## Section I: Involving Some Quantum Mechanics

### I-1 Proton-Antiproton

Consider a system of ~~a~~ proton and an antiproton fixed at distance ~~r~~ $d$ apart. ~~Find~~ Evaluate the magnetic interaction energy for this system in terms of the magnetic moment $\vec{\mu}_0$ of the proton, for the eigenstates of total spin ~~S~~. In general, two magnetic dipoles have

$$V(\vec{r}) = \left\{\vec{\mu}_1\vec{\mu}_2 - 3(\vec{\mu}_1\vec{r})(\vec{\mu}_2\vec{r})/r^2\right\}/r^3$$

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Choose $\overset{1}{+}\xrightarrow{\ d\ }\overset{2}{+}\longrightarrow z$

Sln

$$\left.\begin{aligned} p:&\quad \mu_1 = 2\mu_0 S_1 \\ \bar{p}:&\quad \mu_2 = -2\mu_0 S_2 \end{aligned}\right\} \rightarrow V = \frac{\mu_0^2}{d^3}\left\{-4\vec{S}_1\vec{S}_2 + 12 S_{1z}S_{2z}\right\}$$

For spin ½ particles: $2\vec{S}_1\vec{S}_2 = S(S+1) - \frac{3}{2}$ ⊗

$$2S_{1z}S_{2z} = S_z^2 - \frac{1}{2}$$

Then: $$V = \frac{\mu_0^2}{d^3}\left\{-2S(S+1) + \cancel{3} + 6S_z^2 - \cancel{3}\right\}$$

Eigenstates of total spin ($p\bar{p}$ syst) $S=0, S_z=0$ and $S=1$
$S_z = 1, 0, -1$

Evaluate: $$\boxed{V_{00} = 0 \qquad V_{1,0} = -\frac{4\mu_0^2}{d^3} \qquad V_{1,\pm1} = \frac{2\mu_0^2}{d^3}}$$

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App. ⊗ $S^2 = S_1^2 + S_2^2 + 2\vec{S}_1\vec{S}_2 \rightarrow 2\vec{S}_1\vec{S}_2 = \hbar^2[S(S+1) - S_1(S_1+1) - S_2(S_2+1)]$
$= \hbar^2[S(S+1) - \frac{3}{2}]$

$S_z^2 = S_{1z}^2 + S_{2z}^2 + 2S_{1z}S_{2z} \rightarrow 2S_{1z}S_{2z} = S_z^2 - \frac{1}{4} - \frac{1}{4}$

## I-2 Spin States

An electron is in a homogeneous magnetic field along the z-direction. At t=0 its spin points in the +x-direction.

a) Give a simple Hamiltonian and an ansatz for the spinwave function.
b) Solve the Schroedinger equation.
c) Calculate the probability for t>0 to find the electron in a state of:
i) $S_x$=1/2 ii) $S_x$ = -1/2 iii) $S_z$ = 1/2.

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Sln a). Take for $e^-$ : $H = \mu_0 B \sigma_z$ , $\mu_0$ = mag. mom. of $e^-$

Pauli : $\sigma_x = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$ $\sigma_y = \begin{pmatrix} & -i \\ i & \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Ansatz : $\psi = A(t) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C(t) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

b) Satisfy Schr: $i \frac{\partial \psi}{\partial t} = H \psi$

init. cond. $t = 0$: $A(0) = e^{i\delta}$ $C(0) = 0$

guarantees at $t = 0$ spin is in eigenstate $S_x = \frac{1}{2}$

eq. of mot. $i \dot{A}(t) = \mu_0 B C(t)$ and
$i \dot{C}(t) = \mu_0 B A(t)$

Sln $A(t) = e^{i\delta} \cos(\mu_0 B t)$
$C(t) = -i e^{i\delta} \sin(\mu_0 B t)$

c)

i) $P(S_x = +\frac{1}{2}) = |A(t)|^2 = \cos^2(\mu_0 B t)$

ii) $P(S_x = -\frac{1}{2}) = |C(t)|^2 = \sin^2(\quad)$

iii) $\langle S_z \rangle = 0 = \frac{P_{+z} + P_{-z}}{2} \rightarrow P(S_z = +\frac{1}{2}) = P(S_z = -\frac{1}{2}) = \frac{1}{2}$

**I-3 Truncated Oscillator**

Find the energy levels in the potential:

$$V(x) = \frac{1}{2} m\omega^2 x^2 \text{ for } x > 0$$
$$= \infty \text{ for } x < 0,$$

using the one dimensional Schroedinger equation.

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1D - Schr: $-\frac{\hbar}{2m}\frac{d}{dx^2} u = \{E - V(x)\} u$ $\quad \psi = u(x)\, e^{-i\frac{E}{\hbar}t}$ separation

Schiff, 60

$\frac{d^2 u}{d\xi^2} + (\lambda - \xi^2)\, u = 0$ with $\xi = \sqrt{\frac{m\omega}{\hbar}}$, $\lambda = \frac{2E}{\hbar\omega}$

$u(\xi) = H(\xi)\, e^{-\frac{\xi^2}{2}}$ : $H'' - 2\xi H' + (\lambda - 1) H = 0$ Hermite Poly.

sln if $\lambda = 2n + 1$

E levels : $E_n = (n' + \frac{1}{2})\hbar\omega$ for full harm. osz. potent.

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Easy $x > 0$ : $U$ = all sln of full osz. with "0 node" at $x = 0$

$n' = 1, 3, 5 \cdots$ = odd eigenfunctions with

Energy Eigenvalues $\boxed{E_n = (2n + \frac{3}{2})\hbar\omega}$ $\quad n = 0, 1, 2 \cdots$

**I-4 Electron in B-field**

An electron moves in free space filled with a homogenous magnetic field $B_0$ in the z direction. Neglecting spin,

(a) write down the Schroedinger equation, and
(b) find the energy levels.
(c) Show the quantization of the magnetic flux enclosed by large orbits.
(Hint: Start with the Bohr Sommerfeld quantization.)

Hint: You may take $\vec{A} = -B_y\hat{y}$. The students were instructed to ignore this error in the exam. It did not seem to cause any confusion (except for 1 student who was graded more leniently).

One can take $\vec{A} = B_0 x \hat{y}$ or alternatively $\vec{A} = -B_0 y \hat{x}$ or $\vec{A} = \frac{B_0}{2}(-y\hat{x} + x\hat{y})$ and obtain the same results.

a) $$\hat{H} = \frac{\hat{P}_x^2 + \hat{P}_z^2}{2m} + \frac{1}{2m}\left(\hat{P}_y \quad \frac{eB_0}{c}x\right)^2$$ $e$ is the charge on the electron

let $\hat{q} = x \quad \frac{c}{eB_0}\hat{P}_y$

$$[\hat{P}_z, \hat{q}] = [\hat{P}_y, \hat{q}] = 0, \quad [\hat{P}_x, \hat{q}] = -i\hbar$$

$$\hat{H} = \frac{\hat{P}_z^2}{2m} + \left(\frac{\hat{P}_x^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2\right)$$ ← Hamiltonian for harmonic oscillator

$$\omega = eB_0/mc$$

b) $$E_n = \frac{P_z^2}{2m} + \hbar\omega(n + 1/2), \quad \hat{H}\psi = E_n\psi$$

c) Bohr - Sommerfeld

$$I \equiv \oint \vec{P} \cdot d\vec{q} = \oint \left(\vec{p} - \frac{e}{c}\vec{A}\right) \cdot d\vec{q} = (n + 1/2)\, h$$

use semiclassical approach with large circular orbits, $\omega = eB/mc$

$$I_1 \equiv \oint \vec{p} \cdot d\vec{q} = \int_0^{2\pi} m r^2 \omega \, d\theta = 2\pi m r^2 \frac{eB}{mc} = 2\frac{e\Phi}{c}$$

where $\Phi = \int_A \vec{B} \cdot d\vec{A} = \pi r^2 B$ = magnetic flux

$$I_2 \equiv -\frac{e}{c} \oint \vec{A} \cdot d\vec{q} = -\frac{e}{c} \int_A (\nabla \times \vec{A}) \cdot d\vec{A}$$

$$= -\frac{e}{c} \int_A \vec{B} \cdot d\vec{A} = -\frac{e}{c}\Phi$$

so $I = I_1 - I_2 = \frac{e}{c}\Phi = (n + 1/2)\, h$

for $\Delta n = 1$, $\Delta \phi = \frac{hc}{e}$ quantized!

Linear Superposition:
one spere $\rho$ ; the small one $-\rho$

$$\vec{r} = \vec{r}_s + \vec{a}$$

Always inside the cavity
from large sphere: $\vec{\mathcal{E}}_L = \frac{4}{3}\pi|r|^3\rho\frac{1}{|r|^2}\frac{\vec{r}}{|r|}$ Gauss Law

$$\mathcal{E}_L = \frac{4}{3}\pi\rho\,\vec{r}$$

small sphere: $-\rho$ $\mathcal{E}_s = \frac{4}{3}\pi|r_s|^3(-\rho)\frac{1}{|r_s|^2}\frac{\vec{r}_s}{|r_s|}$

$$\mathcal{E}_s = \frac{4}{3}\pi(-\rho)\,\vec{r}_s$$

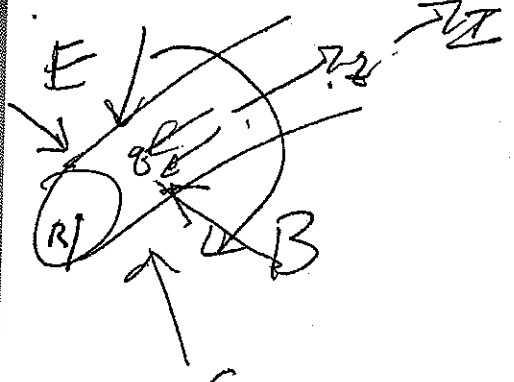
But $\vec{r}_s = \vec{r} - \vec{a}$

By superposition

a) $\mathcal{E}_T = \mathcal{E}_L + \mathcal{E}_s = \frac{4}{3}\pi\rho[\vec{r} - \vec{r}_s] = \frac{4}{3}\pi\rho\,\vec{a}$

constant field in cavity.

$$\frac{\rho\,\vec{a}}{3\epsilon_0} \text{ in MKS}$$




Gauss: $\iint E\,dA = q/\epsilon_0$

$$E\,2\pi R l = \frac{\rho \pi R^2 l}{\epsilon_0} = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0}\frac{1}{2\pi R l}$$

$$q = \rho \pi R^2 l \qquad \pi R^2 \rho v_z = I$$

$$q = \frac{I l}{v_z}$$

$$E = \frac{I}{2\pi \epsilon_0 v_z R}$$

Amp. $\oint B \cdot dl = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi R}$$

Force is Radial $= -eE$ and $-eBv_z$

$$F = \frac{Ie}{2\pi \epsilon_0 R v_z}(1 - \mu_0 \epsilon_0 v_z^2) = \frac{Ie}{2\pi \epsilon_0 R v_z \gamma^2}$$

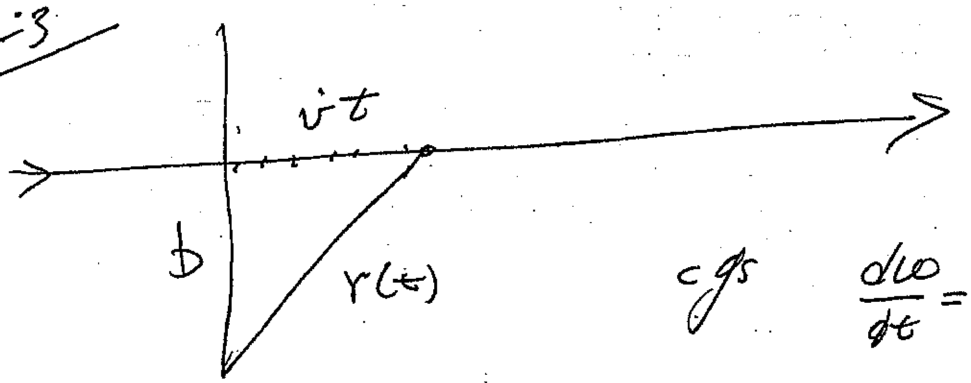
$$\gamma m_0 \frac{dv_r}{dt} = F = \gamma m_0 \frac{d^2R}{dt^2}$$

$$\frac{d^2R}{dt^2} = \frac{Ie}{2\pi \epsilon_0 R\, m_0 (\beta c) \gamma^3}$$

$$\beta = \frac{v}{c} \approx \frac{v_z}{c}$$

error on page of quiz
$d^2R/dz^2 \rightarrow$ instead of $\frac{d^2R}{dt^2}$

II-3




cgs $\quad \frac{dW}{dt} = \frac{2}{3}\frac{e^2}{c^3}|\dot{\vec{v}}|^2$

a)

$$\underline{mks \rightarrow \frac{dW}{dt} = \frac{2}{3}\frac{e^2}{c^3}} \cdot \frac{1}{4\pi\epsilon_0}|\dot{\vec{v}}|^2$$

$$\frac{dW}{dt} = \frac{2e^2}{3c^3}\frac{1}{4\pi\epsilon_0}\left|\frac{Qe}{m4\pi\epsilon_0 |r(t)|^2}\right|^2 \qquad \boxed{\frac{dW}{dt} = \frac{e^2|\dot{\vec{v}}|^2}{6\pi\epsilon_0 c^3}}$$

$$|r(t)|^2 = b^2 + (vt)^2$$

$$W = \int_{-\infty}^{\infty}\frac{dW}{dt}dt$$

$$= \frac{e^4Q^2}{\cancel{4}\times 96\pi^3\epsilon_0^3 m^2c^3}\underbrace{\int_{-\infty}^{\infty}\frac{dt}{(b^2+v^2t^2)^2}}$$

$$x = \frac{vt}{b}$$

$$\frac{1}{vb^3}\int_{-\infty}^{\infty}\frac{d\left(\frac{vt}{b}\right)}{(1+x^2)} \qquad \frac{\pi}{2}$$

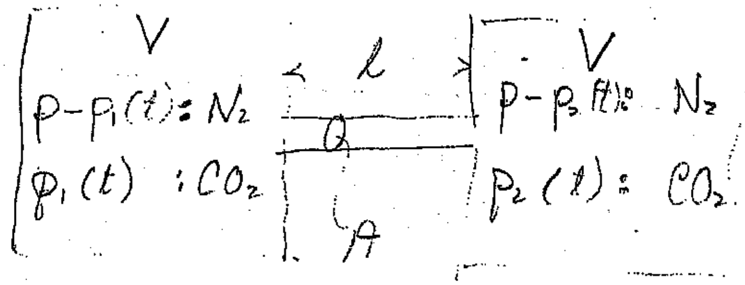
$$W = \frac{e^4Q^2}{192\pi^2\epsilon_0^3m^2c^3}\frac{1}{vb^3}$$

b) $$\int_{b_{min}}^{\infty} W\,N\,v\,2\pi b\,db = \frac{e^4Q^2N}{192\pi^2\epsilon_0^3m^2c^3}\int\frac{v\,2\pi b\,db}{vb^3}$$

$$= \frac{e^4Q^2N}{192\pi^2\epsilon_0^3m^2c^3}2\pi\underbrace{\int_{b_m}^{\infty}\frac{b\,db}{b^3}} \qquad -\frac{1}{b}\Big|_{b_m}^{\infty} = \frac{1}{b_m}$$

$$= \frac{e^4Q^4N}{96\pi\epsilon_0^3m^2c^3\,b_{min}}$$

III-1 Mixing gases




Call $J$ the flux of $CO_2$ in the connecting tube.

By Fick's Law

$$\boxed{J = -D\,\mathrm{grad}\,n.} \qquad 2 \qquad (1)$$

where $n$ is the particle density of $CO_2$ in the tube. Call $n_1$ & $n_2$ the particle densities of $CO_2$ in $V_1$ + $V_2$, respectively. Then

$$n_1(t) + n_2(t) = n_0 \qquad (2)$$

so $$\boxed{\mathrm{grad}\,n = \frac{n_2 - n_1}{l} = \frac{n_0 - 2n_1}{l}}. \qquad 2 \qquad (3)$$

By continuity, $$\boxed{V\frac{dn_1}{dt} = -AJ.} \qquad 2 \qquad (4)$$

Combining (1), (3) & (4) we obtain

$$\boxed{\frac{V\,dn_1}{n_0 - 2n_1} = \frac{DA}{l}\,dt.} \qquad 2 \qquad (5)$$

Initial conditions are $n_1(t=0) = n_0$, $n_2(t=0) = 0$.

Solving (5) with the initial conditions,

$$n_1 = \frac{n_0}{2}\left\{1 + \exp\left(-\frac{2DA}{lV}t\right)\right\} \qquad (6)$$

But $p = nkT$. Thus

$$\boxed{p_1 = \frac{p_0}{2}\left\{1 + \exp\left(-\frac{2DA}{lV}t\right)\right\}} \qquad 2 \qquad (7)$$

III-2 Specific Heat

given $dQ = dU + p\,dV$

a) Definitions: $\boxed{dS = \frac{dQ}{T},\ C_V = \left(\frac{\partial Q}{\partial T}\right)_V,\ C_p = \left(\frac{\partial Q}{\partial T}\right)_p}$ 1

$$dS = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V dT + \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV.$$

Equating cross derivatives: $\left(\frac{\partial}{\partial V}\right)_T \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{\partial}{\partial T}\right)_V \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]$

$$\rightarrow \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Therefore $\boxed{TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV}$ (1) 2

$$dS = \left[\frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_p + \frac{p}{T}\left(\frac{\partial V}{\partial T}\right)_p\right]dT + \left[\frac{1}{T}\left(\frac{\partial U}{\partial P}\right)_T + \frac{p}{T}\left(\frac{\partial V}{\partial P}\right)_T\right]dp$$

Again, equating cross derivatives:

$$\rightarrow \left(\frac{\partial U}{\partial p}\right)_T = -p\left(\frac{\partial V}{\partial P}\right)_T - T\left(\frac{\partial V}{\partial T}\right)_p .$$

Therefore $\boxed{TdS = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dP}$ (2) 2

Subtracting (2) from (1) and setting $dT = \left(\frac{\partial T}{\partial V}\right)_p dV + \left(\frac{\partial T}{\partial P}\right)_V dP$,

$$\left[(C_p - C_v)\left(\frac{\partial T}{\partial p}\right)_V - T\left(\frac{\partial V}{\partial T}\right)_p\right]dP + \left[(C_p - C_v)\left(\frac{\partial T}{\partial V}\right)_p - T\left(\frac{\partial P}{\partial T}\right)_V\right]dV = 0.$$ 2

a. Since $P$ + $V$ are independent variables, the coeff of $dp$ + $dV \equiv 0$.

Therefore $\boxed{C_p - C_V = T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_P}$ QED.

b) given $PV = RT + B(T)P$

$$p\left(\frac{\partial V}{\partial T}\right)_P = R + p\frac{dB}{dT},\quad \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{p} + \frac{dB}{dT}$$

$$V\left(\frac{\partial P}{\partial T}\right)_V = R + B\left(\frac{\partial P}{\partial T}\right)_V + P\frac{dB}{dT},\quad \left(\frac{\partial P}{\partial T}\right)_V = \frac{R + p\frac{dB}{dT}}{V - B}$$

$$\therefore\ C_p - C_V = T\left(\frac{R}{p} + \frac{dB}{dT}\right)\left(\frac{R + p\frac{dB}{dT}}{V - B}\right) = \frac{T}{V-B}\left\{\frac{R^2}{P} + 2R\frac{dB}{dT} + p\left(\frac{dB}{dT}\right)^2\right\}.$$

But $V - B = \frac{RT}{P}$. Therefore $\boxed{C_p - C_V \cong R + 2p\frac{dB}{dT}}$ QED. 4

III-2 Specific Heat (alternative derivation).



given $dQ = dU + PdV$

define $dS = \frac{dQ}{T}$, $C_p = \left(\frac{\partial Q}{\partial T}\right)_p$, $C_v = \left(\frac{\partial Q}{\partial T}\right)_v$

$$C_p - C_v = T\left[\left(\frac{\partial S}{\partial T}\right)_p - \left(\frac{\partial S}{\partial T}\right)_v\right] \qquad 1 \qquad (1)$$

Using the identity $\left(\frac{\partial A}{\partial x}\right)_y = \left(\frac{\partial A}{\partial x}\right)_w + \left(\frac{\partial A}{\partial w}\right)_x \left(\frac{\partial w}{\partial x}\right)_y$

and setting
$A = S$
$x = T$
$y = P$
$w = V$

we obtain $\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_v + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$. $\qquad 2 \quad (2)$

now $F = U - TS$ so $dF = dU - TdS - SdT = -SdT - PdV$.

Thus $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$. $\qquad 2 \qquad (3)$

substituting (3) in (2) & the result in (1)

$$C_p - C_v = T\left[\cancel{\left(\frac{\partial S}{\partial T}\right)_v} + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p - \cancel{\left(\frac{\partial S}{\partial T}\right)_v}\right]$$

or $\boxed{C_p - C_v = T\left(\frac{\partial P}{\partial T}\right)_v \left(\frac{\partial V}{\partial T}\right)_p}$ QED. 2

# III-3 Debye Shielding

MKS

$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0}$ 1

a) $\rho = +e\left[n_0^+ - n^-(r)\right]$

$n^-(r) = n_0^- \exp\left(-\frac{e\Phi}{kT}\right) \approx n_0^-\left(1 - \frac{e\Phi}{kT}\right), \quad \frac{e\Phi}{kT} \ll 1.$

$\rho = +e\left[n_0^+ - n_0^- + \frac{n_0^- e\Phi}{kT}\right].$

cgs $\nabla^2\Phi = -4\pi\rho$

$\lim_{r\to\infty} \rho = 0, \ \Phi \to 0. \ \therefore n_0^- = n_0^+$

$\rho = \frac{n_0^+ e^2}{kT}\Phi$ 3

$\nabla^2\Phi = -\left(\frac{n_0^+ e^2}{\epsilon_0 kT}\right)\Phi$

$\nabla^2\Phi = -\frac{4\pi n_0^+ e^2}{kT}\Phi$

try $\Phi = \frac{A}{r} e^{-r/\lambda}$

$\nabla^2\Phi = \frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\Phi$

$r^2 \frac{d\Phi}{dr} = r^2 \Phi_0 \left[-\frac{1}{\lambda r} - \frac{1}{r^2}\right] e^{-r/\lambda}$

$= \left(-\frac{r}{\lambda} - 1\right)\Phi_0 e^{-r/\lambda}$

a) $\nabla^2\Phi = \frac{A}{r^2}\left\{\left(-\frac{r}{\lambda} - 1\right)\left(-\frac{1}{\lambda}\right) - \frac{1}{\lambda}\right\} e^{-r/\lambda}$

$\lambda = \sqrt{\frac{kT}{4\pi n_0^+ e^2}}$

$= -\frac{1}{\lambda^2}\Phi$

$\therefore \boxed{\lambda = \sqrt{\frac{\epsilon_0 kT}{n_0^+ e^2}}}$ 3

As $r \ll \lambda$, $\Phi \to \frac{A}{r}$

$-\iint \nabla\Phi \cdot d\vec{S} = -\frac{e}{\epsilon_0}$

$+\frac{A}{r^2} 4\pi r^2 = -\frac{e}{\epsilon_0}$

$A = \frac{-e}{4\pi\epsilon_0}$

$-\iint \nabla\Phi \cdot d\vec{S} = -4\pi e$

$\frac{A}{r^2} 4\pi r^2 = -4\pi e$

$A = -e$

$\boxed{\Phi = -\frac{e}{r} e^{-r/\lambda}}$

3

b) $\therefore \boxed{\Phi = -\frac{e}{4\pi\epsilon_0 r} e^{-r/\lambda}}$ QED

IV-1

SECTION IV

7 Jan 95 UB
9.9

Gen I-7

Collapsing Binary stars.

Under the influence of gravitational forces two stars orbit around each other with $\tau$ period. At $t=0$ the motion is stopped and they are allowed to fall into each other.

~~What is the time~~ Expressed in $\tau$ they collide after which time?

- reduced mass $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

E conserv: $\frac{1}{2}\mu \left(\frac{dr}{dt}\right)^2 - G\frac{m_1 m_2}{r} = -G\frac{m_1 m_2}{r_0}$ $(\alpha)$

Centrif. eq.: $F = \mu\frac{v^2}{r_0} = G\frac{m_1 m_2}{r_0^2}$

Kepler III : $r_0^3 = G\frac{m_1 + m_2}{4\pi^2}\tau^2$ $(\beta)$

$(\alpha)$ : $t = C\int_0^{r_0} \frac{dr'}{\sqrt{(1/r - 1/r_0)}}$ $\quad C = \sqrt{\frac{\mu}{2G m_1 m_2}}$

$r = r_0 \sin^2\theta$: $t = 2r_0^{3/2} C \int_0^{\pi/2} \sin^2\theta\, d\theta = 2C\frac{\pi}{4} r_0^{3/2} = \frac{\pi}{2}\sqrt{\frac{\mu r_0^3}{2G m_1 m_2}}$

$(\beta)$ : $\boxed{t = \frac{\tau}{4\sqrt{2}}}$

over

IV-2



Solution $E = \frac{1}{4\pi\varepsilon_0} \frac{qd}{\left(x^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0} \frac{Qd}{R^3}$

$$V_{dipole} = \frac{Qd^2}{4\pi\varepsilon_0 R^3} \longrightarrow \ddot{Q}_1 + \frac{Q_1}{LC} = \frac{Q_2 d^2}{4\pi\varepsilon_0 R^3 L}$$

Kirchhoff's Law:

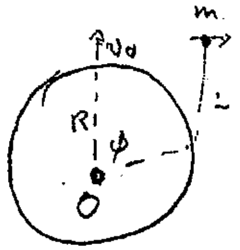
$$\frac{d^2Q_1}{dt^2} + \omega_0^2 Q_1 = \frac{Q_2 d^2 C}{4\pi\varepsilon_0 R^3}\,\omega_0^2$$

and
$$\frac{d^2Q_2}{dt^2} + \omega_0^2 Q_2 = \frac{Q_1 d^2 C}{4\pi\varepsilon_0 R^3}\,\omega_0^2$$

solve $\omega = \omega_0\sqrt{[1 \pm \alpha]}$ $\alpha = \frac{Cd^2}{4\pi\varepsilon_0 R^3}$

$\frac{Q}{V} = C$ $\frac{Q}{LC} = V/L$ ✓

Mech I - B




A mass point m is at the end of a weightless cord initially wrapped around a cylinder with radius R, so that m rests on the cylinder. At $t=0$ m receives a kick radially outward.

a) Find the eq of mot. in suitable generalized coordin.
b) Give a sln satisf. the initial condition.
c) Using b find the angular momentum wrt. O the cyl. axis.

- Use $\mathcal{L}$ to eliminate considerations of T in the rope. Convenient coordinate L = length of cord unwound.

$$\mathcal{L} = \frac{m}{2}v^2 = \frac{1}{2} m L^2 \dot\phi^2 = \frac{mL^2}{2R^2}\dot L^2$$

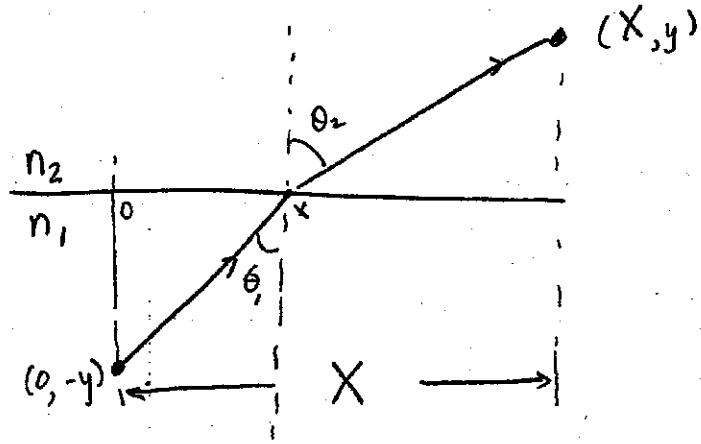
3 a) Eq of mot: $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \frac{dL}{dt}}\right) - \frac{\partial \mathcal{L}}{\partial L} = 0$ or $\boxed{\frac{d}{dt}(L\dot L) = 0}$ or $\frac{d}{dt}(\phi\dot\phi) = 0$

3 b) Gen sln: $\boxed{L^2 = 2Rv_0 t}$ boundary $L=0$, $v=v_0$ at $t=0$ ok.

2 c) At any time: $J = m v L = \frac{m\dot L L^2}{R} = m\sqrt{2Rv_0^3 t}$

(has other easier slns)

IV - 4




Interface at $y = 0$
find $x$, where ray crosses interface

$$P = \text{path} = \int n(s)\,ds = n_1\sqrt{x^2+y^2} + n_2\sqrt{(X-x)^2+y^2}$$

Minimize: $$\frac{\partial P}{\partial x} = n_1 \frac{x}{\sqrt{x^2+y^2}} - n_2 \frac{(X-x)}{\sqrt{(X-x)^2+y^2}} = 0$$

$$n_1 \sin\theta_1 - n_2 \sin\theta_2 = 0$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$