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DOCTORAL GENERAL EXAMINATION

PART I

January 30th, 2012

FIVE HOURS

1. This examination is divided into five sections, each consisting of four problems. Answer all the problems. Each problem is worth 5 points, thus the maximum score for the exam is 100.
2. For each section, please use the booklets that you have been given according to the section listed. Do not put your name on it, as each booklet has an identification number that will allow the papers to be graded blindly.
3. A diagram or sketch is often useful to the grader and could improve your grade.
4. Read each problem carefully and do not do more work than is necessary. For example "give" and "sketch" do not mean "derive".
5. Calculators may **not** be used.
6. No books, notes or reference materials may be used.

Possibly Useful Fomulae and Constants

A normalized Gaussian distribution with mean μ and standard deviation σ is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\; e^{-(x-\mu)^2/2\sigma^2}$$

A normalized Poisson distribution of mean $\bar{n}$ is

$$p(m) = \frac{\bar{n}^m}{m!}\; e^{-\bar{n}}$$

A normalized binomial distribution of mean $\mu = n\theta$ and standard deviation $\sqrt{n\theta(1-\theta)}$ is

$$p(x) = \frac{n!}{x!(n-x)!}\; \theta^x\; (1-\theta)^{n-x}$$

The magnetic field of a dipole with magnetic moment $\vec{m}$ at a distance r away is given by

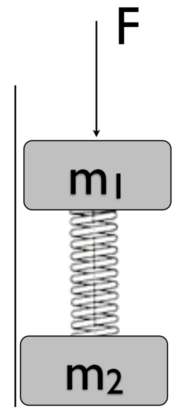
$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}[\frac{3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}}{r^3}]$$

k_B	$=$	1.4×10^{-23}	$\mathrm{J\,K^{-1}}$	$=$	$10^{16}\,\mathrm{erg\,K^{-1}}$
$\hbar$	$=$	1×10^{-34}	J.s	$=$	$10^{-27}\,\mathrm{erg\cdot s}$
c	$=$	3×10^{8}	$\mathrm{m\,s^{-1}}$	$=$	$3 \times 10^{10}\,\mathrm{cm\,s^{-1}}$
e	$=$	1.6×10^{-19}	Coulomb	$=$	4.8×10^{-10} statcoulomb
m_e	$=$	9.1×10^{-31}	kg	$=$	$0.5\,\mathrm{MeV/c^2}$
$\hbar c$	$=$	3.2×10^{-26}	$\mathrm{J\cdot m}$	$\simeq$	$1.97 \times 10^{-13}\,\mathrm{MeV\cdot m}$
G	$=$	6.7×10^{-11}	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	$=$	$6.7 \times 10^{-8}\,\mathrm{cm^3\,g^{-1}\,s^{-2}}$
ε_0	$=$	8.9×10^{-12}	$\mathrm{F\,m^{-1}}$	$=$	1 (cgs)
μ_0	$=$	$4\pi \times 10^{-7}$	$\mathrm{N\,A^{-2}}$	$=$	1 (cgs)

GROUP I

I-1 Pogo Stick

Two masses m_1 and m_2 are connected by a massless spring with spring constant k (see diagram). Initially m_1 is pushed down with force F. After some time, the force is removed. How big must F have been, if the subsequent motion causes m_2 to be lifted from the ground?



I-2 Electromagnetic Pendulum

Suppose a small sphere of charge $-q$ is suspended along the positive $\hat{z}$-axis by a massless string above a very large plate of charge Q and area A (where A is much larger than the distance from the charged surface to the sphere). Suppose the mass is moving horizontally with a velocity $\vec{v} = -v_0\hat{x}$ with respect to the plate. What is the electromagnetic force acting on the sphere in the reference frame of the sphere? You should not assume that $v_0 \ll c$, but you may ignore any radiative losses.

I-3 1D Potential

A non-relativistic particle of mass m moves in a 1-dimensional potential

$$V(x) = A|x|,$$

where A is a positive constant. Estimate the minimum total energy of the particle by using the Heisenberg uncertainty principle and explicitly minimizing an expression for the energy.

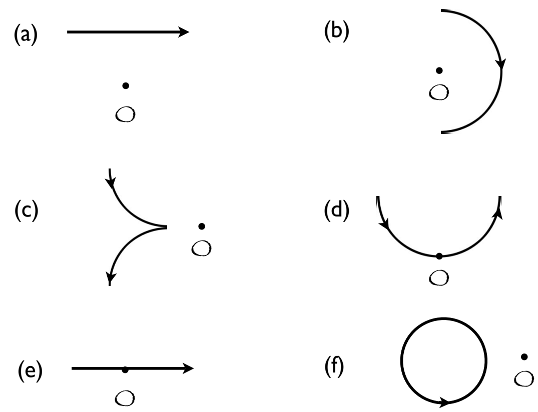
I-4 Relativistic Fermi Gas

Consider an ideal Fermi gas of N uncharged particles with spin 1/2 and rest mass m (each) confined to a small three dimensional volume V at temperature $T \ll \epsilon_F/k$. Find the Fermi energy ϵ_F of this gas in the extreme relativistic limit where $\epsilon_F \gg mc^2$.

GROUP II

II-1 Angular Momenta Along Trajectories

For each of the trajectories (a)-(f), state whether or not the angular momentum $\vec{\mathbf{L}}$ about the origin (denoted by $\mathcal{O}$) could be conserved. If it is not, explain why.



II-2 Three Bosons

Three non-identical spin-1 particles with spin operators $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ interact via a Hamiltonian

$$H = \frac{E_0}{\hbar^2}(\vec{s}_1 \cdot \vec{s}_2 + \vec{s}_1 \cdot \vec{s}_3 + \vec{s}_2 \cdot \vec{s}_3).$$

How many independent states are there for the three particle system? What is the energy and the degeneracy of each of the lowest two energy levels?

II-3 Magnetic Monopole

Imagine an electric charge moving in the field of a magnetic monopole (although none has yet been found). Set up the non-relativistic equation of motion for an electric charge q of mass m in the field of a magnetic monopole of strength Γ (a positive constant).

Assume the particle at a particular moment in time is a distance r from the magnetic monopole and that the particle's velocity $\vec{\mathbf{v}}$ is perpendicular to the line between the charged particle and the monopole. Give an expression for the force vector on the particle at this point.

II-4 Decompression of Ideal Gas

An ideal monoatomic gas at a temperature of 20 degrees Celsius expands adiabatically so that the final volume is 8 times the initial volume. What is the final temperature?

GROUP III

III-1 The Sopwith Camel

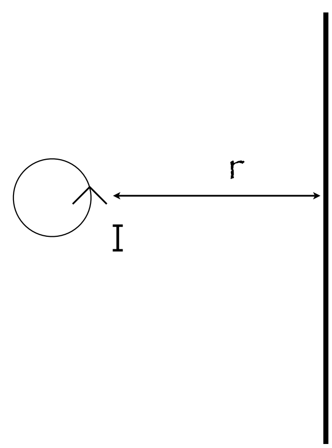
The Sopwith Camel was a single engine fighter plan flown by the British in WWI (and also by the character Snoopy in the Peanuts comic strip). It was powered by a radial engine, and the entire engine rotated with the propeller. The Camel had an unfortunate property: if the pilot turned to the right, the plane tended to go into a dive, while a left turn caused the plane to climb steeply. These tendencies caused inexperienced pilots to crash or stall during take-off. From the perspective of the pilot, who sat behind the engine, did the engine rotate clockwise or counter-clockwise? Explain *briefly* how you arrived at your answer.

III-2 Compton Scattering

In 1923, Compton performed a series of experiments in which he was scattering X-rays from a graphite scatterer. The wavelength of the X-rays emitted by his source was $\lambda = 0.7$ Angstroms. What was the wavelength of the longest wavelength scattered X-rays that he observed?

III-3 Magnetic Flux

A current I flows around the small loop of radius a shown in the figure. Calculate the total magnetic flux that falls to the right of the infinite line shown a distance $r \gg a$ from the loop. The current loop and the line are all in the same plane.



III-4 Lattice Defects

A cubic lattice of N atoms has M interstitial sites (site between lattice sites). An atom can be displaced from its lattice site to any one of the M interstitial sites at a cost of energy ϵ. Find the number of atoms located at interstitial sites as a function of temperature, $n(T)$. You may assume that $n \gg 1$ and that $M, N \gg n$ and for any large x you may use the approximation $x! \approx x \log x$.

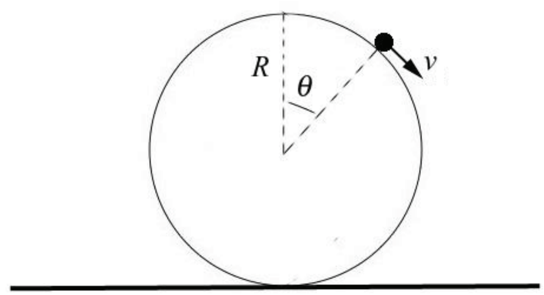
GROUP IV

IV-1 Radiative Transfer Between Plates

Two parallel plates plates are maintained at temperatures T_L and T_R respectively and have emissivities ϵ_L and ϵ_R respectively. Given the Stephan-Boltzmann constant σ, express the net energy transfer rate per area from the left plate (L) to the right plate (R). *Hint:* this problem can be solved by using an infinite series, or by finding the energy transfer rate per area to the right and left, I_R and I_L, respectively.

IV-2 Particle on a Sphere

A particle initially sits on top of a large smooth sphere of radius R as shown in the figure. The particle begins to slide along the surface of the sphere. There is negligible friction between the particle and the surface. Let g denote the gravitational constant. Determine the angle θ_1 with respect to the vertical at which the particle will lose contact with the surface of the sphere.



IV-3 Simple Harmonic Oscillator

A certain particle is free until $t = 0$, at which time a simple harmonic oscillator (SHO) potential is suddenly switched on. For convenience, assume that we have re-scaled the problem so that the time-independent Schrödinger equation for the SHO is

$$-\frac{d^2}{dx^2}\varphi_n(x) + x^2\varphi_n(x) = E_n\varphi_n(x),$$

from which the normalized ground state wavefunction is

$$\varphi_0(x) = (\pi)^{-1/4}e^{-\frac{1}{2}x^2},$$

with energy $E_0 = 1/2$. If the free particle's wavepacket at time $t = 0$ is proportional to

$$e^{-\frac{1}{2}x^2}e^{ikx},$$

where k is real, what is the probability that a measurement of the energy at a later time will *not* yield $E = 1/2$?

IV-4 Pluto

To a good approximation, the surface temperatures of planets and dwarf planets in our solar system can be estimated by assuming that they are thermodynamically ideal blackbodies that absorb all of the incident radiation from the Sun and re-radiate all of this energy as thermal radiation. If the Earth's surface temperature is 300 K, estimate the temperature on the surface of Pluto. Possibly useful facts are that Pluto is 40 times more distant from the sun than the Earth is, and Pluto's radius is one-fifth that of the Earth's.

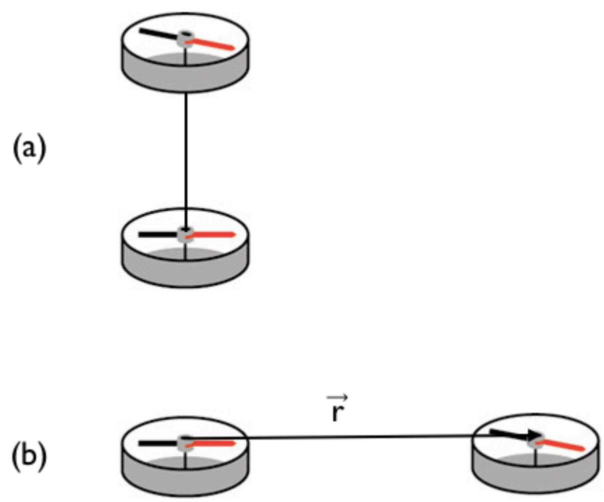
GROUP V

V-1 Needles

Consider two identical compass needles that are placed at a distance $\vec{r}$ relative to each other (with $|r| \gg$ the length of the needles). Determine the relative orientation of the needles for the following two cases (neglecting Earth's magnetic field):

1. The needles are mounted on a common axis running through their respective centers, such that each needle can rotate freely in a plane perpendicular to this axis.

2. The needles are displaced horizontally from each other, such that they both can rotate in the same horizontal plane



V-2 Faulty Cup
Certain cups used for soups have the unfortunate design flaw that they tend to tip over when filled if tilted at a small angle, causing burns. Imagine such a cup with a base radius R_1 and top radius R_2 (with $R_1 < R_2$) and filled to a depth h. At what angle with respect to the table surface do you need to tilt the cup for it to spill? You may treat the contents of the cup as one solid rigid object and the walls of the cup to be of negligible mass.

V-3 Discharging a Plate A very large metal plate carries a charge of $Q = -1$ C. The work function for the metal is $\phi = 3$ eV. The plate is illuminated by a 60 Watt light source with a wavelength λ of 330 nm. How long does it take to completely discharge the plate? How does the answer change for a light source with $\lambda = 660$ nm?

V-4 Mixing Gases Consider two containers with volume V_1 and V_2, respectively. V_1 contains oxygen at pressure p_1, while V_2 contains nitrogen at pressure p_2. At constant temperature T, a valve connecting the two containers is opened.

1. What is the final pressure p in the two containers?

2. What is the change in entropy once the gases are mixed completely?

3. Would the change in entropy be different if both containers were filled with nitrogen initially? Give an argument, but no calculation is necessary.