L)$ generator that goes into the ν_μ Dirac mass [28].

The parameters σ, η, ϵ are found to be

$$\sigma \simeq -0.110\ \eta_{cb}\ ,\eta \simeq -0.151\ \eta_{cb}\ ,\epsilon \simeq 0.095\ \eta_\epsilon\ . \tag{21}$$

($\eta_\epsilon = \pm 1$ is the phase of ϵ.) The left–handed mixing angle from the charged lepton sector is $\theta_{\mu\tau}^\ell \simeq -3\epsilon + \eta \simeq -0.437\ \eta_\epsilon$ (for $\eta_{cb}/\eta_\epsilon = +1$).

Given the bizarre pattern of quark and lepton masses and mixing, we regard the overall fit to all of them, good to within 10%, using the pattern shown in Eq. (15), as reason to take this pattern seriously.

Note that owing to the asymmetric mass matrix, the square–root formula for the mixing angle $\theta_{\mu\tau}^\ell$ receives a correction given by the factor $[(-3\epsilon + \eta)/(3\epsilon + \eta)]^{1/2}$. For the values of η and ϵ determined in Eq. (21), this factor is about 1.6 for $\eta_\epsilon/\eta_{cb} = +1$.

Although all the entries for the Dirac mass matrix are now fixed, to obtain the parameters for the light neutrinos one needs to specify the Majorana mass matrix. Unfortunately, here there is much less information to guide our hypotheses. For concreteness, let us imagine that this too takes the type A form:

$$M_\nu^R = \begin{pmatrix} 0 & y \\ y & 1 \end{pmatrix} M_R \; , \tag{22}$$

where we allow $y = \eta_y |y|$ to have either sign, i.e., $\eta_y = \pm 1$. Note that Majorana mass matrices are constrained to be symmetric by Lorentz invariance. The seesaw mass matrix $(-N(M_\nu^R)^{-1} N^T)$ for the light $(\nu_\mu - \nu_\tau)$ system is then

$$M_\nu^{\text{light}} = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} \frac{m_U^2}{M_R} \; , \tag{23}$$

where $A \simeq (\sigma^2 - 9\epsilon^2)/y$ and $B \simeq -(\sigma + 3\epsilon)(\sigma + 3\epsilon - 2y)/y^2$. With $A \ll B$, this yields

$$m_{\nu_3} \simeq B \frac{m_U^2}{M_R}; \quad \frac{m_{\nu_2}}{m_{\nu_3}} \simeq -\frac{A^2}{B^2}; \quad \tan\theta_{\mu\tau}^\nu = \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \; . \tag{24}$$

Correspondingly,

$$y\eta_\epsilon \simeq \frac{\pm\sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}(3|\epsilon| - |\sigma|)}{3|\epsilon| + |\sigma| \pm 2\sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}} \; , \tag{25}$$

where $\eta_\epsilon = \pm 1$ and we have used the fact that ϵ and σ are relatively negative (for $\eta_{cb}/\eta_\epsilon = +1$, see Eq. (21)). For a given choice of the sign of y relative to that of ϵ, and for a given mass ratio m_{ν_2}/m_{ν_3}, we can now determine $y\eta_\epsilon$ using Eq. (25) and the values of ϵ and σ obtained in Eq. (21). Corresponding to $m_{\nu_2}/m_{\nu_3} = (1/10, 1/15, 1/20, 1/25, 1/30)$, we calculate $(y\eta_\epsilon)_+ = (0.0543, 0.0500, 0.0468, 0.0444, 0.0424)$ and $(y\eta_\epsilon)_- = (0.2359, 0.3781, 0.7681, 8.4095, -1.0519)$, where the subscripts $\pm$ correspond to $\eta_y\eta_\epsilon = \pm 1$. The case of $\eta_y\eta_\epsilon = +1$ typically requires smaller values of $|y|$ than for the case of $\eta_y\eta_\epsilon = -1$. The former is more in accord with the idea of the type A matrix and flavor symmetries as the origin of hierarchical masses.

We obtain for the neutrino oscillation angle:

$$\theta_{\mu\tau} \simeq \theta_{\mu\tau}^\ell - \theta_{\mu\tau}^\nu \simeq 0.437 \pm \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \; . \tag{26}$$

If $\eta_y/\eta_\epsilon = +1$, the relative $+$ sign should be chosen, and for $\eta_y/\eta_\epsilon = -1$ the relative $-$ sign should be chosen. Taking the relative $+$ sign, i.e., $\eta_y\eta_\epsilon = +1$, and using the more precise expression given in Eq. (12) (and replacing $\sqrt{m_\mu/m_\tau}$ by the numerical value of the charged lepton mixing angle $\simeq 0.437$), we obtain

$$\sin^2 2\theta_{\mu\tau} = (0.96, 0.91, 0.86, 0.83, 0.81)$$
$$\text{for} \quad m_{\nu_2}/m_{\nu_3} = (1/10, 1/15, 1/20, 1/25, 1/30) \; . \tag{27}$$

As previously advertised, we see that one can derive rather plausibly a large $\nu_\mu - \nu_\tau$ oscillation angle $\sin^2 2\theta_{\mu\tau} \geq 0.8$, together with an understanding of hierarchical masses and mixing of the quarks and the charged leptons, while maintaining a large hierarchy in the seesaw derived masses ($m_{\nu_2}/m_{\nu_3} = 1/10 - 1/30$) of ν_μ and ν_τ, all within a unified framework including both quarks and leptons. In the example exhibited here, the mixing angles for the mass eigenstates of the neutrinos and for the charged leptons are both fairly small, $\theta^\ell_{\mu\tau} \simeq 0.437 \simeq 23^0$ and $\theta^\nu_{\mu\tau} \simeq 0.2 \approx 12^0$, yet the oscillation angle obtained by combining the two is near-maximal. This contrasts with most previous work, in which a large oscillation angle is obtained either entirely from the neutrino sector (with nearly degenerate neutrinos) or entirely from the charged lepton sector. In our case, the mass eigenstates of the neutrinos and the charged leptons are approximately also the respective gauge eigenstates.

V. INCLUSION OF THE FIRST FAMILY: $\nu_E - \nu_\mu$ OSCILLATION

There are several alternative ways to include the first family, without upsetting the successful predictions of the 2-3 sector. In the absence of a deeper understanding, the theoretical uncertainties in analyzing the masses and mixings of the first family are much greater than for the heavier families, simply because the masses of the first family are so small, that relatively tiny perturbations can significantly affect their values. With this warning, we will now briefly consider, as an illustrative example and "proof of principle", a minimal extension to the first family, inspired by the type A pattern.

The 3×3 Dirac mass matrices in the four sectors take the form:

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon + \sigma \\ 0 & -\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' + \eta' & 0 & \epsilon + \eta \\ 0 & -\epsilon + \eta & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon + \sigma \\ 0 & 3\epsilon + \sigma & 1 \end{pmatrix} m_U, \quad L = \begin{pmatrix} 0 & -3\epsilon' + \eta' & 0 \\ 3\epsilon' + \eta' & 0 & -3\epsilon + \eta \\ 0 & 3\epsilon + \eta & 1 \end{pmatrix} m_D. \quad (28)$$

At the level of underlying primary couplings, the ϵ' term is very similar to the ϵ term, arising through a Yukawa coupling $a_{12}\mathbf{16_1 16_2 45_H 10_H}$. The η' term arises through the coupling $g_{12}\mathbf{16_1 16_2 16_H 16_H}$. With $\epsilon, \sigma, \eta, m_U$ and m_D determined by considerations of the second and the third families, we now have just two new parameters in Eq. (28) – i.e. ϵ' and η' – which describe five new observables: m_u, m_d, m_e, θ_C and V_{ub}. Thus with m_u and m_e taken as inputs to fix ϵ' and η', one can calculate the other three observables. In addition, $m^D_{\nu_e}$ will be determined.

At the outset, it is worth noting that this specific pattern (Eq. (28)) is consistent with the empirical unification scale relations: $m_d \simeq 3m_e, m_s \simeq m_\mu/3$ [29] and $m_b \simeq m_\tau$, which in turn imply $m_d m_s m_b \approx m_e m_\mu m_\tau$, and thus $\text{Det}(D) \simeq \text{Det}(L)$. Eq. (28) obeys this relation to a good approximation, for the following reason. From $\text{Det}(U) = \epsilon'^2 m^3_U$, one obtains $\epsilon' \simeq \sqrt{m_u/m_c}(m_c/m_t) \approx 2 \times 10^{-4}$. Using $\text{Det}(L) = (9\epsilon'^2 - \eta'^2)m^3_D = m_e m_\mu m_\tau$, and neglecting the ϵ'^2 term (justified *a posteriori*), one obtains $|\eta'| \simeq \sqrt{m_e/m_\mu}(m_\mu/m_\tau)$ $\simeq 4.4 \times 10^{-3}$. Since $|\epsilon'| \ll |\eta'|$, one gets $\text{Det}(D) \simeq \text{Det}(L)$, as described. Similarly, one also obtains the relation $\theta_C \simeq |\sqrt{m_d/m_s} - e^{i\phi}\sqrt{m_u/m_c}|$, which is known to be fairly successful.

The third prediction, $|V_{ub}/V_{cb}| \simeq \sqrt{m_u/m_c} \simeq 0.07$ is also quite successful. The Dirac mass of ν_e is given by $m_{\nu_e}^D \simeq 9(\epsilon')^2/(9\epsilon^2 - \sigma^2)m_U \simeq 0.4$ MeV.

Turning to the Majorana mass matrix, one could in general introduce (1,1), (1,2) and (1,3) entries involving the first family. We choose the (1,2) element to be zero, both because by itself (i.e., with $(1,2) \neq 0$, but $(1,1) = (1,3) = 0$) it would upset the success of the $\nu_\mu - \nu_\tau$ sector obtained above, and also for economy in parameters. We therefore consider the following pattern:

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R . \tag{29}$$

Consistent with our presumption that hierarchical masses have their origin in the type A pattern, we will assume that $x \leq z^2$ and $z \leq y < 1$ and that the zero in the (2,2) entry only reflects that this entry is small compared to y^2.

Note that all entries in the Majorana matrix arise through $f_{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}_\mathbf{H}}\,\overline{\mathbf{16}_\mathbf{H}}$, which however do not contribute to the Dirac mass matrices. As we have stressed before, they therefore need not have the same flavor structure as the Dirac mass matrices of Eq. (28). Given the Dirac matrix N, we can work out the light neutrino mass matrix after the seesaw diagonalization. It is given by:

$$M_\nu^{\text{light}} = \frac{m_U^2}{M_R}\frac{1}{xy^2} \times \tag{30}$$

$$\begin{pmatrix} 9\epsilon'^2(x - z^2) & 3\epsilon'y\{-3\epsilon'z + (-3\epsilon + \sigma)x\} & 3\epsilon'\{xy - (3\epsilon + \sigma)(x - z^2)\} \\ 3\epsilon'y\{-3\epsilon'z + (-3\epsilon + \sigma)x\} & -9\epsilon'^2y^2 & (3\epsilon + \sigma)y\{3\epsilon'z - (-3\epsilon + \sigma)x\} \\ 3\epsilon'\{xy - (3\epsilon + \sigma)(x - z^2)\} & (3\epsilon + \sigma)y\{3\epsilon'z - (-3\epsilon + \sigma)x\} & (3\epsilon + \sigma)\{-2xy + (3\epsilon + \sigma)(x - z^2)\} \end{pmatrix}$$

Three simplifications arising from Eq. (30) are especially noteworthy. First, that the parameter x figures prominently only in the mass of ν_e, which is given by

$$m_{\nu_e} \simeq \frac{81\epsilon'^4}{xy^4(9\epsilon^2 - \sigma^2)^2}\left(\frac{m_U^2}{M_R}\right) \simeq \frac{2 \times 10^{-10} \text{ eV}}{x} , \tag{31}$$

where we have inserted the mass of $\nu_\tau \simeq 1/10$ eV. For $x = (10^{-5} - 10^{-3})$, $m_{\nu_e} \approx (2 \times 10^{-5} - 2 \times 10^{-7})$ eV. (x cannot be much larger than 10^{-3}, for that would lead to a conflict with proton lifetime, see later.) Second, that the $\mu - \tau$ sector discussion is corrected by terms that are proportional to ϵ', which is tiny, or by terms proportional to z. As long as $z^2 \leq x$ (from the (3,3) entry) and $z \leq 10^3x$ (from the (2,3) entry), the discussion of the (2,3) sector will remain essentially unaltered. And third, that the $\nu_e - \nu_\mu$ mixing angle arising from the light neutrino mass matrix itself (as opposed to the contribution from charged lepton mixing) is tiny, but still significant for the small angle MSW explanation of the solar neutrino puzzle. The $\nu_e - \nu_\tau$ mixing is also small, since $\epsilon' \simeq 2 \times 10^{-4}$ is extremely small.

As an example, let us take $x = 10^{-4}, y = 0.05$ and $z = -0.002$ along with the "central values" of σ, η, ϵ given in Eq. (21) and $\epsilon' = 2 \times 10^{-4}, \eta' = 4.4 \times 10^{-3}$ (from a fit to the u–quark and electron masses). The neutrino mass eigenvalues are then $m_{\nu_\tau}, m_{\nu_\mu}, m_{\nu_e} =$

$\{5.1, 0.35, 2.9 \times 10^{-7}\} m_U^2 / M_R$ and the mixing angles coming from the neutrino sector are: $(\theta^\nu_{\mu\tau}, \theta^\nu_{e\mu}, \theta^\nu_{e\tau}) = \{0.25, 0.009, 0.0034\}$.

Though the mixing angles involving the first family are small, $\theta^\nu_{e\mu}$ does play an important role for solar neutrinos. Combining $\theta^\nu_{e\mu}$ with the corresponding mixing angle from the charged lepton sector, $\theta^\ell_{e\mu} \simeq 0.85 \sqrt{m_e/m_\mu} \simeq 0.06$, where the factor 0.85 is a correction factor from the asymmetric ϵ' term in Eq. (28), we find that $\theta^{\rm osc}_{e\mu} \simeq 0.05$ if the two contributions subtract. This is just about the right value to explain the solar neutrino data via the small angle MSW mechanism [21]. Note that without the (1,3) entry z in Eq. (29), $\theta^\nu_{e\mu}$ would have been less than 0.002, which would have resulted in $\theta^{\rm osc}_{e\mu}$ being larger than the required value for MSW by about 15-20%. This was part of our motivation to introduce the (1,3) entry z into Eq. (29). Since the discussion of the (2-3) sector is little affected so long as $z \leq 10^3 x$ and $z^2 \leq x$, we will allow a range $z = (0.003 - 0.03)$, corresponding to $x = (10^{-3} - 10^{-5})$. This will be significant for the discussion of proton decay, especially for final states involving charged leptons.

Another motivation for including z is consideration of cosmological baryon asymmetry. Note that for $x = 10^{-5} - 10^{-3}$, the mass of ν_e^R is in the range $10^9 - 10^{11}$ GeV, which is of the right magnitude for inducing lepton asymmetry in ν_e^R decay, that is subsequently converted to baryon asymmetry via the electroweak sphalerons [30]. For the lepton number violating decay of ν_e^R to proceed efficiently, it is desirable that ν_e^R mixes with ν_μ^R and ν_τ^R so that no approximately conserved quantum number persists.

While we cannot claim that our several choices have been unique, we have demonstrated that a rather simple pattern for the four Dirac mass matrices, motivated and constrained by the group structure of $SO(10)$, is consistent within 10 to 20% with all observed masses and mixing of the quarks and the charged leptons. This fit is significantly overconstrained, as we have discussed. The same pattern, supplemented with a similar structure for the Majorana mass matrix, quite plausibly accommodates both the SuperKamiokande result with the large $\nu_\mu - \nu_\tau$ oscillation angle required for the atmospheric neutrinos and a small $\nu_e - \nu_\mu$ oscillation angle relevant for theories of the solar neutrino deficit.

VI. NEUTRINO MASSES AND PROTON DECAY

In an earlier paper [12] we pointed out that the theory of neutrino masses can significantly affect expectations for proton decay, both as regards its rate and especially its branching ratios. This happens because in supersymmetric unified theories a new set of color triplet fields is needed to generate heavy Majorana masses for the right-handed (RH) neutrinos, as required for the seesaw mechanism. Exchange of its superpartners generates new dimension 5 operators, that appear in addition to the "standard" $d = 5$ operators in the effective Lagrangian. The standard operators arise through the exchange of color triplets that are related to the electroweak doublets, such as those appearing in the $\mathbf{5} + \overline{\mathbf{5}}$ of $SU(5)$ or the $\mathbf{10}$ of $SO(10)$ [13,31–33].

The standard $d = 5$ operators are estimated to dominate over the gauge boson mediated $d = 6$ proton decay operators. Furthermore, owing to a combination of color antisymmetry, Bose symmetry of the superfields, and hierarchical Yukawa couplings of the fermions, they predict that the dominant modes are $\overline{\nu}_\mu K^+$ and $\overline{\nu}_\mu \pi^+$ modes, while $e^+ \pi^0$ and $e^+ K^0$ and

even $\mu^+\pi^0$ and μ^+K^0 modes are highly suppressed, at least for small and moderate values of $\tan\beta$ (≤ 15) [13,31–33].

In Ref. [12] we argued that for neutrino masses in a plausible range the new $d = 5$ operators could compete favorably with the standard ones, and induce potentially observable, though not yet excluded, proton decay rates. Within the new contribution charged lepton decay mode amplitudes, including those for $\mu^+K^0, \mu^+\pi^0$ and possibly e^+K^0 and $e^+\pi^0$, are not terribly small compared to the $\overline{\nu}K^+$ and $\overline{\nu}\pi^+$ modes, even for low $\tan\beta$. Thus charged lepton modes of nucleon decay potentially furnish an independent signature for the interactions that play a central role in the theory of neutrino masses.

As mentioned earlier, in our previous quantitative work we had to hypothesize values for the neutrino masses. Now, after the SuperK observations, we can revisit the quantitative aspect with some crucial experimental information reliably in hand. Changes are induced not only for the new $d = 5$ operators that are directly related to neutrino masses, but also for the standard $d = 5$ operators.

To address these issues in detail, we must specify the nature of the color triplet Higgsino couplings. This issue is closely tied to the mechanism of doublet–triplet splitting in $SO(10)$, to which we now turn.

A. Natural doublet-triplet splitting in $SO(10)$

In supersymmetric $SO(10)$, a natural doublet–triplet splitting can be achieved by coupling the adjoint Higgs $\mathbf{45_H}$ to a $\mathbf{10_H}$ and a $\mathbf{10'_H}$, with $\mathbf{45_H}$ acquiring a unification–scale VEV in the $B - L$ direction [23]: $\langle\mathbf{45_H}\rangle = (a, a, a, 0, 0) \times \tau_2$ with $a \sim M_U$. As discussed in Section II, to generate CKM mixing for fermions we require an $\langle\mathbf{16_H}\rangle_d$ that acquires an electroweak scale vacuum expectation value. To insure accurate gauge coupling unification, the effective low energy theory should not contain split multiplets beyond those of MSSM. Thus the MSSM Higgs doublets must be linear combinations of the $SU(2)_L$–doublets in $\mathbf{10_H}$ and $\mathbf{16_H}$. A simple set of superpotential terms that ensures this and incorporates doublet–triplet splitting is:

$$W_H = \lambda\mathbf{10_H}\mathbf{45_H}\mathbf{10'_H} + M_{10}\mathbf{10'_H}^2 + \lambda'\overline{\mathbf{16_H}\mathbf{16_H}}\mathbf{10_H} + M_{16}\mathbf{16_H}\overline{\mathbf{16_H}} \ . \qquad (32)$$

A complete superpotential for $\mathbf{45_H}, \mathbf{16_H}, \overline{\mathbf{16_H}}, \mathbf{10}_H, \mathbf{10'}_H$ and possibly other fields, which ensure that $\mathbf{45_H}, \mathbf{16_H}$ and $\overline{\mathbf{16_H}}$ acquire unification scale VEVs with $\langle\mathbf{45_H}\rangle$ being along the $(B - L)$ direction, that exactly two Higgs doublets (H_u, H_d) remain light, with H_d being a linear combination of $(\mathbf{10_H})_d$ and $(\mathbf{16_H})_d$, and that there are no unwanted pseudoGoldstone bosons, can be constructed [34–38]. [2] The various possibilities generate different predictions for threshold corrections in the unification of gauge couplings, for example, but hardly differ in their consequences for neutrino masses and proton decay.

The Higgs doublet and the color triplet mass matrices following from Eq. (32) are, in $SU(5)$ notation,

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[2] It is intriguing that the Higgs fields used in [36] can be neatly incorporated into a single adjoint representation of E_7.

$$(\overline{\mathbf{5}}_{10_H} \quad \overline{\mathbf{5}}_{10'_H} \quad \overline{\mathbf{5}}_{16_H}) \begin{pmatrix} 0 & \lambda \langle \mathbf{45_H} \rangle & \lambda' \langle \overline{\mathbf{16_H}} \rangle \\ -\lambda \langle \mathbf{45_H} \rangle & M_{10} & 0 \\ 0 & 0 & M_{16} \end{pmatrix} \begin{pmatrix} \mathbf{5}_{10_H} \\ \mathbf{5}_{10'_H} \\ \mathbf{5}_{\overline{16_H}} \end{pmatrix} . \tag{33}$$

With the vacuum expectation value $\langle \mathbf{45_H} \rangle$ in the $B - L$ direction it does not contribute to the doublet matrix, so one pair of Higgs doublet remains light, while all triplets acquire unification scale masses. The light MSSM Higgs doublets are

$$H_u = \mathbf{10}_u, \; H_d = \cos \gamma \, \mathbf{10}_d + \sin \gamma \, \mathbf{16}_d , \tag{34}$$

with $\tan \gamma \equiv \lambda' \langle \overline{\mathbf{16_H}} \rangle / M_{16}$. Consequently, $\langle \mathbf{10} \rangle_d = \cos\gamma \, v_d, \langle \mathbf{16}_d \rangle = \sin\gamma \, v_d$, with $\langle H_d \rangle = v_d$ and $\langle \mathbf{16}_d \rangle$ and $\langle \mathbf{10_d} \rangle$ denoting the electroweak VEVs of those multiplets. Note that the H_u is purely in $\mathbf{10_H}$ and that $\langle \mathbf{10}_d \rangle^2 + \langle \mathbf{16}_d \rangle^2 = v_d^2$.

This pattern of gauge symmetry breaking, while motivated on separate grounds, nicely harmonizes with the fermion mass pattern advocated in Secs. IV and V. Specifically, the g_{ij} terms will contribute to the down–flavored fermion masses, while there are no analogous terms in the up–flavor sector. We also note the relation $\tan\gamma \, \tan\beta \simeq m_t/m_b \simeq 60$. Since $\tan \beta$ is separately observable, the angle γ, which will prove relevant for proton decay, is thereby anchored.

B. Baryon number violation

By combining Eqs. (32)-(34) with the Yukawa couplings from Eq. (14), we can now obtain the effective baryon number violating superpotential. Denote the color triplet and anti–triplet in $\mathbf{10_H}$ as $(H_C, \overline{H}_C)$, in $\mathbf{10'_H}$ as $(H'_C, \overline{H}'_C)$ and in $(\overline{\mathbf{16_H}}, \mathbf{16_H})$ as $(\hat{H}_C, \hat{\overline{H}}_C)$. The relevant Yukawa couplings of these color triplets, extracted from Eq. (14) are[3] (i) $h_{ij}(\frac{1}{2} Q_i Q_j H_C + Q_i L_j \overline{H}_C)$, (ii) $-a_{ij}[\frac{1}{2} Q_i Q_j H_C (B - L)_{Q_j} + Q_i L_j \overline{H}_C (B - L)_{L_j}]$, (iii) $(g_{ij} \langle \mathbf{16_H} \rangle / M) Q_i L_j \hat{\overline{H}}_C$, and (iv) $(f_{ij} \langle \overline{\mathbf{16_H}} \rangle / M) Q_i Q_j \hat{H}_C$. Note that for the a_{ij} coupling there are two contractions: $\mathbf{10_H} \times \mathbf{45_H} \supset \mathbf{10} + \mathbf{120}$. Whereas the antisymmetric $\mathbf{120}$ contributes to fermion masses (for $\langle \mathbf{45_H} \rangle \propto B - L$), as in Eq. (15), it is the symmetric $\mathbf{10}$ that leads to the Yukawa couplings of H_C and $\overline{H}_C$. For concreteness, we have assumed that the a_{ij} term arises by integrating out a pair of superheavy $\mathbf{16} + \overline{\mathbf{16}}$ states which couple through the renormalizable interactions $W \supset (\mathbf{16_2} \mathbf{16}) \mathbf{10_H} + (\mathbf{16_3} \overline{\mathbf{16}}) \mathbf{45_H} + M_V \mathbf{16} \overline{\mathbf{16}}$.[4] This is why the $(B - L)$ generator appears in the a_{ij} couplings involving $H_C, \overline{H}_C$. The strength of this coupling is then fixed in terms of the corresponding doublet coupling. Similarly, in the f_{ij} coupling, there are two $SO(10)$ contractions, and we shall assume both to have comparable strength.

Baryon number violating operators of the type $QQQL/M$ are the ones that connect to ordinary quarks and leptons by wino dressing, and which generally dominate (but see below).

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[3] Here we have absorbed the factor $\langle \mathbf{45_H} \rangle / M$ into a_{ij} and the factor $\langle \mathbf{16_H} \rangle / M$ into g_{ij}. We use the same notation (a_{ij} and g_{ij}) for these redefined quantities. As for f_{ij}, we define $\hat{f}_{ij} \equiv f_{ij} \langle \overline{\mathbf{16_H}} \rangle / M$.

[4] A variant is to interchange the indices 2 and 3 in this renormalizable interaction.

Integrating out the color triplet fields, one arrives at the following effective superpotential terms involving these operators:

$$
\begin{aligned}
W_{\text{eff}}^{(L)} =\ & M_{\text{eff}}^{-1}[(u^T\hat{H}d')\left\{u^T\hat{H}V'\ell - d'^T\hat{H}V'\nu'\right\} + 3(u^T\hat{H}d')\left\{u^T\hat{A}V'\ell - d'^T\hat{A}V'\nu'\right\} \\
& - (u^T\hat{A}d')\left\{u^T\hat{H}V'\ell - d'^T\hat{H}V'\nu\right\} - 3(u^T\hat{A}d')\left\{u^T\hat{A}V'\ell - d'^T\hat{A}V'\nu'\right\} \\
& - \tan\gamma(u^T\hat{H}d')\left\{u^T\hat{G}V'\ell - d'^T\hat{G}V'\nu'\right\} + \tan\gamma(u^T\hat{A}d')\left\{u^T\hat{G}V'\ell - d'^T\hat{G}V'\nu'\right\}] \\
& + M_{16}^{-1}(u^T\hat{F}d')\left\{u^T\hat{G}V'\ell - d'^T\hat{G}V'\nu'\right\}\ .
\end{aligned}
\tag{35}
$$

Here $M_{\text{eff}} = (\lambda a)^2/M_{10}$. u and ℓ denote the column matrices of the physical left–handed up quark and charged lepton superfields in the supersymmetric basis (i.e., a basis in which neutral gaugino interactions are flavor diagonal). The d' and ν' fields are related to the physical down quark and light neutrino fields by the CKM matrices for quarks and leptons: $d' = V_{CKM}d$ and $\nu' = V_{CKM}^\ell\nu$, while $V' = V_u^\dagger V_\ell$, where V_u and V_ℓ diagonalize respectively the left–handed up quark and charged lepton mass matrices: $u^{(g)} = V_u u^{(m)}$, where (g) and (m) denote the gauge and mass eigenstates. In writing Eq. (35), the color indices (α, β, γ) on quark fields are suppressed and use is made of the fact that $(u_\alpha^T\hat{H}d'_\beta - d'^T_\alpha\hat{H}u_\beta) = 2u_\alpha^T\hat{H}d'_\beta$, which holds because of antisymmetry under the interchange $\alpha \leftrightarrow \beta$ and because $\hat{H}$ is symmetric. The superscript (L) on $W_{\text{eff}}^{(L)}$ signifies that all the fields in $W_{\text{eff}}^{(L)}$ belong to $SU(2)_L$ doublets. We comment later on the contributions from $W_{\text{eff}}^{(R)}$, involving $RRRR$ operators of the form $u^c u^c d^c e^c$, involving $SU(2)_L$ singlets which can be important in certain range of supersymmetric parameter space.

The 3×3 matrices $(\hat{H}, \hat{A}, \hat{G}, \hat{F})$ operate in the family space and are related to the Yukawa coupling matrices given by the elements $(h_{ij}, a_{ij}, g_{ij}, f_{ij})$ respectively, as follows:

$$
(\hat{H}, \hat{A}, \hat{G}, \hat{F}) = V_u^T(h, a, g, f)V_u\ .
\tag{36}
$$

The matrices (h, a, g, f) are related to those appearing in Eqs. (28)- (29) and are given by:

$$
h = h_{33}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix}\ ,\quad a = h_{33}\begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix}\ ,
$$

$$
g = \frac{h_{33}}{\tan\gamma}\begin{pmatrix} 0 & \eta' & 0 \\ \eta' & 0 & \eta - \sigma \\ 0 & \eta - \sigma & 0 \end{pmatrix}\ ,\quad f = \hat{f}_{33}\begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix}\ ,
\tag{37}
$$

where $\hat{f}_{ij} = f_{ij}v_R/M$. Values of $\sigma, \eta, \epsilon, \eta', \epsilon'$ and y have been obtained in Sec. IV-V from considerations of fermion masses and mixings. From the neutrino sector we have estimated $x \sim (10^{-5} - 10^{-3})$ and $z \sim (0.003 - 0.03)$. V_u can be worked out using Eq. (28) and (21), and is found to be close to an identity matrix. Its largest off–diagonal entry is the (1,2) element $\simeq \epsilon'/(\epsilon^2 - \sigma^2) \simeq -0.06$. However the matrix V_ℓ that diagonalizes the charged lepton mass matrix L is far from trivial, and so $V' = V_u^\dagger V_\ell$ picks up a substantial (2,3) element $\simeq -3\epsilon + \eta \simeq -0.437$, reflecting sizable $\mu - \tau$ mixing. The numerical values of the matrices V_u, V_ℓ and V' are given in Appendix A, as are the matrices $(\hat{H}, \hat{A}, \hat{G}, \hat{F})$.

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Even if one is skeptical of our particular pattern of fermion mass matrices, it is difficult to avoid the general conclusion[5] that if neutrino masses are hierarchical, a large $\nu_\mu - \nu_\tau$ oscillation angle (the SuperK result) requires a sizable $\mu - \tau$ mixing angle (say ≥ 0.3). This has significant implications for proton decay, as we now discuss.

After wino dressing, which dominates over gluino dressing for $\tan\beta \leq 20$, each of the seven terms in Eq. (35) leads to twelve four fermion operators for proton decay into $\overline{\nu} + X$, a subset of which was exhibited in Ref. [12]. In Appendix B.1 we give the complete expression for the neutrino as well as the charged lepton decay modes of the proton. Representative contributions to the proton decay amplitudes are analyzed in more detail for the dominant modes in Appendix B.2. We now briefly summarize the results of a lengthy investigation of the net effect of all these terms.

To evaluate the strength of each term in Eq. (35), we need h_{33} and $\hat{f}_{33}$ (see Eq. (37)) at $M_{\rm U}$. h_{33} is determined using $h_{33}v_u \simeq m_t(M_{\rm U}) \simeq 100 - 120$ GeV, which yields $h_{33} \simeq 1/2$. $\hat{f}_{33}$ can be determined as follows. Using Eq. (23) for the mass matrix of the light $\nu_\mu - \nu_\tau$ sector, we have

$$m_{\nu_3} \simeq B m_U^2/M_R, \tag{38}$$

where

$$B = -(\sigma + 3\epsilon)(\sigma + 3\epsilon - 2y) \simeq 5. \tag{39}$$

Here we have put σ and ϵ from Eq. (21) and used $y = 0.047$, corresponding to $m_{\nu_2}/m_{\nu_3} = 1/15$. Putting $m_{\nu_3} = (1/20 \text{ eV})\zeta$, where $\zeta = 2$ to $1/2$, corresponding to SuperK results, and $m_U \simeq m_t(M_{\rm U})$, we get $M_R \simeq 10^{15}$ GeV$/\zeta$. Using $M_R = f_{33}v_R^2/M_{\rm Planck}$ (see Eq. (4)), with $v_R = \langle \mathbf{16_H} \rangle \approx 2 \times 10^{16}$ GeVκ_R and $\kappa_R \approx 1/2$ to 2, we find

$$\hat{f}_{33} = f_{33}v_R/M_{\rm Planck} \simeq (1/20)(1/\kappa_R\zeta). \tag{40}$$

We will use $\hat{f}_{33} \approx 1/20$ with the understanding that it is uncertain by a factor of 2-3 either way. Note that $\hat{f}_{33}$ is considerably larger, by about a factor of 200–700, than the value estimated in Ref. [12]. This results partly from lowering of m_{ν_τ} from a few eV (used in Ref. [12]) to about 1/20 eV (the SuperK value) and in part from interplay between the mixings in the Dirac and the Majorana mass matrices via the seesaw mechanism. The latter has the net effect of enhancing B by a factor ≈ 5, and thereby $M_R \simeq B m_U^2/m_{\nu_3}$, and in turn $\hat{f}_{33}$.

Note that the net value of B (see Eq. (39)) depends on the parameters of the Majorana mass matrix as well as on σ and ϵ from the Dirac mass matrix of the neutrinos, which in turn are determined within $SO(10)$ by the masses and mixings of quarks and charged leptons. That is why our expectations for proton decay are significantly affected by our understanding of the masses and mixings of quarks and charged leptons.

The full set of contributions to the proton decay amplitudes (from all the operators in Eq. (35) involving all possible combinations of family indices) was obtained numerically using

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[5]Barring near-complete accidental cancellation between diagonal and off–diagonal contributions to ν_μ.

Mathematica and is listed in Appendix B.1. Calculations of a few representative (dominant) contributions to the amplitudes are exhibited in detail in Appendix B.2, where estimates of the amplitudes allowing for uncertainties in the relative phases of different contributions are presented. A general discussion of proton decay rate is given in Appendix C. Based on the result of these appendices, we now discuss the general constraint on the proton decay amplitude and thereby on the mass scale M_{eff} and $M_{16} \tan \gamma$ corresponding to the numerical estimate of the amplitude given in Appendix B. These constraints arise from existing lower limits on the proton lifetime.

C. Constraints on the proton decay amplitude from proton lifetime

In Appendix C we have shown that with a certain (apparently) reasonable choice of supersymmetric spectrum, the lifetime of the proton decaying into neutrinos is:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+) \approx (2.2 \times 10^{31}) \text{ yrs} \times$$

$$\left(\frac{.67}{A_S}\right)^2 \left[\frac{0.006 \text{ GeV}^3}{\beta_H}\right]^2 \left[\frac{(1/6)}{(m_{\tilde{W}}/m_{\tilde{q}})}\right]^2 \left[\frac{m_{\tilde{q}}}{1 \text{ TeV}}\right]^2 \left[\frac{2 \times 10^{-24} \text{ GeV}^{-1}}{\hat{A}(\overline{\nu})}\right]^2 . \quad (41)$$

Here $\hat{A}(\overline{\nu}) = A(\overline{\nu})/(2\overline{f})$, where $A(\overline{\nu})$ is the strength of the four fermion proton decay amplitude and $\overline{f}$ is the average wino–dressing function (see Eq. (78) in Appendix B and Eq. (90)-(91) in Appendix C). The quantity $\hat{A}(\overline{\nu})$ is simply the product of all the vertex factors in the wino–dressed Higgsino exchange diagram divided by the effective mass of the relevant color triplet Higgsino. For normalization purpose we can define $\hat{A}(\overline{\nu})^{\text{SU(5)}} = (\lambda_c \lambda_s \theta_C^2)/M_{H_C}$, where λ_i stand for the Yukawa couplings of the quarks at M_{U}. (For clarity of discussions, only the second generation contribution is kept here.) The quantity $A(\overline{\nu})$ is the full amplitude, including the loop factor associated with the wino dressing. If we substitute $\hat{A}(\overline{\nu})^{\text{SU(5)}}$ defined above into Eq. (41), we will reproduce the results given in Ref. [33]. (We have allowed for a factor of 4 enhancement in the lifetime relative to [33], corresponding to an apparent slip by a factor of $\frac{1}{2}$ in going from Eq. (3.7) to Eq. (3.8) of that paper.)

Note that in writing Eq. (41), the short distance renormalization of the $d = 5$ operator in going from M_{U} to M_{SUSY} (denoted by A_S in Eq. (41), see Ref. [33]) as well as the running factor (A_L) to go from M_{SUSY} to 1 GeV have been included. A_S has a central value of about 0.67, which we shall adopt even for the $SO(10)$ model.

We see that with $\hat{A}(\overline{\nu}) \approx 2 \times 10^{-24}$ GeV^{-1}, a reasonable "central value" for the partial lifetime $\Gamma^{-1}(p \to \overline{\nu} K^+)^{-1}$ is 2.2×10^{31} yrs., which corresponds to the hadronic matrix element $\beta_H = 0.006$ GeV^3 (this is the central value quoted in one lattice calculation [39]), $(m_{\tilde{W}}/m_{\tilde{q}}) \approx 1/6$ and $m_{\tilde{q}} \approx 1$ TeV. This is however 25 times smaller than the empirical lower limit [40]

$$\tau(p \to \overline{\nu} K^+)_{\text{expt}} \geq 5.6 \times 10^{32} \text{ yr.} \quad (42)$$

Allowing that both β_H and the ratio of superpartner masses $(m_{\tilde{W}}/m_{\tilde{q}})$ might well be smaller by factor of 2 (say) than the value quoted above, and that $m_{\tilde{q}}$ could be (say) 1.4 TeV rather than 1 TeV, the theoretical value of the lifetime could plausibly increase by a factor of 32 compared to the "central value" 2.2×10^{31} yr. Comparing with the limit in Eq. (42), we find the limit

$$\hat{A}(\overline{\nu}_i) \leq 2.3 \times 10^{-24} \text{ GeV}^{-1} \tag{43}$$

should be satisfied for every neutrino flavor ν_i. For squark masses not exceeding about 1.5 TeV, we take Eq. (43) as a conservative upper bound. It should however be noted that the lifetime depends quartically on the squark mass, so increasing $m_{\tilde{q}}$ by a factor of 2 to about 3 TeV, would lengthen proton lifetime by a factor of 16. This would relax the bound on $\hat{A}(\overline{\nu})$ quoted in Eq. (43) by a factor of 4 to $\hat{A}(\overline{\nu}) \leq 9 \times 10^{-24}$ GeV^{-1}. Such a heavy spectrum ($m_{\tilde{q}} \sim 3$ TeV) for all three generations of squarks would require severe fine–tuning of parameters in order to keep the vacuum expectation value of the light Higgs field at the electroweak scale. We shall assume a relatively light squark spectrum ($m_{\tilde{q}} \leq 1.5$ TeV), for which there is no need for such an adjustment of parameters. In this case, the bound in Eq. (43) will have to be satisfied, which we shall assume. In writing Eq. (43), we have allowed for the fact that both $\overline{\nu}_\tau K^+$ and $\overline{\nu}_\mu K^+$ modes are expected to be prominent (as discussed in Appendix B).

D. Constraints on M_{eff} and proton decay via standard operators

In Appendix B, Eq. (86), we show that within the concrete $SO(10)$ model, the decay amplitude from the standard $d = 5$ operator is given by:

$$A(\overline{\nu}_\tau K^+)_{\text{std}} \simeq \left[\frac{2h_{33}^2 \hat{f}(c,d)}{M_{\text{eff}}} \right] \eta_{cd} \eta_{\epsilon'} \eta_{cb} (2.8 \times 10^{-5})(\frac{1}{2} \text{ to } \frac{3}{2}) \epsilon_{\alpha\beta\gamma} (d^\alpha u^\beta)(s^\gamma \nu_3) . \tag{44}$$

There is an analogous expression for the new neutrino mass–related $d = 5$ operator, that will be discussed in the next subsection (E).

We now compare the upper limit (Eq. (43)) on the amplitude from proton decay searches against theoretical expectations based on the concrete $SO(10)$ model. This leads to constraints on M_{eff} from the standard $d = 5$ operator (and on $M_{16} \tan \gamma$ from the new operator). The upper bound (Eq. (43)) on $\hat{A}(\overline{\nu}_i)$ applies to the net amplitude, which is given by the sum of the contributions from the standard (Eq. (44)) and the new operators (Eq. (51) below). For the sake of clarity, we will derive constraints on M_{eff} ($M_{16} \tan \gamma$) under the assumption that the standard (respectively, the new) operator dominates. Indeed, near-complete accidental cancellation between the two contributions is unlikely to occur for both $\overline{\nu}_\tau K^+$ and $\overline{\nu}_\mu K^+$ modes.

Using the net contribution from the standard operators to the amplitude given by Eq. (44), and the definition of $\hat{A}(\overline{\nu}_i)$, we obtain (putting $h_{33} \simeq 1/2$)

$$\hat{A}(\overline{\nu}_\tau)_{\text{std}} \approx \frac{(7 \times 10^{-6})}{M_{\text{eff}}} (1/2 \text{ to } 3/2) . \tag{45}$$

Comparing with the empirical upper limit on $\hat{A}(\overline{\nu}_i)$ (Eq. (43)) obtained as above, we get:

$$M_{\text{eff}} \geq 3.0 \times 10^{18} \text{ GeV } (1/2 \text{ to } 3/2) . \tag{46}$$

Thus we see that M_{eff} has to be rather large compared to the MSSM unification scale of 2×10^{16} GeV in order that the standard operators may not run into conflict with the

observed limits on proton lifetime. In effect, this reflects a net enhancement – by almost two orders of magnitude – of the standard $d = 5$ proton decay operators for realistic $SO(10)$, compared to those in minimal $SU(5)$, with low $\tan \beta \leq 3$. In the latter case, one need only require that the color triplet mass exceed 2×10^{16} GeV.

Now, as mentioned in Appendix C, there are theoretically attractive mechanisms whereby the mass of $\mathbf{10'}_H$, denoted by M_{10} (see Eq. (32)), can be suppressed relative to the unification scale M_U. In this case, $M_{\text{eff}} \equiv (\lambda a)^2/M_{10}$ can be larger than $\lambda a \sim M_U$. Very large values of $M_{\text{eff}} \gg M_U$ could however lead to large *positive* corrections to $\alpha_3(M_Z)$, just from the doublet–triplet mechanism, above and beyond the value expected on the basis of simple coupling unification. For the doublet–triplet splitting mechanism described by Eq. (33) the shift in $\alpha_3(m_Z)$ from this sector alone is found to be

$$\Delta \alpha_3(m_Z)|_{DT} = \frac{[\alpha_3(m_Z)]^2}{2\pi} \frac{9}{7} \ln \left(\frac{M_{\text{eff}} \cos \gamma}{M_U} \right) \ . \tag{47}$$

This generalizes the expression given in Ref. [12,41], where the MSSM Higgs doublets were assumed to be contained entirely in $\mathbf{10}_H$ and not in $\mathbf{16}_H$, corresponding to $\cos \gamma = 1$. The argument of the logarithm in Eq. (47) is simply the ratio: (product of the three color triplet masses)/ (product of the two superheavy doublet masses $\times M_U$). From the determinant of Eq. (33) we see that the product of the color triplet masses is equal to $M_{\text{eff}} M_{10} M_{16}$, while the two heavy doublets have masses given by M_{10} and $M_{16}/\cos \gamma$. Note that the second heavy doublet has a mass larger than M_{16}. This is the reason for the presence of the $\cos \gamma$ factor in Eq. (47).

To evaluate the RHS of Eq. (47), we use the value of M_{eff} that is suggested by proton lifetime constraints, viz., $M_{\text{eff}} \geq (1-5) \times 10^{18}$ GeV (see Eq. (46)), and the MSSM unification scale of $M_U \simeq 2 \times 10^{16}$ GeV. We should also specify the value of $\cos \gamma$. It is obtained in terms of $\tan \beta$ as follows. From $m_t \simeq h_{33} \langle \mathbf{10_H} \rangle_u = h_{33} v_u$ and $m_b \simeq h_{33} \langle \mathbf{10_H} \rangle_d = h_{33} \cos \gamma v_d$, we have $m_t/m_b \simeq (v_u/v_d)(1/\cos \gamma)$. Inserting $m_t/m_b \simeq 60$, we thus obtain $\cos \gamma \simeq (\tan \beta/60)$. (This can also be expressed as $\tan \beta \tan \gamma \simeq m_t/m_b$, which is valid for $\tan \gamma \geq 3$.) We see that $\cos \gamma$ is a small number for small and moderate values of $\tan \beta$. We will argue in Sec. VI.G that $\tan \beta$ cannot be much larger than about $(10 - 20)$ in this class of models, or else proton lifetime arising through the charged Higgsino dressing diagram (which scales as $(\tan \beta)^{-2}$) will be too short and will conflict with experimental limits. Thus we take as a reasonable range $\tan \beta = (3-20)$, which corresponds to $\cos \gamma = (1/20 - 1/3)$, or equivalently to $\tan \gamma = (20 - 3)$.

Now, let us recall that, in the absence of unification–scale threshold and Planck–scale effects, the MSSM value of $\alpha_3(m_Z)$ in the $\overline{\text{MS}}$ scheme, obtained by assuming gauge coupling unification, is given by $\alpha_3^0(m_Z)|_{MSSM} = 0.125 - 0.13$ [7,42]. This is about 5-10% higher than the observed value: $\alpha_3(m_Z) = 0.118 \pm 0.003$ [25]. Substituting a value of $\alpha_3(M_Z)$ intermediate between these two – i.e., $\alpha_3(M_Z) = 0.12$ – and the MSSM unification scale of $M_U = 2 \times 10^{16}$ GeV, one obtains for $\cos \gamma = 1/20$: $\Delta \alpha_3(m_Z)|_{DT} \simeq 0.0027 - 0.0074$, for $M_{\text{eff}} \approx (1 - 5) \times 10^{18}$ GeV. Corresponding to $\cos \gamma = 1/3$ we find $\Delta \alpha_3(m_Z)|_{DT} \simeq 0.0083 - 0.0130$, for the same range of $M_{\text{eff}} = (1 - 5) \times 10^{18}$ GeV. Thus the constraint on M_{eff} from proton lifetime amounts to having in MSSM a net value $\alpha_3(m_Z)|_{net} = \alpha_3(M_Z)^{(0)}|_{MSSM} + \Delta \alpha_3(M_Z)^{MSSM}_{DT} + \Delta'_3 \simeq (0.128 - 0.132) + \Delta'_3$, for $\cos \gamma = 1/20$, where Δ'_3 denotes other unification scale threshold and Planck scale effects. For $\cos \gamma = 1/3$

the net value is $\alpha_3(m_Z)|_{net} = \alpha_3(M_Z)^{(0)}|_{MSSM} + \Delta\alpha_3(M_Z)^{MSSM}_{DT} + \Delta'_3 \simeq (0.133 - 0.138) + \Delta'_3$. Thus if coupling unification should hold, one must assume, for the case of MSSM embedded in $SO(10)$, that other unification scale threshold and Planck scale effects denoted by Δ'_3 provide a nearly 7-10% (11-15%) *negative* contribution to $\alpha_3(M_Z)$ corresponding to $\cos\gamma = 1/20$ (1/3). Threshold corrections of such magnitude from unification scale and Planck scale effects are quite natural, especially the corrections required for small values of $\cos\gamma$ ($\sim 1/20$).

It should be emphasized that the presence of $\cos\gamma$ inside the logarithm of Eq. (47) has played a significant role in diminishing the positive threshold correction to $\alpha_3(m_Z)$. In its absence, negative threshold corrections as large as 15-20% would have been required from other unification scale and Planck scale effects.

For larger values of $\cos\gamma$ ($\sim 1/3$) (corresponding to large $\tan\beta \sim 20$) and for large $M_{eff} \sim 5 \times 10^{18}$ GeV, a 15% negative threshold correction is required from other sources. Although such a large correction is in principle possible, (see e.g. Ref. [43]), it diminishes the luster of the observed agreement of simple coupling unification, by making it appear somewhat fortuitous. In this connection it is noteworthy that extra vector–like matter – specifically a $\mathbf{16 + \overline{16}}$ as proposed for ESSM (Extended Supersymmetric Standard Model) [44] – at the TeV scale could greatly ease this problem which may arise for special choices of parameters as mentioned above, while leaving our discussion of ordinary fermion masses essentially unaltered. In this case, α_{unif} is raised to nearly 0.25 to 0.3, compared to 0.04 in the MSSM. Owing to increased two–loop effects, the scale of unification M_U is raised to $(1-2) \times 10^{17}$ GeV [44,45], while $\alpha_3(M_Z)$ is lowered to about $0.112 - 0.118$. With increased M_U the correction $\Delta\alpha_3(M_Z)|_{DT}$ is also lowered. As a result, even for $\cos\gamma = 1/3$ and $M_{eff} \approx 5 \times 10^{18}$ GeV, one obtains, for the case of ESSM, the net value of $\alpha_3(m_Z) = \alpha_3(m_Z)^{(0)}|_{ESSM} + \Delta\alpha_3(m_Z)|_{DT} + \Delta'_3 \simeq (0.118 - 0.126) + \Delta'_3$. Thus for ESSM embedded in $SO(10)$, proton lifetime constraint on M_{eff} requires only a small correction to $\alpha_3(M_Z)$ ($\Delta'_3/\alpha_3(m_Z) \leq 5\%$) from threshold and Planck scale effects, even for this extreme choice of parameters ($M_{eff} \simeq 5 \times 10^{18}$ GeV, $\tan\beta \simeq 20$).

For the case of MSSM, assuming that the correction to $\alpha_3(m_Z)$ from threshold and Planck scale effects are less than about 5-8%, and allowing for $\cos\gamma \simeq 1/20$, we find that $M_{eff} \leq 4 \times 10^{18}$ GeV. For the case of ESSM, with $\cos\gamma \simeq 1/3$, we would obtain the same limit, $M_{eff} \leq 4 \times 10^{18}$ GeV, if the threshold corrections are smaller than about 8%. Now, substituting $M_{eff} \leq 4 \times 10^{18}$ GeV in Eq. (45), we find:

$$\hat{A}(\overline{\nu}_\tau)_{st} \geq (1.7 \times 10^{-24} \text{ GeV}^{-1})\{\frac{1}{2} \text{ to } \frac{3}{2}\} . \tag{48}$$

Substituting Eq. (48) into expression for proton lifetime (Eq. (41)), we thus obtain:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+)_{std} \leq (1.4 \times 10^{32} \text{ yrs.})\{1 \text{ to } \frac{1}{9}\}[32 \text{ to } \frac{1}{32}] . \tag{49}$$

Here the curly bracket corresponds to uncertainty in $\hat{A}(\overline{\nu}_\tau)_{std}$ (see Eq. (41)), while the square bracket corresponds to variations in $\beta_H, (m_{\tilde{W}}/m_{\tilde{q}})$ and $m_{\tilde{q}}$ by factors of 2, 2 and $\sqrt{2}$ respectively, either way, around the central values, reflected in Eq. (41). We thus see that for ESSM, and even more so for MSSM, the inverse partial proton decay rate should satisfy:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+)_{\text{std}} \leq 1.4 \times 10^{32^{+1.5}_{-2.5}} \text{yrs},$$

$$\leq 4.5 \times 10^{33} \text{ yrs.} \tag{50}$$

Improving the current limit ($\Gamma^{-1}(p \to \overline{\nu} K^+)_{\text{expt}} \geq 5.6 \times 10^{32}$ yrs. [40]) by a modest factor, 5 to 10, should either reveal proton decay or essentially exclude the existence of the standard $d = 5$ interaction that arises in $SO(10)$.

E. Constraint on $M_{16} \tan\gamma$ and proton decay via the new operator

In Appendix C we show, in the context of $SO(10)$ or an analogous G_{224} model, that the proton decay amplitude from the new $d = 5$ operator related to neutrino masses must satisfy $\hat{A}(\overline{\nu}) \leq 2.3 \times 10^{-24}$ GeV^{-1}. The decay amplitude for the new operator is given by:

$$A(\overline{\nu}_\tau K^+)_{\text{new}} \simeq \left(\frac{\hat{f}_{33} h_{33}}{M_{16} \tan\gamma} \right) \left[(1.3 \times 10^{-6})(1/2.5 \text{ to } 2.5) \right] \left[f(t,d) + f(t,l) \right] \times$$

$$\epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) . \tag{51}$$

From the definition of $\hat{A}(\overline{\nu})$ we obtain:

$$\hat{A}(\overline{\nu}_\tau)_{\text{new}} \approx \left[\frac{\hat{f}_{33} h_{33}}{M_{16} \tan\gamma} \right] \left[(1.3 \times 10^{-6})(1/2.5 \text{ to } 2.5) \right] \tag{52}$$

$$\approx \left[\frac{1/(40\kappa_R \zeta)}{M_{16} \tan\gamma} \right] \left[(1.3 \times 10^{-6})(1/2.5 \text{ to } 2.5) \right] . \tag{53}$$

In going from Eq. (52) to Eq. (53), we have substituted $h_{33} \simeq 1/2$ and the value of $\hat{f}_{33} \equiv f_{33} v_R/M_{\text{Pl}} \simeq (M_R/v_R) \simeq 1/(20\kappa_R \zeta)$ obtained in Sec. V, where $\kappa_R = (1/2 \text{ to } 2)$ and $\zeta = (1/2 \text{ to } 2)$ denote the uncertainties in v_R and in m_{ν_3} – i.e., $v_R = (2 \times 10^{16} \text{ GeV})\kappa_R$ and $m_{\nu_3} = \zeta(1/20)$ eV. Using the upper limit given by Eq. (43), and assuming that the new operator dominates, we obtain:

$$M_{16} \tan\gamma \geq (1.4 \times 10^{16} \text{ GeV}) \left[\frac{(1/2.5 \text{ to } 2.5)}{\kappa_R \zeta} \right] . \tag{54}$$

Thus we see that the new operators can be compatible with limits on proton lifetime for $M_{16} \tan\gamma \approx 2 \times 10^{16}$ GeV(1 to 3). The relation $M_{16} \tan\gamma = \lambda' v_R = (2 \times 10^{16} \text{ GeV})(\lambda' \kappa_R)$, combined with the perturbativity requirement on λ' ($\lambda' \leq 1$), and the numerical estimate $\kappa_R \zeta = 2 \text{ to } 4^6$ suggests that $M_{16} \tan\gamma \simeq 2 \times 10^{16}$ GeV (1 to 3), where we have allowed a

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[6]By using $\hat{f}_{33} \equiv f_{33} v_R/M_{\text{Pl}}$ and $M_{16} \tan\gamma = \lambda' v_R$, one gets $\psi \equiv \hat{f}_{33}/(M_{16} \tan\gamma) \approx (f_{33}/\lambda')(2 \times 10^{18} \text{ GeV})^{-1}$. Alternatively, using $\hat{f}_{33} = 1/(20\kappa_R \zeta)$ and $M_{16} \tan\gamma = \lambda' v_R = (\lambda' \kappa_R)(2 \times 10^{16} \text{ GeV})$, one obtains $\psi = [\hat{f}_{33}/(M_{16} \tan\gamma)] \approx (\lambda' \kappa_R^2 \zeta)^{-1}(4 \times 10^{17} \text{ GeV})^{-1}$. Equating the two values of ψ thus obtained, one finds $f_{33} \approx 5/(\kappa_R^2 \zeta)$. On the one hand, values of $f_{33} \gg 1$ are implausible, thus we expect $\kappa_R \sqrt{\zeta} > 1$. On the other hand, from the success of our estimate of m_{ν_τ}, we expect f_{33} to be not much smaller than one either. Taking $f_{33} \approx (1 \text{ to } 1/3)$, we obtain $\kappa_R^2 \zeta \approx (5 \text{ to } 15)$. Keeping ζ at its central value of unity (corresponding to $m_{\nu_3} \approx 1/20$ eV) we expect $\kappa_R \zeta = (2 \text{ to } 4)$.

range $\lambda' = (0.7 - 1)$. (For smaller values of λ' the new $d = 5$ operators will be even more significant.) To illustrate the lifetimes that the new operators by themselves might generate we insert in Eq. (54) $M_{16} \tan \gamma \approx 2 \times 10^{16}$ GeV(1 to 3) and $\kappa_R \zeta =2$ to 4. Then

$$\hat{A}(\overline{\nu}_\tau)_{\text{new}} \approx (10^{-24} \text{ GeV})^{-1}(2.5 \text{ to } 1/15) \tag{55}$$

and using Eq. (41) we find:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+)_{\text{new}} \approx (6 \times 10^{32} \text{ yrs}) \left[(1/6 \text{ to } 200) \times 10^{\pm 1.5} \right] . \tag{56}$$

Here the first factor inside the square bracket corresponds to the uncertainty in the amplitude reflected by Eq. (41) and the second factor $10^{\pm 1.5}$ corresponds to varying β_H and $(m_{\tilde{W}}/m_{\tilde{q}})$ by a factor of two around 0.006 GeV^3 and 1/6 respectively, and to letting $m_{\tilde{q}}$ vary within a factor of 1.5 either way around 1 TeV.

For nearly "central values" of all relevant entries – i.e., (a) the amplitude $\hat{A}$ (see Appendix B), (b) the masses of the supersymmetric partners: $m_{\tilde{q}} \approx 1$ TeV, $m_{\tilde{W}} \approx (1/6)m_{\tilde{q}}$, and (c) $\beta_H \approx 0.006$ GeV^3, the new operators related to neutrino masses lead by themselves to proton decay lifetimes centered near current experimental limits ($\Gamma^{-1}(p \to \overline{\nu}K^+) \geq 5.6 \times 10^{32}$ yrs [40]). Thus if the new operators were the only source of proton decay – i.e., if the standard $d = 5$ operators were somehow absent – one could be comfortably compatible with existing limits, but optimistic regarding future observation. We now briefly explore this possibility.

F. The neutrino mass related operator as the sole source of proton decay

As we have seen, straightforward minimal embedding of the MSSM in $SO(10)$, with informed hypotheses about the fermion-Higgs couplings, leads to contributions from the standard $d = 5$ operators that are disturbingly large. One can relieve this difficulty by accepting a large M_{eff}, as discussed previously. Still, one is motivated to wonder whether the standard operators might not be present. This possibility is realized, if the higher gauge symmetry is G_{224} (or $G_{2113} = SU(2)_L \times I_{3R} \times (B - L) \times SU(3)^c$) rather than $SO(10)$. Such gauge symmetries have been proposed to appear in solutions of string theory (see Ref. [46] for G_{224} and Ref. [50] for G_{2113}). Such possibilities retain some attractive features of $SO(10)$, notably the unification of quark-lepton families into single multiplets including the right-handed neutrino ν^R as indicated for the neutrino seesaw. One sacrifices a simple group-theoretic explanation for the observed unification of coupling, but if the models derive from an underlying string theory, one might still expect such unification at the string scale. Plausible mechanisms to reconcile the string and the MSSM unification scale have been proposed [48]. The color triplets related to the electroweak doublets, which generate the standard $d = 5$ proton decay operators, need not exist. The standard source of $d = 5$ operators can be absent in such models [49].

It is possible that after projecting out appropriate fields the couplings of those that remain reflect the original higher symmetry[7]. This is what is believed to occur for the gauge

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[7]For example, in a class of string solutions leading to G_{2113}, the cubic level top and ν_τ Yukawa couplings are claimed to be equal at the string scale despite $SU(4)_C$ breaking [50].

couplings, as previously mentioned. If it also occurs for the Higgs superpotential couplings, our considerations on fermion and neutrino masses in Sec. II-IV and their relationships to proton decay will remain valid.

G. Baryon number violation from the $RRRR$ operator

So far we have focused on the charged wino dressing of the effective superpotential $W_{\text{eff}}^{(L)}$ of Eq. (35). For small values of $\tan\beta$ and μ, this gives the dominant contribution to the proton decay amplitude. However, if $\mu\tan\beta$ is large, (≥ 2 TeV), dressing of the effective baryon number violating operator involving only the right–handed fields, $u^c u^c d^c e^c$, by the charged Higgsino becomes important [51–53]. The ratio of this amplitude to the usual wino contribution scales as $\tan\beta(\mu/m_{\tilde{W}})$ (for $m_{\tilde{q}} \gg m_{\tilde{W}}$). In minimal supersymmetric $SU(5)$, the Higgsino dressing becomes more important than the wino dressing when $\tan\beta \geq 9(m_{\tilde{W}}/\mu)$ [53]. This estimate takes into account the differences in the renormalization of the $(RRRR)$ compared to the $(LLLL)$ operator owing to running from M_{U} to M_{SUSY}, and the difference between their strong matrix elements.

In the $SO(10)$ model, once the quark and lepton masses and mixings are fixed, the strength of the $RRRR$ operator, which we denote as $W_{\text{eff}}^{(R)}$ will be determined, just as was $W_{\text{eff}}^{(L)}$. It is given by:

$$
\begin{aligned}
W_{\text{eff}}^{(R)} = &-M_{\text{eff}}^{-1}[2(u^{c\,T}\tilde{H}\tilde{V}'e^c)(u^{c\,T}\tilde{H}Kd^c) + (u^{c\,T}\tilde{H}\tilde{V}'e^c)(u^{c\,T}\tilde{A}Kd^c) \\
&- 3(u^{c\,T}\tilde{A}\tilde{V}'e^c)(u^{c\,T}\tilde{H}Kd^c) - 3(u^{c\,T}\tilde{A}\tilde{V}'e^c)(u^{c\,T}\tilde{A}Kd^c) \\
&- \tan\gamma(u^{c\,T}\tilde{H}\tilde{V}'e^c)(u^{c\,T}\tilde{G}Kd^c) + 3(u^{c\,T}\tilde{A}\tilde{V}'e^c)(u^{c\,T}\tilde{G}Kd^c)] \\
&+ M_{16}^{-1}(u^{c\,T}\tilde{F}\tilde{V}'e^c)(u^{c\,T}\tilde{G}Kd^c) \ .
\end{aligned}
\tag{57}
$$

Here

$$
(\tilde{H}, \tilde{A}, \tilde{G}, \tilde{F}) \equiv \tilde{V}_u^T (h, a, g, f)\tilde{V}_u,
\tag{58}
$$

with $\tilde{V}_u$ the unitary matrix that rotates the u^c fields $u^{c(g)} = \tilde{V}_u u^{c(m)}$ from the gauge basis to the mass basis, $\tilde{V}_e$ is the unitary matrix that similarly rotates the right–handed electron field, and $\tilde{V}' \equiv \tilde{V}_u^\dagger \tilde{V}_e$. K is the right–handed analog of V_{CKM}, $K \equiv \tilde{V}_u^\dagger \tilde{V}_d$.

The contribution to proton decay amplitude from Eq. (57) is estimated as follows. Since a charged Higgsino is involved in the dressing, the internal scalars have to be from the third generation (other diagrams will be suppressed by small Yukawa couplings). This uniquely picks out the $\tilde{t}_R$ and $\tilde{\tau}_R$ as the internal scalars. The external quark fields are then fixed to be u and either a d or an s.

This suggests that the combination of indices $(ij)(kl)$ in Eq. (57) must be $(33)(11)$, $(33)(12)$, $(13)(32)$ or $(13)(31)$. Among these four, the combination $(33)(12)$ can proceed without utilizing any of the right handed mixing angles. (Though the right–handed mixing angles in the (23) sector of both u and d are ~ 0.2, not terribly small.) We find the amplitude, after Higgsino dressing, from this dominant contribution to be

$$
\hat{A}[(u^c s^c)^\dagger(d\nu_\tau)] \simeq M_{\text{eff}}^{-1}h_{33}^2\eta' V_{td} + (M_{16}\tan\gamma)^{-1}h_{33}\hat{f}_{33}\eta' V_{td} \ .
\tag{59}
$$

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Other contributions are not much bigger. The full amplitude $A[(u^c s^c)^\dagger (d\nu_\tau)]$ will be obtained by multiplying the above expression by a loop function analogous to the function for the wino dressing (see Eq. (78)). There are two differences in this function however: (i) The factor $(\alpha_2/4\pi)$ will be replaced by $(\lambda_t \lambda_\tau)/(16\pi^2)$, where the Yukawa couplings are to be evaluated at the momentum scale M_{SUSY}. (ii) The mass parameter $m_{\tilde{W}}$ is replaced by μ. The presence of λ_τ brings in a $\tan\beta$ dependence in the Higgsino dressing relative to the wino dressing. (In minimal supersymmetric $SU(5)$, the wino dressing has a $\tan\beta$ dependence, but the Higgsino dressing will have a $(\tan\beta)^2$ dependence.) Inserting $\eta' \simeq 4.4 \times 10^{-3}$, $V_{td} \simeq 0.006$, and $h_{33} \simeq 1/2$, we find that the amplitude is $2.5 \times 10^{-7} \tan\beta (\mu/m_{\tilde{W}})$. This can be significant, relative to the wino contribution, for $\tan\beta \geq 10$ and for $\mu \simeq m_{\tilde{W}}$. For larger values of μ, this contribution can dominate over the wino dressing. Then the $RRRR$ contribution will provide limits on $\mu \tan\beta$.

The charged Higgsino dressing results in decay modes of the proton containing final state neutrinos, not charged leptons. Apart from the direct charged Higgsino exchange, there are diagrams involving $\tilde{t}_L - \tilde{t}_R$ mixing and $\tilde{\tau}_L - \tilde{\tau}_R$ mixing (the latter being proportional to $\tan\beta$) followed by the exchange of charged wino. This contribution is smaller than the direct charged Higgsino exchange by roughly a factor v_u/M_{SUSY}. Similar arguments apply to mixed contributions involving charged Higgsino–wino mixing.

H. Charged lepton decay mode

In minimal supersymmetric $SU(5)$ and many of its variants, charged lepton decay of the proton is suppressed as long as $\tan\beta \leq 20$ or so. This is because of a GIM-type cancellation in the wino dressing diagrams which brings in a suppression factor in this amplitude proportional to the small u–quark mass. Gluino dressing can lead to charged lepton decay of the proton, but owing to flavor conservation of the primary gluino vertex and the flavor antisymmetry of the effective superpotential, this contribution is suppressed for small values of $\tan\beta$ (≤ 20). For large values of $\tan\beta$ (≥ 20), flavor mixing in the up–squark sector becomes significant and the gluino graph begins to be important [54]. However, for minimal supersymmetric $SU(5)$, such large values of $\tan\beta$ (≥ 20) are highly disfavored owing to limits on proton lifetime (see for e.g. Ref. [33]). Furthermore, as discussed in the previous subsection, for large $\tan\beta$, dressing of the $RRRR$ operator by charged Higgsino becomes the dominant source of proton decay, again resulting in modes with final state neutrinos. So in minimal supersymmetric $SU(5)$, and many of its variants, charged lepton decay mode of the proton is highly suppressed relative to the neutrino mode, for *all* values of $\tan\beta$.

The situation is different in our $SO(10)$ model, for two reasons. First, the contribution to $p \to \mu^+ K^0$ arising from the standard $d = 5$ wino dressing diagram in this model is not small. Since we have a realistic spectrum of quark and lepton masses, especially with $m_s \neq m_\mu$ and $m_d \neq m_e$ at M_{U}, the wino contribution does not experience a GIM cancellation. Second, the new $d = 5$ operator related to neutrino masses lends comparable strength to the neutrino and the charged lepton modes.

From Eq. (73) of Appendix B, by allowing the larger terms of the standard $d = 5$ operator to add constructively, but keeping track of the correlation $\eta_{ts}\eta_{cb} = -1$, we find that $\hat{A}[(us)(u\mu)] \simeq M_{\mathrm{eff}}^{-1} \times h_{33}^2 \times 3 \times 10^{-6}$. This is not terribly small compared to the strength

of the standard $d = 5$ neutrino amplitude (see Eq. (45)). Thus typically, we would expect the rate for $p \to \mu^+ K^0$ arising from the standard $d = 5$ operator to be within an order of magnitude or so of the one for $p \to \overline{\nu} K^+$.

The neutrino mass related $d = 5$ operator is seen to contribute with roughly equal strength to the charged lepton and neutrino modes (compare terms proportional to $(M_{16} \tan \gamma)^{-1}$ in Eq. (70) and (73)). This is true for the terms proportional to $(1, y, z, x)$ in the Majorana neutrino mass matrix. For $x \geq 10^{-3}$ and $z \geq 10^{-2}$ the respective contributions proportional to x and z can actually dominate. One also sees from Eq. (70) that x cannot be much bigger than 10^{-3} (for $M_{16} \tan \gamma \leq 5 \times 10^{17}$ GeV), otherwise the bound in Eq. (43) will be violated.

If the standard $d = 5$ operators are strongly suppressed (see Sec. VI.F), then proton decay will still proceed via the new $d = 5$ operators. The charged lepton mode in this case has roughly the same strength as the neutrino mode. As noted above, the terms proportional to (x, z) can dominate charged lepton decay amplitude for $x \geq 10^{-4}$ and $z \geq 10^{-2}$.

Because of the hierarchical nature of the Yukawa couplings in the Dirac as well as the Majorana sectors, proton decay into $e^+ K^0$ and $e^+ \pi^0$ are highly suppressed in our scenario compared to the neutrino mode $\overline{\nu} K^+$ or to the μ mode $\mu^+ K^0$.

VII. SUMMARY AND CONCLUDING REMARKS

One major goal of this paper has been to understand the masses and mixings of the neutrinos, suggested by the atmospheric and the solar neutrino anomalies, *in conjunction with* those of the quarks and charged leptons. Adopting familiar ideas of generating eigenvalues through off–diagonal mixings, we find that the bizarre pattern of masses and mixings observed in the charged fermion sector can be adequately described (with $\sim 10\%$ accuracy) within an economical $SO(10)$ framework. A concrete proposal was presented that provides five successful predictions for the masses and mixings in the quark and the charged lepton systems. The same description provides, in agreement with the SuperK result on atmospheric neutrinos, a large $\nu_\mu - \nu_\tau$ oscillation angle ($\sin^2 2\theta_{\mu\tau}^{\text{osc}} \simeq 0.82 - 1.0$), despite highly non–degenerate masses for the light neutrinos.

The other major goal of this paper has been to revisit the previously noted link between neutrino masses and nucleon decay [12] in the light of the SuperKamiokande result. We find that the mass of ν_τ ($\sim 1/20$ eV), together with the large $\nu_\mu - \nu_\tau$ oscillation angle implied by the SuperK result, suggest a significant enhancement in the standard as well as in the new (neutrino mass related) $d = 5$ proton decay operators, relative to previous estimates, including those of Ref. [12]. There are many uncertainties in the prediction for proton decay rate, including ones arising from uncertainties in the SUSY–spectrum, in the hadronic matrix elements, and in the relative phases of the many different contributions (see discussion in Appendix B and in Sec. VI D and E). Nevertheless, we found that the standard operator contributions severely constrain the underlying model.

Specifically, we found that for MSSM embedded in $SO(10)$, the standard operators, with generous allowance for the uncertainties, lead to lifetime estimate $\Gamma^{-1}(p \to \overline{\nu} K^+)_{\text{std}}^{\text{MSSM}} \leq 4.5 \times 10^{33}$ yrs corresponding to small values of $\tan \beta \sim 3$. For larger values of $\tan \beta \sim 20$, one can turn to the Extended Supersymmetric Standard Model (ESSM) embedded in $SO(10)$ which allows for two extra (vector-like) families at the TeV scale (and has been motivated

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on other grounds [44]) where the standard $d = 5$ operators still represent perfectly viable sources for proton decay. In this case, we expect $\Gamma^{-1}(p \rightarrow \overline{\nu}K^+)_{\text{std}}^{\text{ESSM}} \leq 4 \times 10^{33}$ yrs for $\tan \beta \simeq 20$ (Sec. VI.D). If $\tan \beta$ were even larger than 20, charged Higgsino exchange would dominate the proton decay amplitude leading to a lifetime shorter than the experimental limit.

As observed in our earlier work [12], one very important consequence of quark–lepton unification is the likely existence of new $d = 5$ proton decay operators that are related to neutrino masses. We have remarked in Sec. VI.F that in some string-inspired models leading to G_{224} or G_{2113} symmetries (rather than intact $SO(10)$), the color triplets related to electroweak doublets get projected out of the spectrum altogether, and thus the standard $d = 5$ operators do not contribute. In that case, the new $d = 5$ operators related to neutrino masses will survive and dominate. We found (Sec. VI.F) that (given the SuperK result) these new operators by themselves lead to proton lifetime $\Gamma^{-1}(p \rightarrow \overline{\nu}K^+)$ with a "central value" of 3×10^{32} yrs, and a range that is perfectly compatible with the observed limit. Assuming that not all the uncertainties (i.e., those in $(m_{\tilde{W}}/m_{\tilde{q}})$, $m_{\tilde{q}}$, and the matrix element β_H) work in the same direction, the new operators by themselves lead to a proton lifetime $\leq 10^{34}$ yrs. Thus for MSSM (or ESSM) embedded in $SO(10)$ we would expect proton lifetime to be shorter than 10^{34} yrs., either from the new (neutrino mass related) or the standard $d = 5$ operators.

A distinctive feature of the $SO(10)$ framework discussed here is the general prominence of the charged lepton mode $p \rightarrow \mu^+ K^0$. In minimal $SU(5)$, this mode becomes prominent only for $\tan \beta \geq 20$ when gluino dressing becomes significant. But for such large values of $\tan \beta$, the Higgsino dressing of the baryon number violating $RRRR$ operator becomes dominant, leading anyhow to dominance of the neutrino mode. Thus in conventional $SU(5)$–like supersymmetric unified models, charged lepton decay of the proton is (even relatively!) scarce. In the $SO(10)$ framework, on the contrary, the decay $p \rightarrow \mu^+ K^0$ competes favorably with the neutrino mode, both for the new neutrino mass related $d = 5$ operators and for the standard $d = 5$ operators. Thus, observation of the $\mu^+ K^0$ decay mode of the proton would reinforce the circle of ideas discussed here. Owing to the hierarchical Yukawa couplings of the Majorana neutrinos suggested by the solar and the atmospheric neutrino data, the modes $p \rightarrow e^+ K^0$ and $p \rightarrow e^+ \pi^0$ are predicted to be highly suppressed relative the $p \rightarrow \overline{\nu}K^+$.

While we focused on a specific $SO(10)$ example, the results obtained here are likely to be more general. In any scenario with just three light neutrinos, a simultaneous resolution of the solar and atmospheric neutrino anomalies argues for hierarchical neutrino masses. In a framework that also unifies quarks and leptons (with modest mixing angles), a simple way to generate the near-maximal neutrino oscillation angle needed for the atmospheric neutrino anomaly is to attribute it partly to the $(\mu - \tau)$ sector and partly to the $(\nu_\mu - \nu_\tau)$ sector. The precise way this division is made is model dependent. As one goes beyond minimal supersymmetric $SU(5)$ to a unified framework where small neutrino mass is a compelling feature, the predicted lifetime of the proton tends to decrease. In such unified models there are various contributing factors to the shorter lifetime (see Appendix C) including (i) an enhanced coupling of the muon to the color triplets (relative to the coupling of the strange quark), (ii) an enhanced up–quark coupling to the color triplet which scales as $\sqrt{m_c m_t}$ rather than m_c, (iii) the presence of several terms arising from a realistic quark–lepton mass spectrum and (iv) the presence of new operators related to neutrino mass.

Proton decay has been anticipated for quite some time in the context of unified theories. Recent data from SuperK on neutrino mass makes the case for observable proton decay still more compelling. With improved observations, either proton decay will be revealed, or some promising and otherwise remarkably successful ideas on unification will require revision.

APPENDIX A

Numerical values of various matrices

Here we give the numerical values of the matrices V_u, V_ℓ and $V' = V_u^\dagger V_l$, where V_u diagonalises the up quark matrix U of Eq. (28): $u^{(g)} = V_u u^{(m)}$ with (g) and (m) denoting the gauge and mass eigenstates, V_ℓ diagonalises the charged lepton mass matrix. We also give the numerical values of the matrices $(\hat{H}, \hat{A}, \hat{G}, \hat{F})$ as well as their matrix product with V', these are relevant for proton decay amplitude calculations. From the fit to the fermion masses discussed in Sec. IV-V, we have determined the (approximate) values of the parameters $(\sigma, \eta, \epsilon, \epsilon', \eta';)$. For numerical purposes we shall choose their "central values":

$$
\begin{aligned}
\sigma &= -0.1096 \ \eta_{cb}, \quad \eta = -0.1507 \ \eta_{cb}, \quad \epsilon = 0.0954 \ \eta_\epsilon, \\
\epsilon' &= 1.76 \times 10^{-4} \ \eta_{\epsilon'}, \quad \eta' = 4.14 \times 10^{-3} \ \eta_{\eta'} \ .
\end{aligned} \tag{60}
$$

With these values, the matrix V_u is given by:

$$
V_u \simeq \begin{pmatrix} 1 & \frac{\epsilon'}{\epsilon^2 - \sigma^2} & \epsilon'(-\epsilon + \sigma) \\ -\frac{\epsilon'}{\epsilon^2 - \sigma^2} & 1 & \epsilon + \sigma \\ \frac{\epsilon'}{\epsilon - \sigma} & -(\epsilon + \sigma) & 1 \end{pmatrix}
$$

$$
\simeq \begin{pmatrix} 1 & -0.061\eta_{\epsilon'} & -3.61 \times 10^{-5}\eta_{cb}\eta_{\epsilon'} \\ 0.061\eta_{\epsilon'} & 1 & -0.0142\eta_{cb} \\ 8.6 \times 10^{-4}\eta_{cb}\eta_{\epsilon'} & 0.0142\eta_{cb} & 1 \end{pmatrix} \ . \tag{61}
$$

An analogous expression for V_ℓ is obtained by the replacement $\sigma \to \eta, \epsilon \to -3\epsilon$ and $\epsilon' \to -3\epsilon'$ in the first part of Eq. (61). The numerical values of V_ℓ and $V' = V_u^\dagger V_\ell$ are:

$$
V_\ell \simeq \begin{pmatrix} 1 & 0.07\eta_{\eta'} & 5.6 \times 10^{-4}\eta_{cb}\eta_{\eta'} \\ -0.07\eta_{\eta'} & 0.905 & -0.4369\eta_{cb} \\ -0.031\eta_{cb}\eta_{\eta'} & 0.4369\eta_{cb} & 0.905 \end{pmatrix} \ ,
$$

$$
V' \simeq \begin{pmatrix} 1 & 0.052\eta_{\epsilon'} + 0.07\eta_{\eta'} & -0.026\eta_{cb}\eta_{\epsilon'} \\ -(0.052\eta_{\epsilon'} + 0.07\eta_{\eta'}) & 0.911 & -0.4227\eta_{cb} \\ -0.030\eta_{cb}\eta_{\eta'} & 0.4227\eta_{cb} & 0.911 \end{pmatrix} \ . \tag{62}
$$

Numerical values of the matrices $(\hat{H}, \hat{A}, \hat{G}, \hat{F})$ and their products with V' are:

$$\hat{H} \simeq h_{33} \begin{pmatrix} -1.1 \times 10^{-5} & -1.8 \times 10^{-4} \eta_{\epsilon'} & -5.8 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} \\ -1.8 \times 10^{-4} \eta_{\epsilon'} & -2.9 \times 10^{-3} & -0.0954 \eta_{cb} \\ -5.8 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} & -0.0954 \eta_{cb} & 1 \end{pmatrix} ,$$

$$\hat{H} V' \simeq h_{33} \begin{pmatrix} 1.8 \times 10^{-4} \eta_{\epsilon'} \eta_{\eta'} & -2.6 \times 10^{-3} \eta_{\epsilon'} & -5.8 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} \\ 3.0 \times 10^{-3} \eta_{\eta'} & -0.043 & -0.0954 \eta_{cb} \\ 2.9 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} - 0.024 \eta_{cb} \eta_{\eta'} & 0.3273 \eta_{cb} & 1 \end{pmatrix} , \quad (63)$$

$$\hat{A} \simeq h_{33} \begin{pmatrix} 3.1 \times 10^{-5} & 3.4 \times 10^{-4} \eta_{\epsilon'} & 5.8 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} \\ 3.4 \times 10^{-4} \eta_{\epsilon'} & 2.7 \times 10^{-3} & 0.0954 \eta_{cb} \\ 5.8 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} & 0.0954 \eta_{cb} & -2.7 \times 10^{-3} \end{pmatrix} ,$$

$$\hat{A} V' \simeq h_{33} \begin{pmatrix} -1.9 \times 10^{-4} \eta_{\epsilon'} \eta_{\eta'} + 3.5 \times 10^{-5} & 2.8 \times 10^{-3} \eta_{\epsilon'} & 5.8 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} \\ 3.0 \times 10^{-3} \eta_{\eta'} + 5.6 \times 10^{-4} \eta_{\epsilon'} & 0.043 & 0.0954 \eta_{cb} \\ -6.7 \times 10^{-3} \eta_{cb} \eta_{\eta'} + 8.6 \times 10^{-4} \eta_{cb} \eta_{\epsilon'} & 0.0954 \eta_{cb} & -0.043 \end{pmatrix} , \quad (64)$$

$$\tan \gamma \hat{G} \simeq h_{33} \begin{pmatrix} 5.4 \times 10^{-4} \eta_{\epsilon'} \eta_{\eta'} & 4.1 \times 10^{-3} \eta_{\eta'} & -2.5 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} \\ 4.1 \times 10^{-3} \eta_{\eta'} & -1.2 \times 10^{-3} & -0.0411 \eta_{cb} \\ -2.5 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} & -0.0411 \eta_{cb} & 1.2 \times 10^{-3} \end{pmatrix} ,$$

$$\tan \gamma \hat{G} V' \simeq h_{33} \begin{pmatrix} \{3.7 \times 10^{-4} \eta_{\epsilon'} \eta_{\eta'} & \{-1.1 \times 10^{-3} \eta_{\epsilon'} & \{-2.5 \times 10^{-3} \eta_{cb} \eta_{\epsilon'} \\ -2.9 \times 10^{-4}\} & +4.1 \times 10^{-3} \eta_{\eta'}\} & -1.8 \times 10^{-3} \eta_{cb} \eta_{\eta'}\} \\ 5.4 \times 10^{-3} \eta_{\eta'} & -0.019 & -0.0411 \eta_{cb} \\ -3.7 \times 10^{-4} \eta_{cb} \eta_{\epsilon'} + 2.9 \times 10^{-3} \eta_{cb} \eta_{\eta'} & -0.0411 \eta_{cb} & 0.019 \end{pmatrix} \quad (65)$$

$$\hat{F} \simeq \hat{f}_{33} \begin{pmatrix} x + 7.3 \times 10^{-7} + 1.0 \times 10^{-4} \eta_{cb} y & 1.7 \times 10^{-3} \eta_{\epsilon'} \eta_{cb} y - 0.06 \eta_{\epsilon'} x & 0.06 \eta_{\epsilon'} y \\ 1.7 \times 10^{-3} \eta_{\epsilon'} \eta_{cb} y - 0.06 \eta_{\epsilon'} x & 0.0284 \eta_{cb} y & y + 0.0142 \eta_{cb} \\ 0.06 \eta_{\epsilon'} y & y + 0.0142 \eta_{cb} & 1 \end{pmatrix} \quad (66)$$

APPENDIX B

B.1. Numerical evaluation of the full amplitudes for proton decay

In converting the superpotential given in Eq. (35) into a proton decay amplitude, the first step is to dress two of the sfermions among the four superfields in each term of Eq. (35) *i.e.*, to convert them into fermions. The dominant contribution to the amplitude arises from the dressing involving the charged wino. Here we list the 12 terms that arise in dressing each one of the 7 terms of Eq. (35) with a final state neutrino. The four fermion amplitude after wino dressing of a generic superpotential term

$$W = M_{\text{eff}}^{-1}(u^T F d')\{u^T G \ell - d'T G \nu'\} \tag{67}$$

is

$$
\begin{aligned}
A(p \to \overline{\nu} X) = M_{\text{eff}}^{-1} \epsilon_{\alpha\beta\gamma} \times \; & [F_{11}G_{2l}(u^\alpha d'^\beta)(s'^\gamma \nu_l')\,(f(c,l) + f(u,d)) \\
& - F_{22}G_{1l}(u^\alpha s'^\beta)(s'^\gamma \nu_l')\,(f(c,l) + f(c,d)) - F_{12}G_{1l}(u^\alpha d'^\beta)(s'^\gamma \nu_l')\,(f(c,l) + f(u,d)) \\
& + F_{12}G_{2l}(u^\alpha s'^\beta)(s'^\gamma \nu_l')\,(f(c,l) + f(d,c)) - F_{13}G_{1l}(u^\alpha d'^\beta)(b'^\gamma \nu_l')\,(f(t,l) + f(u,d)) \\
& + F_{13}G_{2l}(u^\alpha b'^\beta)(s'^\gamma \nu_l')\,(f(c,l) + f(d,t)) + F_{13}G_{3l}(u^\alpha b'^\beta)(b'^\gamma \nu_l')\,(f(t,l) + f(d,t)) \\
& - F_{23}G_{1l}(u^\alpha b'^\beta)(s'^\gamma \nu_l')\,(f(c,l) + f(t,d)) - F_{23}G_{1l}(u^\alpha s'^\beta)(b'^\gamma \nu_l')\,(f(c,d) + f(t,l)) \\
& - F_{33}G_{1l}(u^\alpha b'^\beta)(b'^\gamma \nu_l')\,(f(t,l) + f(t,d)) + F_{11}G_{3l}(u^\alpha d'^\beta)(b'^\gamma \nu_l')\,(f(t,l) + f(u,d)) \\
& + F_{12}G_{3l}(u^\alpha s'^\beta)(b'^\gamma \nu_l')\,(f(t,l) + f(d,c))] \left(\frac{\alpha_2}{4\pi}\right) \; . \tag{68}
\end{aligned}
$$

The dressing functions $f(a, b)$ are defined later in Eq. (78). In writing Eq. (68), use has been made of the symmetric nature of F ($F_{ij} = F_{ji}$). As described in the text, the notation $d' = V_{CKM}d$ and $\nu' = V_{CKM}^\ell \nu$ has been adopted.

A simpler expression can be obtained for proton decay amplitude with a charged lepton in the final state (for the same generic superpotential term as in Eq. (67)):

$$
\begin{aligned}
A(p \to \ell^+ X) = M_{\text{eff}}^{-1} \epsilon_{\alpha\beta\gamma} \times \; & [(u^\alpha d^\beta)(u^\gamma \ell_l^-)[V_{cd}(G_{1l}F_{21} - G_{2l}F_{11}) + V_{td}(G_{1l}F_{31} - G_{3l}F_{11}) \\
& + (u^\alpha s^\beta)(u^\gamma \ell^-)[V_{cs}(G_{1l}F_{21} - G_{2l}F_{11}) + V_{ts}(G_{1l}F_{31} - G_{3l}F_{11})] \; . \tag{69}
\end{aligned}
$$

Note that the down type quark fields appearing above (without primes) are the physical ones.

With the numerical values of the Yukawa coupling matrices relevant for the color triplet exchange, we can use Eqs. (68) and (69) to compute the decay amplitude for any given channel. Since there are seven terms in Eq. (35), for the neutrino mode, there will be a total of $7 \times 12 = 84$ terms to be summed. This is most efficiently done numerically using *Mathematica*. We now present some of the dominant amplitudes. For these estimates we define $V_{cd} = 0.22\eta_{cd}, V_{td} = 0.006\eta_{td}, V_{ts} = 0.04\eta_{ts}$ and $V_{ud} = V_{cs} = 1$. The η_{ij} are the unknown phase factors, they can also be used to vary the central values of the CKM mixing angles adopted. We drop terms which are smaller by more than an order of magnitude compared to the leading term in each set. The amplitude with a hat is the full four–Fermion amplitude divided by the loop function $(2f)$, defined in Eq. (90) and (91) of Appendix C.

$$
\begin{aligned}
\hat{A}[(ud)(s\nu_\tau)] \simeq \; & M_{\text{eff}}^{-1} h_{33}^2 [1.9 \times 10^{-5}\eta_{cd}\eta_{ts}\eta_{\epsilon'} - 9.0 \times 10^{-6}\eta_{cd}\eta_{cb}\eta_{\epsilon'} \\
& - 6.1 \times 10^{-6}\eta_{td}\eta_{ts}\eta_{cb}\eta_{\epsilon'} + 4.8 \times 10^{-6}\eta_{ts} - 2.5 \times 10^{-6}\eta_{cb} \\
& + 2.1 \times 10^{-6}\eta_{td}\eta_{\epsilon'} + 3.0 \times 10^{-6}\eta_{cd}\eta_{ts}\eta_{\eta'} - 2.2 \times 10^{-6}\eta_{cd}\eta_{cb}\eta_{\eta'}] \\
& + (M_{16}\tan\gamma)^{-1}h_{33}\hat{f}_{33}[3.1 \times 10^{-7}\eta_{cd}\eta_{ts}\eta_{\epsilon'} + 6.0 \times 10^{-7}\eta_{td}\eta_{ts}\eta_{cb}\eta_{\epsilon'} \\
& - 2.2 \times 10^{-7}\eta_{cd}\eta_{cb}\eta_{\epsilon'} + 4.3 \times 10^{-7}\eta_{td}\eta_{ts}\eta_{cb}\eta_{\eta'} + 1.5 \times 10^{-7}\eta_{td}\eta_{\eta'} \\
& + 2.3 \times 10^{-7}\eta_{cd}\eta_{ts}\eta_{\eta'} - 0.0411\eta_{cb}x + 3.1 \times 10^{-5}\eta_{cd}\eta_{\epsilon'}y \\
& + 2.2 \times 10^{-5}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\epsilon'}y + 1.1 \times 10^{-5}\eta_{td}\eta_{cb}\eta_{\eta'}y \\
& + 1.6 \times 10^{-5}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\eta'}y - 1.1 \times 10^{-5}\eta_{cd}\eta_{\eta'}y \\
& + z\{-2.47 \times 10^{-4}\eta_{td}\eta_{cb} - 1.28 \times 10^{-4}\eta_{cd} \\
& + 9.94 \times 10^{-5}\eta_{ts}\eta_{cb}\eta_{\epsilon'} + 7.23 \times 10^{-5}\eta_{ts}\eta_{cb}\eta_{\eta'}\}] \; . \tag{70}
\end{aligned}
$$

$$\hat{A}[(ud)(d\nu_\tau)] \simeq M_{\text{eff}}^{-1} h_{33}^2 [2.9 \times 10^{-6} \eta_{cd}\eta_{td}\eta_{\epsilon'} - 9.6 \times 10^{-6} \eta_{cb}\eta_{\epsilon'}]$$
$$+ (M_{16}\tan\gamma)^{-1}\hat{f}_{33}h_{33}[4.7 \times 10^{-8}\eta_{cd}\eta_{td}\eta_{\epsilon'} + 8.9 \times 10^{-8}\eta_{cb}\eta_{\epsilon'}$$
$$- 4.9 \times 10^{-8}\eta_{cb}\eta_{\eta'} + 1.3 \times 10^{-8}\eta_{td} + 6.5 \times 10^{-8}\eta_{cb}\eta_{\eta'}$$
$$+ 6.8 \times 10^{-8}\eta_{cd}\eta_{td}\eta_{\eta'} - 1.8 \times 10^{-8}\eta_{cb}\eta_{\eta'} - 9.0 \times 10^{-3}\eta_{cd}\eta_{cb}x$$
$$- 6.8 \times 10^{-6}\eta_{\epsilon'}y + 3.3 \times 10^{-6}\eta_{cd}\eta_{td}\eta_{cb}\eta_{\epsilon'}y$$
$$+ 4.8 \times 10^{-6}\eta_{cd}\eta_{td}\eta_{cb}\eta_{\eta'}y - 2.5 \times 10^{-6}\eta_{\eta'}y$$
$$+ z\{-5.42 \times 10^{-5}\eta_{cd}\eta_{td}\eta_{cb} - 2.82 \times 10^{-5} + 1.49 \times 10^{-5}\eta_{td}\eta_{cb}\eta_{\epsilon'}$$
$$+ 1.08 \times \eta_{td}\eta_{cb}\eta_{\eta'}\}] \tag{71}$$

$$\hat{A}[(ud)(s\nu_\mu)] \simeq M_{\text{eff}}^{-1} h_{33}^2 [-3.5 \times 10^{-6}\eta_{td}\eta_{ts}\eta_{\epsilon'} + 8.5 \times 10^{-6}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\epsilon'}$$
$$- 2.0 \times 10^{-5}\eta_{cd}\eta_{\epsilon'} + 2.1 \times 10^{-6}\eta_{ts}\eta_{cb} - 4.7 \times 10^{-6}\eta_{td}\eta_{cb}\eta_{\epsilon'}$$
$$- 7.0 \times 10^{-6}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\eta'} + 5.1 \times 10^{-6}\eta_{cd}\eta_{\eta'} - 2.1 \times 10^{-6}\eta_{\epsilon'}\eta_{\eta'}$$
$$- 1.9 \times 10^{-6}\eta_{ts}\eta_{cb}\eta_{\epsilon'}\eta_{\eta'}] + (M_{16}\tan\gamma)^{-1}\hat{f}_{33}h_{33}[2.6 \times 10^{-7}\eta_{td}\eta_{ts}\eta_{\epsilon'}$$
$$+ 1.4 \times 10^{-7}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\epsilon'} - 9.9 \times 10^{-7}\eta_{td}\eta_{ts}\eta_{\eta'}$$
$$- 3.5 \times 10^{-7}\eta_{td}\eta_{cb}\eta_{\eta'} - 5.2 \times 10^{-7}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\eta'} + 1.8 \times 10^{-7}\eta_{cd}\eta_{\eta'}$$
$$- 1.4 \times 10^{-7}\eta_{ts}\eta_{cb}\eta_{\epsilon'}\eta_{\eta'} - 1.6 \times 10^{-3}\eta_{ts}\eta_{cb}x - 1.8 \times 10^{-2}x$$
$$+ 9.2 \times 10^{-6}\eta_{cd}\eta_{ts}\eta_{\epsilon'}y - 1.4 \times 10^{-5}\eta_{cd}\eta_{cb}\eta_{\epsilon'}y + 2.5 \times 10^{-6}\eta_{ts}y$$
$$- 2.5 \times 10^{-5}\eta_{td}\eta_{\eta'}y - 3.6 \times 10^{-5}\eta_{cd}\eta_{ts}\eta_{\eta'}y + 2.6 \times 10^{-5}\eta_{cd}\eta_{cb}\eta_{\eta'}y$$
$$- 1.0 \times 10^{-5}\eta_{ts}\eta_{\epsilon'}\eta_{\eta'}y - 7.1 \times 10^{-6}\eta_{cb}\eta_{\eta'}\eta_{\epsilon'}y$$
$$+ z\{-9.86 \times 10^{-6}\eta_{td}\eta_{ts}\eta_{cb} - 1.11\eta_{td}$$
$$- 5.79 \times 10^{-5}\eta_{cd}\eta_{cb} + 4.20 \times 10^{-5}\eta_{ts}\eta_{\epsilon'}\}] \tag{72}$$

Similarly, the amplitudes $\hat{A}[(us)(d\nu_\tau)]$, $\hat{A}[(us)(d\nu_\mu)]$ and $\hat{A}[(ud)(d\nu_\mu)]$ can be computed. We do not display these results here, since the short distance matrix elements for these operators are always somewhat smaller than the ones displayed. Furthermore, the long distance matrix element for the former two turn out to be suppressed by about a factor of 3.

Turning now to the charged lepton decay amplitude,

$$\hat{A}[(us)(u\mu^-)]) \simeq M_{\text{eff}}^{-1} h_{33}^2 [V_{cs}\{-8.4 \times 10^{-7} - 2.1 \times 10^{-6}\eta_{\epsilon'}\eta_{\eta'}\}$$
$$+ V_{ts}\{5.2 \times 10^{-5} - 4.8 \times 10^{-5}\eta_{cb}\eta_{\epsilon'}\eta_{\eta'}\}$$
$$+ (M_{16}\tan\gamma)^{-1}\hat{f}_{33}h_{33}[\{V_{cs}\{-5.0 \times 10^{-8}\eta_{\epsilon'}\eta_{\eta'} + 1.8 \times 10^{-2}x$$
$$- 7.1 \times 10^{-6}y\eta_{cb}\eta_{\eta'}\eta_{\epsilon'}\} + V_{ts}\{9.3 \times 10^{-7}\eta_{cb} - 3.6 \times 10^{-6}\eta_{cb}\eta_{\eta'}\eta_{\epsilon'}$$
$$- 0.0411\eta_{cb}x + 6.4 \times 10^{-5}y + 2.5 \times 10^{-4}y\eta_{\eta'}\eta_{\epsilon'}\}$$
$$+ z\{4.2 \times 10^{-5}\eta_{ts}\eta_{\epsilon'} - 1.59 \times 10^{-5}\eta_{\epsilon}\eta_{\epsilon'}$$
$$- 1.66 \times 10^{-4}\eta_{ts}\eta_{\eta'} - 5.88 \times 10^{-5}\eta_{\epsilon}\eta_{\eta'}\}] \,. \tag{73}$$

$$\hat{A}[(us)(ue^-)] \simeq h_{33}^2 M_{\text{eff}}^{-1} [V_{cs}\{8.4 \times 10^{-8}\eta_{\eta'} + 1.5 \times 10^{-7}\eta_{\epsilon'}\}$$
$$+ V_{ts}\{8.3 \times 10^{-7}\eta_{cb}\eta_{\epsilon'} - 6.9 \times 10^{-6}\eta_{cb}\eta_{\eta'} + 3.3 \times 10^{-6}\eta_{cb}\eta_{\epsilon'}\}$$

$$
\begin{aligned}
&+ (M_{16}\tan\gamma)^{-1}\hat{f}_{33}h_{33}[V_{cs}\{3.5\times10^{-9}\eta_{\epsilon'} - 1.7\times10^{-4}\eta_{\epsilon'}x \\
&+ 5.3\times10^{-3}\eta_{\eta'}x + 5.0\times10^{-7}\eta_{cb}\eta_{\epsilon'}y\} + V_{ts}\{8.3\times10^{-9}\eta_{cb}\eta_{\epsilon'} \\
&+ 3.1\times10^{-7}\eta_{cb}\eta_{\eta'} + 2.5\times10^{-7}\eta_{cb}\eta_{\epsilon'} + 3.7\times10^{-4}\eta_{cb}\eta_{\eta'}x \\
&+ 2.9\times10^{-3}\eta_{cb}\eta_{\eta'}x + 2.2\times10^{-5}\eta_{\eta'}y + 1.8\times10^{-5}\eta_{\epsilon'}y\} \\
&+ z\{-1.44\times10^{-5}\eta_{ts}\eta_{\epsilon'}\eta_{\eta'}\}] \,.
\end{aligned} \tag{74}
$$

The decay amplitude for $p \to \pi^0\mu^+$ can be obtained from the above by the replacement $V_{cs} \to V_{cd}$ and $V_{ts} \to V_{td}$. The amplitude for $p \to e^+\pi^0$ may be obtained from Eq. (74) by the replacement: $V_{cs} \to V_{cd}, V_{ts} \to V_{td}$.

B.2 Representative contributions to proton decay amplitudes and their estimated magnitudes:

In this subsection, we exhibit in detail the evaluation of a few representative (dominant) contributions to proton decay amplitudes. The full set of contributions are listed in the preceding subsection. We also estimate the magnitudes of the full amplitudes allowing for uncertainty in the relative phase of the different contributions.

To obtain some of the leading terms in the results exhibited above, consider the part containing the ν' field – in the first term of $W^{(L)}$ (Eq. (35)), which we specify by subscript $I\nu'$.

$$
W^{(L)}_{I\nu'} = -M_{\rm eff}\epsilon_{\alpha\beta\gamma}\hat{H}_{ij}(\hat{H}V')_{kl}(u_i^\alpha d_j'^\beta)(d_k'^\gamma\nu_l') \,. \tag{75}
$$

Here (α, β, γ) denote color indices. The transpose symbols on the relevant fields are dropped henceforth. The first and the second terms in Eq. (70) arise by making the choices (A and B) and (C and D) respectively, for the indices as given below:

$$
\begin{aligned}
(ij)(kl) &= (21)(23) &&\text{(Choice A)} \\
&= (22)(13) &&\text{(Choice B)} \\
&= (23)(13) &&\text{(Choice C)} \\
&= (21)(33) &&\text{(Choice D)}
\end{aligned} \tag{76}
$$

Note that A and B are related by the interchange of $j \leftrightarrow k$, and similarly C and D.

Choice A: (ij)(kl) = (21)(23): Starting with the operator $(u_2^\alpha d_1'^\beta)(d_2'^\gamma\nu_3)$ in the superpotential, two fields need to be utilized for wino dressing. Which ones remain external gets determined as follows. The field $d_2'^\gamma$ must remain external, yielding a strange quark s^γ, accompanied by the CKM factor $V_{cs} \simeq 1$.[8] (If $\tilde{d}_2'$ were dressed instead, it would yield an external charm quark, which is kinematically disallowed.) Thus both u_2^α and $d_1'^\beta$ must be utilized for dressing, which, after conversion of $\tilde{c}$ and $\tilde{d}_1'$ at the wino vertex, respectively,

---

[8]If d_2' is used to yield an external d–quark, and $\tilde{u}_2$ (after dressing) an s–quark, one would obtain the four–fermion operator $(su)(d\nu_3)$. It turns out that the matrix element of this operator is suppressed by about a factor of 2-3 compared to that of $(du)(s\nu_3)$. [See [33].]

yield $V_{cd}d^\alpha \equiv (\eta_{cd}\theta_C)d^\alpha$ and $V_{ub}u^\beta$, where $\theta_C \simeq 0.22$. As discussed before, $\eta_{cd} = \pm 1$. η_{ud} and η_{us} are chosen to be $+1$ by convention. Thus, the corresponding contribution to the four–fermion proton decay operator leading to $\overline{\nu}_3$ emission is given by

$$
\begin{aligned}
A^{(1)}_{I,\nu'} &\simeq \left[\frac{-\hat{f}(c,d)}{M_{\text{eff}}}\right] (\theta_C \eta_{cd})\epsilon_{\alpha\beta\gamma}\hat{H}_{21}(\hat{H}V')_{23}(d^\alpha u^\beta)(s^\gamma \nu_3) \\
&\simeq -\left[\frac{h^2_{33}}{M_{\text{eff}}}\right]\hat{f}(c,d)(3.8 \times 10^{-6})\eta_{cd}\eta_{\epsilon'}\eta_{cb}\epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3)
\end{aligned}
\tag{77}
$$

where

$$
\hat{f}(a,b) = \left(\frac{\alpha_2}{4\pi}\right)\frac{m^2_{\tilde{W}}}{m^2_a - m^2_b}\left(\frac{m^2_a}{m^2_a - m^2_{\tilde{W}}}\ln\frac{m^2_a}{m^2_{\tilde{W}}} - [a \leftrightarrow b]\right) \equiv \left(\frac{\alpha_2}{4\pi}\right)f(a,b) \ . \tag{78}
$$

In getting Eq. (77), we have used the numerical values of the elements of $\hat{H}$ and $\hat{H}V'$ given in Appendix A (Eq. (63)). The phase factors $\eta_{\epsilon'}$ and η_{cb} are ± 1. Likewise, confining still to the index combination A, the contributions from the remaining five standard operators of $W^{(1)}$ labeled by the subscripts (II-VI) are found to be:

$$
\left(A^{(1)}_{II,\nu'}, \ A^{(1)}_{III,\nu'}, \ A^{(1)}_{IV\nu'}, A^{(1)}_{V\nu'}, \ A^{(1)}_{VI\nu'}\right) \simeq (-3, \ +2, \ -6, \ -0.4, \ -0.8)A^{(1)}_{\nu'} \tag{79}
$$

Combining the contributions (77) and (79), the total contribution of the terms containing ν' in the first six operators (I to VI) in W^L is given by

$$
A^{(1)}_{I \text{ to } VI,\nu'} \simeq \left[\frac{h^2_{33}\hat{f}(c,d)}{M_{\text{eff}}}\right]\eta_{cd}\eta_{\epsilon'}\eta_{cb}(2.74 \times 10^{-5})\epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \ . \tag{80}
$$

Next consider choice B. In this case, since one starts with the operator $\epsilon_{\alpha\beta\gamma}(u^\alpha_2 d'^\beta_2)(d'^\gamma_1 \nu_3)$, using the dressing as in Case A, one gets an extra minus sign compared to A, owing to the color factor. Following the same procedure as above, one obtains

$$
B^{(1)}_{I \text{ to } VI,\nu'} \simeq \left[\frac{h^2_{33}\hat{f}(c,d)}{M_{\text{eff}}}\right]\eta_{cd}\eta_{\epsilon'}\eta_{cb}(-1.82 \times 10^{-5})\epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3). \tag{81}
$$

Adding Eq. (80) and (81), the total contribution of terms containing ν' in the first six operators of $W^{(L)}$ for the index combinations A and B, leading to $\overline{\nu}_3 K^+$ emission, is given by:

$$
(A^{(1)} + B^{(1)})_{I \text{ to } VI,\nu'} \simeq \left[\frac{h^2_{33}\hat{f}(c,d)}{M_{\text{eff}}}\right]\eta_{cd}\eta_{\epsilon'}\eta_{cb}(0.9 \times 10^{-5})\epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \ . \tag{82}
$$

This checks with the second term in Eq. (70) obtained numerically – upto an overall sign, which is due to the difference in the sign of Eq. (81) compared to Eq. (35). Likewise, the corresponding contributions for the index combinations C and D from the first six operators in $W^{(L)}$ is found to be:

$$
(C^{(1)} + D^{(1)})_{I \text{ to } VI} \simeq \left[\frac{h^2_{33}\hat{f}(c,d)}{M_{\text{eff}}}\right]\eta_{cd}\eta_{\epsilon'}\eta_{cb}(-1.9 \times 10^{-5})\epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \tag{83}
$$

This is the first term of Eq. (70). Note that Eqs. (82)-(83) add constructively because $\eta_{ts} = -\eta_{cb}$. We still have to include (i) the contribution that arises by interchanging $i \leftrightarrow j$ while keeping (k, l) fixed (see Eq. (68)) and (ii) that from the terms containing charged lepton field ℓ in $W^{(L)}$. The latter contribute to neutrino emission by dressing the charged slepton fields ($\tilde{\ell}$). Including contributions from both (i) and (ii), one can verify that the net contribution of each kind – such as A, B, C, D listed above – is obtained by simply making the following substitution in Eq. (82)-(83), etc:

$$\hat{f}(c,d) \to \hat{f}(c,d) + \hat{f}(c,\ell) \ . \tag{84}$$

In arriving at this result, use has been made of the symmetric nature of the matrices $\hat{H}, \hat{A}, \hat{G}, \hat{F}$, as well as color antisymmetry. Thus, adding Eq. (82)-(83), and making the above substitutions, the net contribution to $\overline{\nu}_3 K^+$ decay mode from the first six terms of $W^{(L)}$, for the index combination $[A+B+C+D + (i \leftrightarrow j)]$ is given by:

$$[A + B + C + D + (i \leftrightarrow j)]_{I \text{ to } VI} \simeq \left[\frac{h_{33}^2 \left[\hat{f}(c,d) + \hat{f}(c,\ell) \right]}{M_{\text{eff}}} \right] \eta_{cd} \eta_{\epsilon'} \eta_{cb} \times$$
$$(2.8 \times 10^{-5}) \epsilon_{\alpha\beta\gamma} (d^\alpha u^\beta)(s^\gamma \nu_3) \ . \tag{85}$$

Having shown how the first two terms of the first bracket of Eq. (70) arise, let us now consider all the other terms in this bracket, which are proportional to M_{eff}^{-1}. It is clear that the joint contribution of the first two terms (Eq. (70)) dominates over the other terms in the same bracket by factors of 5 to 10. Although the relative signs (phases) of these subleading terms are uncertain (since we do not know the η_{ij}), we can see that their combined magnitude even if they all add constructively, is less than above (1.5×10^{-5}), which is about half of that given by the first two terms of Eq. (70). Not knowing the relative signs (phases) we conclude that the net contribution from the standard operator (first six terms of W) is given by:[9]

$$A(\overline{\nu}_\tau K^+)_{\text{std}} \simeq \left[\frac{h_{33}^2 2 \hat{f}(c,d)}{M_{\text{eff}}} \right] \eta_{cd} \eta_{\epsilon'} \eta_{cb} (2.8 \times 10^{-5})(\frac{1}{2} \text{ to } \frac{3}{2}) \epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \ . \tag{86}$$

Contribution to $\overline{\nu}_\tau K^+$ mode from the operator related to neutrino masses (the seventh term in $W^{(L)}$):
Confining first to the part containing ν' in the seventh term of $W^{(L)}$ (see Eq. (35)), we have:

$$W_{VII,\nu'}^{(L)} = -\frac{\epsilon_{\alpha\beta\gamma}}{M_{16}} \hat{F}_{ij} (\hat{G} V')_{kl} (u_i^\alpha d_j'^\beta (d_k'^\gamma \nu_l') \ . \tag{87}$$

Since $\hat{F}_{33}$ is the leading element of $\hat{F}$, first consider the following combination of indices:

---

[9]The reader will note that these unknown phases (or signs) are just the analogs of those which enter into $\tilde{t}$ versus $\tilde{c}$ contributions in minimal $SU(5)$, which is represented by a parameter y_{tK} etc, see Ref. [32,33]

Choice E: $(ij)(kl) = (33)(13)$: Starting with the operator $(u_3^\alpha d_3'^\beta)(d_1'^\gamma \nu_3)$, it is clear that $d_3'^\beta$ must be external. This yields $V_{ts} s^\beta$ (if $\tilde{d}_3$ is dressed, it would lead to external top, which is forbidden); thus $\tilde{u}_3^\alpha$ and $\tilde{d}_1^\gamma$ must be dressed to respectively yield $V_{td} d^\alpha$ and u^γ. Thus one finds:

$$
\begin{aligned}
E_{I,\nu'}^{(1)} &\simeq \frac{\hat{F}_{33}(\hat{G}V')_{13}}{M_{16}} V_{td} V_{ts} f(t,d) \epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \\
&\simeq \left(\frac{\hat{f}_{33} h_{33}}{M_{16} \tan\gamma} \right) \left[-(6\eta_{cb}\eta_{\epsilon'} + 4.3\eta_{cb}\eta_{\eta'})\eta_{td}\eta_{ts} \times 10^{-7} \right] \times \\
&\quad \zeta_{KM} f(t,d) \epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \ .
\end{aligned}
\tag{88}
$$

Here, we have used values of $\hat{F}_{ij}$ and $(\hat{G}V')_{kl} \tan\gamma$ listed in Appendix A (Eq. (65)-(66)), and have put $V_{td} = 0.006\eta_{td}$, $V_{ts} = 0.04\eta_{ts}$. The factor $\zeta_{KM} = 1/2$ to 1 denote the combined uncertainty in the two CKM elements. Note that the two terms in Eq. (88) check with the second and the fourth terms in Eq. (70) proportional to $(M_{16} \tan\gamma)^{-1}$. Some of the other leading contributions arise by choosing $(ij) = (23)$ or (32), varying (kl). These are proportional to y and is given by the eighth through the twelfth terms in the second bracket of Eq. (70). Note that the eleventh and the twelfth terms add (because $\eta_{ts}\eta_{cb} = -1$), and thus have a combined magnitude $\approx 2.7 \times 10^{-5} y$, while the eighth and the ninth terms subtract to have a net magnitude of $0.9 \times 10^{-5} y$. Thus, allowing for unknown phases of each of these contributions and also of the 11th term, we estimate that the total contribution from the y dependent terms is $\approx (2.7 \pm 2) \times 10^{-5} y \simeq (1.3 \pm 1) \times 10^{-6}$, where we have put $y \simeq 1/20$ (see Eq. (25)). Now the y–independent terms in the second square bracket of Eq. (70), including that exhibited in Eq. (71)), and the term proportional to $x \simeq 10^{-5}$ (say), are each individually smaller by factors of 2 to 4 compared to 1.3×10^{-6}. Although the relative phases (signs) of the various contributions, which depend on η_{ij}, are not known, allowing for possible cancellation between some of the smaller terms, it seems most plausible (as a conservative estimate) that the second square bracket in Eq. (70) has a net magnitude $\approx (1.3 \times 10^{-6})(1/2.5$ to $2.5)$. Now, including the contributions from interchange of $i \leftrightarrow j$, keeping (k,l) fixed, as appropriate, and that involving charged lepton dressing, as before, the net contribution from the new operator (the seventh term in $W^{(L)}$), related to the neutrino masses is thus given by:

$$
\begin{aligned}
A(\overline{\nu}_\tau K^+)_{\text{new}} &\simeq \left(\frac{\hat{f}_{33} h_{33}}{M_{16} \tan\gamma} \right) \left[(1.3 \times 10^{-6})(1/2.5 \text{ to } 2.5) \right] [f(t,d) + f(t,l)] \\
&\quad \epsilon_{\alpha\beta\gamma}(d^\alpha u^\beta)(s^\gamma \nu_3) \ .
\end{aligned}
\tag{89}
$$

Charged lepton decay modes:

The amplitude for proton decay into charged lepton, $p \to \ell^+ X$, is given in Eq. (69) for a generic superpotential of the form Eq. (67). The number of terms are fewer compared to the amplitude for neutrino mode, so they can be easily summed. The full result is given in Eqs. (73)-(74) for the muonic and electronic modes. We will briefly describe the origin of the dominant terms in the amplitude involving charged muons (Eq. (73)).

The standard $d = 5$ amplitude receives two contributions, one from the exchange of $\tilde{c} - \tilde{d}'$ squarks (the term proportional to V_{cs} in Eq. (73)), and one from the exchange of

$\tilde{t}\tilde{d}'$ (the V_{ts} term). They are comparable numerically. Note that unlike in minimal $SU(5)$ models, where these contributions undergo a unitarity cancellation, here the amplitude is not much suppressed compared to the neutrino mode. Allowing for all contributions to add constructively, but keeping track of the correlation $\eta_{cb}\eta_{ts} = -1$, we find that $\hat{A}(\mu^+) = 3.0 \times h_{33}^2 M_{\text{eff}}^{-1}$ from the standard $d = 5$ operators. This should be compared to the strength of the neutrino mode (Eq. (43)), which is not much larger.

The neutrino–mass–related $d = 5$ operator also receives two contributions, from the second and the third generation squarks. There are four types of terms in this case, proportional to $(1, y, x, z)$. Recall that $y \simeq 0.05$ from a fit to the neutrino masses and $x \simeq 10^{-3} - 10^{-5}$, while $z \simeq 0.003 - 0.03$ is a reasonable range. We see that all the amplitudes are of comparable size. The net contribution can be written as:

$$\hat{A}(\mu^+)|_{\text{new}} \simeq (M_{16} \tan\gamma)^{-1} \left(5 \times 10^{-7}(x/10^{-3}) + 10^{-7}(z/0.02) + 1.2 \times 10^{-7} \right) .$$

For $M_{16} \tan\gamma \sim 10^{17}$ GeV, this leads to observable rates.

APPENDIX C. CONTRIBUTION TO PROTON DECAY RATE FROM THE $D = 5$ OPERATOR

In this Appendix we give a general discussion of the numerical value for the $d = 5$ proton decay amplitude.

Define

$$\hat{A}(\overline{\nu}_i) = A_5(p \to \overline{\nu}K^+)/(2\overline{f}) . \tag{90}$$

A_5 denotes the strength of the respective four–fermion proton decay amplitudes arising from the $d = 5$ operators in W. One representative contribution to A_5 is given by Eq. (82). The net value of A_5 includes contributions from all the $d = 5$ operators, allowing for all possible combinations of the indices $(ij)(kl)$. It also includes the wino dressing factor and the relevant CKM mixing factors at all vertices. The net values are listed in Appendix B. The quantity $\overline{f}$ is the average of the two relevant dressing functions. For instance, for the amplitude in Eq. (90):

$$\overline{f} \equiv \left[\hat{f}(c,d) + \hat{f}(c,l) \right]/2 . \tag{91}$$

Assuming approximate degeneracy of the sfermions (for simplicity, see however, remarks later), $\hat{f}$ is the same for all contributions to the amplitude. Then $\hat{A}$ defined as above is just the net strength of the corresponding $d = 5$ operator in the superpotential, multiplied by the relevant CKM factors which occur at the vertices involving the color triplet and the wino exchanges. Since $\overline{f}$ has dimension of $(\text{mass})^{-1}$, $\hat{A}$ has dimension 5.

For comparison purposes, it is useful to note that for minimal SUSY $SU(5)$ involving exchange of color triplet H_C between $(\tilde{c}s)$ and $(\tilde{d}\nu_\mu)$ pairs, the $d = 5$ operator has strength $= (h_{22}^u h_{12}^d / M_{H_C}) \simeq (m_c m_s \sin\theta_C/(v_u v_d))/M_{H_C} \simeq (m_c m_s \sin\theta_C/v_u^2)(\tan\beta/M_{H_C}) \approx 9 \times 10^{-8}(\tan\beta/M_{H_C})$, where $\tan\beta = v_u/v_d$ and we have put $v_u = 174$ GeV and the extrapolated values of the fermion masses at the unification scale – i.e., $m_c \simeq 300$ MeV, and

$m_s \simeq 40$ MeV. Multiplying further by an additional factor of $\sin\theta_C$ due to conversion of $\tilde{c}$ to d at the wino vertex, for the case of $SU(5)$,

$$\hat{A}_{\tilde{c}\tilde{d}}(SU(5)) \simeq (1.9 \times 10^{-9})(\tan\beta/M_{H_C}) \ , \tag{92}$$

corresponding to exchange of the pair $(\tilde{c}, \tilde{d})$. There is a similar contribution involving the exchange of the pair $(\tilde{t}, \tilde{d})$ with the substitution $m_c \sin\theta_C \to m_t V_{td}$.

In calculating the proton decay rate, we will assume the following spectrum of supersymmetric particles as a guide: Squarks are nearly degenerate, with masses $m_{\tilde{q}} \approx 1$ TeV (1 to 1.5) and wino is lighter than the squarks – i.e., $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/6$ (1/2 to 2). Consistent with light gaugino masses, we will furthermore assume that $m_{1/2}$ (the common gaugino mass) is small compared to m_0 (the common scalar mass). In this case, starting with universal masses for the scalars at the unification scale, we would expect the slepton masses to be nearly degenerate with the squark masses at the electroweak scale.[10] The two f–functions that enter into the amplitude are then nearly equal (eg: in Eq. (84)), $\hat{f}(c,d) \simeq \hat{f}(c,l))$ and $\hat{f} \approx (m_{\tilde{W}}/m_{sq}^2)(\alpha_2/4\pi)$. A SUSY spectrum as described, i.e., $m_{\tilde{W}} \ll m_{\tilde{q}} \approx m_{\tilde{l}} \sim 1$ TeV is needed anyway, *a posteriori*, because without it SUSY grand unified models based on $SU(5)$ or $SO(10)$ are likely to run into conflict with the experimental limits on proton lifetime. A spectrum of this type is plausible in several scenarios of SUSY breaking.[11]

With the strength of $(du)(s\nu_\tau)$ operator being given by $A_5 = \hat{A}(2\overline{f})$, as in Eq. (90), the inverse decay rate for $p \to \overline{\nu}_\tau K^+$ is given by

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+) \approx (2.2 \times 10^{31}) \text{ yrs } \times$$
$$\left(\frac{.67}{A_S}\right)^2 \left[\frac{0.006\text{GeV}^3}{\beta_H}\right]^2 \left[\frac{(1/6)}{(m_{\tilde{W}}/m_{\tilde{q}})}\right]^2 \left[\frac{m_{\tilde{q}}}{1 \text{ TeV}}\right]^2 \left[\frac{2 \times 10^{-24} \text{ GeV}^{-1}}{\hat{A}(\overline{\nu})}\right]^2 \ . \tag{93}$$

Here β_H denotes the hadronic matrix element defined by $\beta_H u_L(\vec{k}) \equiv \epsilon_{\alpha\beta\gamma} \left\langle 0|(d_L^\alpha u_L^\beta)u_L^\gamma|p, \vec{k}\right\rangle$. While the range $\beta_H = (0.003 - 0.03)$ GeV^3 has been used in the past [33], given that one lattice calculations yield $\beta_H = (5.6 \pm 0.5) \times 10^{-3}$ GeV^3 [39], we will take as a plausible range: $\beta_H = (0.006$ GeV$^3)(1/2$ to 2). A_S stands for the short distance renormalization factor of the $d = 5$ operator, in minimal $SU(5)$ it has a central value of 0.67. Although this factor is slightly different in the $SO(10)$ model, we shall adopt $A_S = 0.67$ for the $SO(10)$ model as well.

Note that the familiar factors that appear in the expression for proton lifetime – i.e., M_{H_C}, $(1 + y_{tK})$ representing the interference between the $\tilde{t}$ and $\tilde{c}$ contributions and $\tan\beta$

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[10]Allowing for sleptons to be lighter than the squarks by a factor of 2 to 3 while keeping the squark masses to be about 1 TeV would amount to increasing $\overline{f}$ by about 50%, compared to its value for the case of nearly degenerate masses ($m_{\tilde{q}} \simeq m_{\tilde{l}}$), and thereby enhancing the rate by a factor of 2. Such enhancement is of course perfectly possible.

[11]For example, models of SUSY breaking based on contributions from a family universal anomalous $U(1)$ D–term, superposed with subdominant dilaton F–term contributions [55] would lead to such a spectrum.

– are all effectively contained in $\hat{A}(\overline{\nu})$. In fact, the analog of M_{H_C} for $SU(5)$ is given in our case by two mass scales: M_{eff} and M_{16}, representing contributions of the standard and the neutrino mass–related operators, respectively. The analog of $(1 + y_{tK})$ is reflected by the uncertainty in the net value of the many terms in Eq. (70), which depends on their unknown relative phases (see discussion in Appendix B, where a reasonable estimate of the uncertainty owing to the phases is given (Eq. (86) and (89)).

In minimal SUSY $SU(5)$, $\hat{A}$ is proportional to $(\sin 2\beta)^{-1} = 1/2(\tan\beta + 1/\tan\beta)$, and thus approximately to $\tan\beta$ for $\tan\beta \geq 3$ or so. Corresponding to a realistic treatment of fermion masses as in Sec. IV and V, this approximate proportionality to $\tan\beta$ does not however hold for $SO(10)$. The reason is this: If the fermions acquire masses only through the $\mathbf{10}_H$ in $SO(10)$, as is well known, the up and the down quark Yukawa couplings will be equal. This would give the familiar $t - b - \tau - \nu_\tau^D$ unification. By itself, it would also lead to a large value of $\tan\beta = m_t/m_b \simeq 60$, and correspondingly to a large enhancement in proton decay amplitude. Furthermore, it would also lead to the bad relations: $m_c/m_s = m_t/m_b$ and $V_{CKM} = 1$. However, in the presence of additional Higgs multiplets contributing to fermion masses, such as $\mathbf{16}_H$, which (a) distinguish between the up and the down sectors and (b) correct the bad relation mentioned above (see Sec. IV and V), $\tan\beta$ can get lowered considerably – for instance to values like 10 - 20. Now, with the contributions from $\langle \mathbf{10}_H \rangle$ and $\langle \mathbf{16}_H \rangle_d$ to fermion masses, as in Sec. IV and V, the simple proportionality of $\hat{A}$ to $\tan\beta$ disappears. In this case it is more useful to write A_5 or $\hat{A}$ simply in terms of the relevant Yukawa couplings. That is what we have done in writing W (see Eq. 34) in terms of the products of Yukawa coupling matrices like $(\hat{H})(\hat{H}V')$ etc, and likewise for the amplitude.

Although in the presence of $\langle \mathbf{10}_H \rangle$ and $\langle \mathbf{16}_H \rangle_d$, $\hat{A}$ is not directly proportional to $\tan\beta$ in $SO(10)$, the effect of a moderately large effective $\tan\beta \approx 20 - 30$, is still implicitly contained in $\hat{A}$.

It is instructive to compare a typical amplitude for $SO(10)$ to that for $SU(5)$. This ratio is given by:

$$\frac{\hat{A}(\overline{\nu}_\mu K^+)^{SO(10)}_{\mathrm{std}}}{\hat{A}(\overline{\nu}_\mu K^+)^{SU(5)}_{\mathrm{std}}} \approx \frac{h_{33}^2}{M_{\mathrm{eff}}} \frac{2 \times 10^{-5}}{1.9 \times 10^{-9}(\tan\beta/M_{H_c})}$$
$$\approx (m_{H_C}/M_{\mathrm{eff}})(88)(3/\tan\beta) . \tag{94}$$

We have put $h_{33}^2 \simeq 1/4$ in going from the first line to the second in Eq. (94). Thus we see that M_{eff} has to be $(88 - 53)$ times larger (for $\tan\beta = 3$ to 5) so that the $SO(10)$ amplitude may be comparable to that of $SU(5)$. In other words, if M_{eff} were equal to M_{H_C}, one would have a net enhancement by about two orders of magnitude of the $d = 5$ amplitude for proton decay in $SO(10)$ compared to that of $SU(5)$. This large enhancement of the amplitude in $SO(10)$ – barring the factor of M_{H_c}/M_{eff} – has come about due to a combination of several factors: (i) the large off–diagonal coupling of the up-type quarks with the color triplets which scale as $\sqrt{m_c m_t}$ in the $SO(10)$ model in contrast to the diagonal m_c in minimal $SU(5)$, (ii) the larger μ Yukawa coupling to color triplets (relative to the strange quark Yukawa in $SU(5)$) and (iii) the presence of several terms associated with a realistic quark–lepton spectrum (see Eq. (35)).

$SO(10)$, however, has a possible source of suppression of the $d = 5$ amplitude, because of the nature of the doublet–triplet splitting mechanism in it. The suppression would arise

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if the mass M_{10} of $\mathbf{10}'_H$ is considerably smaller than the scale of $\lambda \langle \mathbf{45}_H \rangle = \lambda a$, as discussed in Section VI.A. In this case, $M_{\text{eff}} \equiv (\lambda a)^2/M_{10}$ can far exceed λa. Since $\langle \mathbf{45}_H \rangle$ breaks $SO(10)$ to $SU(3)_C \times SU(2)_L \times I_{3R} \times (B-L)$, one of course naturally expects λa to be nearly equal to or somewhat larger than the unification scale $M_U \approx 2 \times 10^{16}$ GeV. But M_{10} can in general be one or even two orders of magnitude smaller than M_U. For instance, $\mathbf{10}'_H$ may carry a charge that would forbid a mass term $(\mathbf{10}'_H)^2$ at the renormalizable level. Such a mass term could still effectively arise through nonrenormalizable operators by utilizing the VEVs of certain fields ϕ_i which do not conserve the respective charge. In this case, the mass term M_{10} may be suppressed by relevant powers of $[\langle \phi_i \rangle /M]$ where one may expect $\langle \phi_i \rangle /M \sim 1/10$. If $M_{10} \sim (1/10 - 1/100)\lambda a$, M_{eff} can exceed M_U and thus M_{H_C} by one or two orders of magnitude. The importance of such a large value of M_{eff} for coupling unification is discussed in the text.

More precise constraints on M_{eff} and $M_{16} \tan \gamma$ are obtained directly by using limits on proton lifetime. They are discussed in the text in Sec. VI.

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