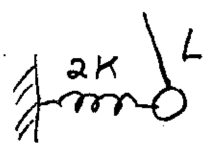
Part I Solutions - Spr 1999

I-1

ONE CAN ALWAYS SET UP 2 EQ, IN 2 UNKNOWNS AND SOLVE THE EIGENVALUE PROBLEM, BUT THERE IS A SIMPLER APPROACH.

IF $X_L = X_R$ THE SPRING DOES NOT FLEX AND EXHERTS NO FORCE, THEN $\omega_a = \sqrt{g/L}$

IF $X_R = -X_L$ THE CENTER OF THE SPRING DOES NOT MOVE AND THE PROBLEM REDUCES TO



$$m\ddot{x} = -m\left(\frac{x}{L}\right)g - (2K)x = -m\left(\frac{g}{L} + \frac{2K}{m}\right)x$$

$$\Rightarrow \omega_s = \sqrt{\frac{g}{L} + \frac{2K}{m}}$$

THEN IN GENERAL

$$X_L = a\sin(\omega_a t + \phi_a) + b\sin(\omega_s t + \phi_s)$$
$$X_R = a\sin(\omega_a t + \phi_a) - b\sin(\omega_s t + \phi_s)$$

$$\dot{X}_L = a\omega_a\cos(\omega_a t + \phi_a) + b\omega_s\cos(\omega_s t + \phi_s)$$
$$\dot{X}_R = a\omega_a\cos(\omega_a t + \phi_a) - b\omega_s\cos(\omega_s t + \phi_s)$$

$X_L = X_R = 0$ AT $t=0 \Rightarrow \phi_a = \phi_s = 0$

AT $t = 0+$
$$\left.\begin{aligned} I_o/m &= a\omega_a + b\omega_s \\ 0 &= a\omega_a - b\omega_s \end{aligned}\right\} \quad \begin{aligned} b\omega_s &= a\omega_a \\ a &= \frac{I_o}{2m\omega_a} \end{aligned}$$

$$\underline{X_L(t) = \frac{I_o}{2m\omega_a}\left[\sin(\omega_a t) + \frac{\omega_a}{\omega_s}\sin(\omega_s t)\right]}$$

I-2 APPLY GAUSS'S LAW TO A CHARGE DISTRIBUTION UNIFORM FOR $r < R$.

$$E_r(r) = \frac{|e|}{r^2} \quad r > R, \qquad r\,\frac{|e|}{R^3} \quad r < R$$

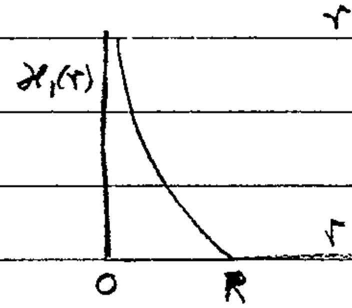
INTEGRATING $\vec{E} = -\nabla\phi$ GIVES

ϕ = continuous at $r = R$

$$\phi(r) = \frac{|e|}{r} \quad r > R, \qquad \frac{1}{2}\frac{|e|}{R^3}\left(3R^2 - r^2\right) \quad r < R$$

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 \qquad \mathcal{H}_1 = -|e|\,\phi(r) - \left(-\frac{e^2}{r}\right) \quad r < R$$

$$= 0 \quad r > R$$



$$\mathcal{H}_1 = \frac{1}{2}\frac{e^2}{R^3}\left(r^2 - 3R^2\right) + \frac{e^2}{r}$$

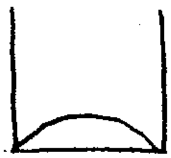
$\psi_{1s} \approx \frac{1}{\sqrt{\pi}}\, a_0^{-3/2}$ IN VICINITY OF NUCLEUS { NOTE THIS WAS GIVEN INCORRECTLY IN PROBLEM

$$\Delta E = \langle 1s|\mathcal{H}_1|1s\rangle = \frac{4}{a_0^3}\int_0^R \mathcal{H}_1 r^2\,dr$$

$$\int_0^R \mathcal{H}_1 r^2\,dr = \underbrace{\frac{1}{2}\frac{e^2}{R^3}\int_0^R r^4\,dr}_{\frac{1}{10}e^2R^2} \underbrace{- \frac{3}{2}\frac{e^2}{R}\int_0^R r^2\,dr}_{-\frac{1}{2}e^2R^2} + \underbrace{e^2\int_0^R r\,dr}_{\frac{1}{2}e^2R^2} = \frac{1}{10}e^2R^2$$

$$\underline{\Delta E = \frac{2}{5}\frac{e^2}{a_0}\left(\frac{R}{a_0}\right)^2} \qquad \text{NOTE: } E_0 = -\frac{1}{2}\frac{e^2}{a_0}$$

I-3

a) 

$\psi \propto \sin k_n x$, $k_n L = n\pi$, $k_n = n\frac{\pi}{L}$

$$E_n = \frac{\hbar^2 k_n^2}{2m} \propto \frac{1}{m}$$

$$\underline{\underline{RATIO = \frac{m_{PROTON}}{m_{ELECTRON}}}} \sim 2000$$

b) $E_n = -\frac{\mu e^4}{2\hbar^2 n^2}$ $\mu_{POSITRONIUM} = \frac{m_e m_e}{m_e + m_e} = \frac{1}{2} m_e$

$\mu_{HYDROGEN} = \frac{m_p m_e}{m_p + m_e} \approx m_e$ $\underline{\underline{RATIO \approx \frac{1}{2}}}$

c) $\hat{\mathcal{H}} = \frac{\hat{L}^2}{2I_0}$ $E_l = \frac{\hbar^2}{2I_0} l(l+1)$ $I_0 \propto M$

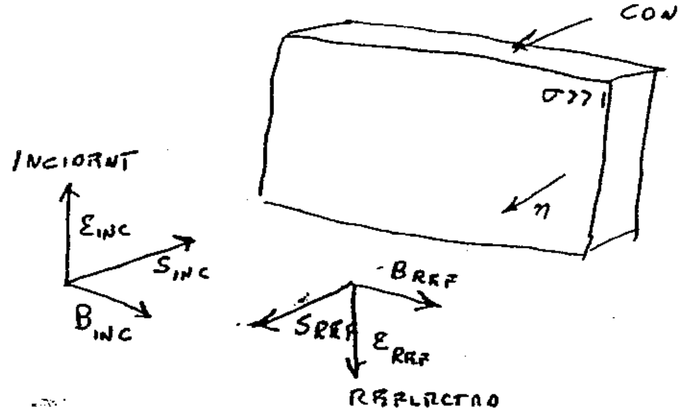
$$\Rightarrow \underline{\underline{RATIO = \frac{15}{14}}}$$

d) $\hat{\mathcal{H}}_{VIB}\Big|_{ONE\ NORMAL\ MODE} = \frac{\hat{p}^2}{2\mu} + \frac{1}{2} K_{EFF} \hat{Q}^2$

$E_n = (n+\frac{1}{2})\hbar\omega$ $\omega = \sqrt{\frac{K_{EFF}}{\mu}}$ AND $\mu \propto M$

$$\Rightarrow \underline{\underline{RATIO = \sqrt{\frac{15}{14}}}}$$

CLASSICAL RADIATION PRESSURE



a) INCIDENT AND REFLECTED WAVE ELECTRIC FIELDS MUST CANCEL AT THE SURFACE

IMAGINE AN IMAGE WAVE COMING FROM BEHIND THE SURFACE TO MAKE REFLECTED WAVE

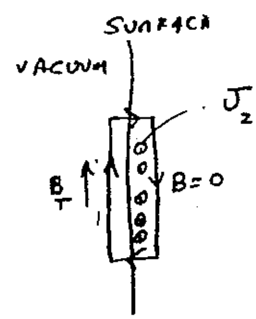
$$\mathcal{E}_T = \mathcal{E}_R + \mathcal{E}_{INC} = 0$$

AT THE SURFACE
SATISFIES $\mathcal{E}_T \to 0$ INSIDE THE CONDUCTOR

$$B_T = B_R + B_{INC} = 2B_{INC}$$

b) AMPERE'S LAW AT THE SURFACE

$$\oint \vec{B}\cdot d\vec{s} = \frac{4\pi}{c}\iint \vec{J}\cdot d\vec{A}$$



ONLY CONTRIBUTION TO CIRCUITAL INTEGRAL IS IN VACUUM

$$\underset{\overset{\|}{2B_{INC}\,l}}{B_T\, l} = \frac{4\pi}{c}\,\underbrace{J_z\,\delta}_{\overset{\|}{J_s}}\, l$$

J_s = SURFACE CURRENT SHEET

$$J_s = \frac{c\,B_{INC}}{2\pi}$$

c) LORENTZ PRESSURE ON THE SURFACE

$$P = \frac{F}{A} = \frac{\vec{B}_{INC} \times \vec{J}_s}{c} = \frac{B_{INC}^2}{2\pi}$$

THE INTENSITY IN THE PLANE WAVE IS DETERMINED BY S

$$S = \frac{c}{4\pi} \vec{\mathcal{E}}_{INC} \times \vec{B}_{INC} = \frac{c}{4\pi} B_{INC}^2$$

SINCE $B_{INC} = \mathcal{E}_{INC}$ IN THE WAVE

$$\frac{P}{S} = \frac{\text{PRESSURE}}{\text{INTENSITY}} = \frac{2}{c}$$ ← COMES FROM THE REFLECTION

II-1

OK

$$m\frac{dv}{dt} = -cv^2$$

$$v^{-2}\,dv = -\frac{c}{m}\,dt$$

$$-\frac{1}{v} + \frac{1}{v_0} = -\frac{c}{m}t$$ integrating and setting initial constant

$$\frac{1}{v} - \frac{1}{v_0} = \frac{c}{m}t$$

$$\frac{dt}{dx} = \frac{c}{m}t + \frac{1}{v_0}$$

$$\frac{dt}{t + \frac{m}{v_0 c}} = \frac{c}{m}\,dx \Rightarrow x = \frac{m}{c}\left[\ln\left(t + \frac{m}{v_0 c}\right) - \ln\left(\frac{m}{v_0 c}\right)\right]$$

$$= \frac{m}{c}\ln\left(\frac{v_0 c t}{m} + 1\right)$$

(a) A beam of $\bar{\nu}_\mu$ (from accelerator-produced π^-) ~~decay~~ will only produce muons when incident on target; a beam of $\bar{\nu}_e$ will only produce electrons.

(b) Forbidden by lepton flavor conservation

(c) Positronium was moving in lab, when it decayed

(d) CP violation

(e) T violation

II-3

As d decreases, get larger diffraction pattern

$\Delta\theta \simeq \frac{\lambda}{d}$ so image smear $\Delta S_1 = L\Delta\theta \simeq \lambda \frac{L}{d}$

As d increases, get larger geometrical spot, side d

so $\Delta S_2 \simeq d$

Combine quadratically and minimize

$$\Delta S_{TOTAL} = \sqrt{\left(\lambda \frac{L}{d}\right)^2 + d^2}$$

$$\frac{d(\Delta S_T)}{d \cdot d} = \frac{1}{2} \frac{1}{\sqrt{(\;)^2 + (\;)^2}} \left\{ 2d - \frac{2\lambda^2 L^2}{d^3} \right\} = 0$$

so $d^4 = \lambda^2 L^2$

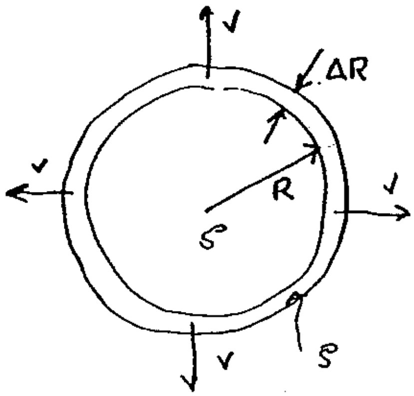
OR $d = \sqrt{\lambda L}$

SAME RESULT IF ONE SIMPLY SETS $\Delta S_1 = \Delta S_2$, WHICH WOULD ALSO BE O.K.

II-4

NEWTONIAN COSMOLOGY

USING HOMOGENEITY AND ISOTROPY ASSUMPTION ESTIMATE THE ENERGY IN A SHELL OF RADIUS R



ASSUME UNIVERSAL SMOOTHED DENSITY IS ρ

HUBBLE LAW RADIAL VELOCITY OF MATTER IN SHELL

$$V_R = HR$$

KINETIC ENERGY OF THE SHELL

$$KE_{SHELL} = 4\pi R^2 \Delta R\, \rho\, \frac{V_R^2}{2} = 2\pi \rho H^2 R^4 \Delta R$$

POTENTIAL ENERGY STORED IN THE SHELL DUE TO THE GRAVITATIONAL INTERACTION WITH THE MATTER BOUNDED BY THE SHELL

$$PE_{SHELL} = -\frac{G M_{SHELL} M_{WITHIN}}{R} = \frac{-G\, 4\pi R^3 \rho}{3R}\, 4\pi R^2 \rho\, \Delta R$$

$$= -\frac{16\pi^2}{3} G \rho^2 R^4 \Delta R$$

TOTAL ENERGY OF THE SHELL

$$E_{TOT} = PE + KE$$

$$E_{TOT} = R^4 \Delta R\, \rho\, 2\pi \left[H^2 - G \frac{8\pi}{3} \rho \right]$$

CRITICAL DENSITY $\quad E_{TOT} = 0$

$$\rho_{CRIT} = \frac{3H^2}{8\pi G}$$

III - 1

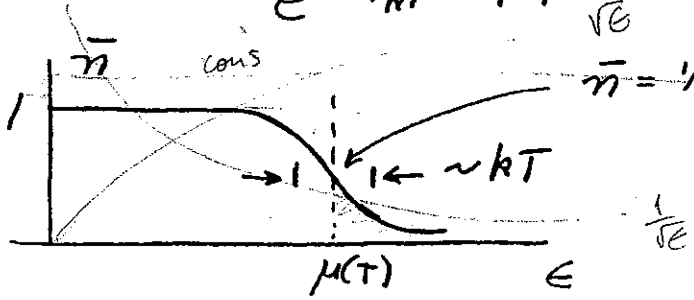
Suppose you place two such hemispheres together so as to form a spherical shell of uniformly distributed charge. By ~~symmetry and~~ Gauss' Law there is no field inside.

If there was an $\mathcal{E}$ field in the plane surface of a single hemisphere then the radial component $\mathcal{E}_r$ would have a divergence $\nabla \cdot \mathcal{E} \neq 0$ when two hemispheres ~~in the complete~~ form a complete sphere, implying $q \neq 0$ inside. But q inside $= 0$ so $\mathcal{E}_r = 0$

An $\mathcal{E}_\theta$ has to be zero as otherwise $\nabla \times \vec{\mathcal{E}} \neq 0$ and $\nabla \times \vec{\mathcal{E}} \equiv 0$ for static $\vec{\mathcal{E}}$ fields.

III-2

a) $$\bar{n}(\epsilon,T) = \frac{1}{e^{\frac{\epsilon-\mu(T)}{kT}}+1}$$



$\mu(T)$ IS THE (TEMPERATURE DEPENDENT) CHEMICAL POTENTIAL. IT IS SET BY REQUIRING THAT THE AVERAGE # OF PARTICLES IS EQUAL TO N:

$$N = \int_0^{\infty} D(\epsilon)\, \bar{n}(\epsilon,T)\, d\epsilon$$

b) THE PARTICLE $(\bar{n})$ HOLE $(1-\bar{n})$ SYMMETRY ABOUT $\epsilon = \mu$ MEANS THAT FOR $kT \ll \mu$ THE <u>TEMPERATURE DEPENDENCE OF μ IS DETERMINED BY THE DERIVATIVE OF THE DENSITY OF STATES AT $\epsilon = \mu$</u>.

IF $\left.\frac{dD(\epsilon)}{d\epsilon}\right|_{\epsilon=\mu} = 0$, μ DOES NOT CHANGE AS T INCREASES

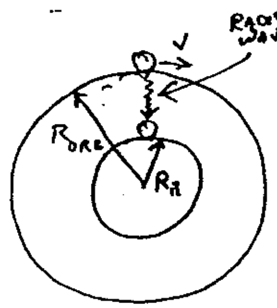
IF $\left.\frac{dD}{d\epsilon}\right|_{\epsilon=\mu} > 0$, μ DECREASES

IF $\left.\frac{dD}{d\epsilon}\right|_{\epsilon=\mu} < 0$, μ INCREASES

CLOCKS IN ORBIT

NEED TO CONSIDER BOTH GRAVITATIONAL RED SHIFT BECAUSE CLOCKS ARE OPERATING AT DIFFERENT GRAVITATIONAL POTENTIALS, AND THE TIME DILATION SINCE THE CLOCKS ARE IN RELATIVE MOTION

THE CLOCK FREQUENCY WHEN FAR FROM THE EARTH IS ν_0, THE PROPER FREQUENCY.



THE CLOCK ON THE EARTH MAINTAINS A RATE

$$\nu_E = \nu_0 \left(1 - \frac{GM_E}{R_E c^2}\right)$$

ASSUMES THAT $\frac{GM_E}{R_E c^2} << 1$

THE CLOCK AT THE GRAVITATIONAL POTENTIAL OF THE ORBIT MAINTAINS A RATE

$$\nu_{ORB} = \nu_0 \left(1 - \frac{GM_E}{R_{ORB} c^2}\right)$$

THE TRANSMISSION OF THE RADIO WAVES FROM THE ORBITING CLOCK TO THE CLOCK ON THE EARTH MUST CONSIDER THE TIME DILATION (SECOND ORDER DOPPLER).

$$\nu_{\substack{EARTH \\ RECEIVED}} = \nu_{ORB} \left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2} \simeq \nu_{ORB} \left(1 - \frac{1}{2}\frac{v^2}{c^2} + \cdots\right)$$

THE DIFFERENCE IN THE FREQUENCY OF THE SIGNAL RECEIVED FROM THE ORBITING CLOCK AND THAT FROM THE CLOCK ON THE GROUND <u>BOTH</u> MEASURED ON THE GROUND

$$\Delta\nu = \nu_{\substack{EARTH \\ RECEIVED}} - \nu_E = \nu_0 \left[1 - \frac{GM_E}{R_{ORB} c^2} - \frac{1}{2}\frac{v^2}{c^2} - \left(1 - \frac{GM_E}{R_E c^2}\right)\right]$$

ASSUME CIRCULAR ORBIT

$$\frac{v^2}{R_{ORB}} = \frac{GM_E}{R_{ORB}^2} \qquad \frac{v^2}{c^2} = \frac{GM_E}{R_{ORB} c^2}$$

$$\Delta\nu = \nu_0 \frac{GM_E}{c^2} \left[\frac{1}{R_E} - \frac{3}{2}\frac{1}{R_{ORB}}\right]$$

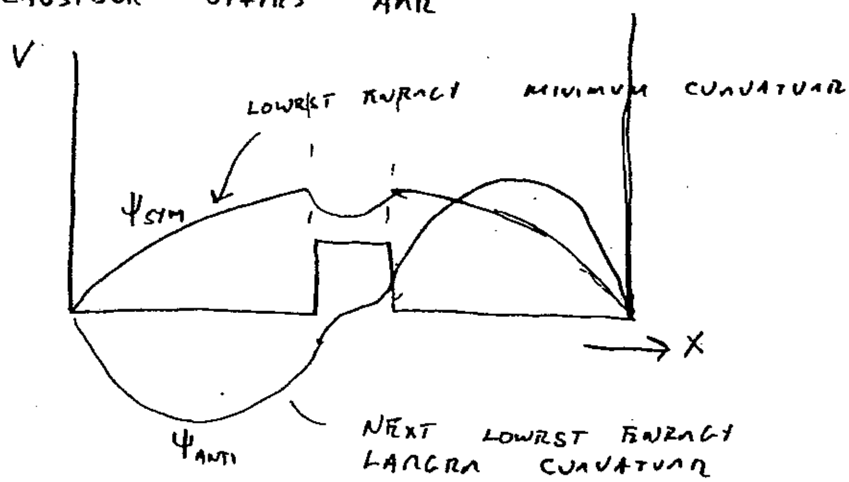
CAN GO TO 0 WHEN $R_{ORB} = 3/\ R$

AMMONIA INVERSION LINE

EIGENSTATES NEED TO HAVE EQUAL PROBABILITY FOR FINDING NITROGEN ON RIGHT OR LEFT

LOWEST ENERGY EIGENSTATES WILL HAVE MINIMUM CURVATURE (SECOND DERIVATIVE)

a) PLAUSIBLE STATES ARE



b) INCLUDE TIME DEPENDENCE OF THE TWO STATES

$$\psi_{SYM}(x,t) = A(x)\, e^{-i\frac{(E_0-B)t}{\hbar}}$$

$$\psi_{ANTI}(x,t) = B(x)\, e^{-i\frac{(E_0+B)t}{\hbar}}$$

A TIME DEPENDENT STATE WITH ENHANCED PROBABILITY FOR FINDING THE NITROGEN ON "SAY" THE RIGHT SIDE WOULD BE SUPERPOSITION OF THE ψ_{SYM} AND ψ_{ANTI} STATES

$$\psi(t) = C(x)\left[e^{-i\frac{(E_0-B)t}{\hbar}} + e^{-\frac{i(E_0+B)}{\hbar}t} \right]$$

(THE SPATIAL FUNCTION IS NOT IMPORTANT IN SHOWING THE TIME DEPENDENCE)

$$\psi(t) = C(x)\, e^{-i\frac{E_0}{\hbar}t}\left[e^{\frac{iBt}{\hbar}} + e^{-\frac{iBt}{\hbar}} \right]$$

$$= C(x)\, e^{-i\frac{E_0}{\hbar}(t)}\; 2\cos\frac{Bt}{\hbar}$$

AMMONIA

THE PROBABILITY OF FINDING THE NITROGEN ON THE RIGHT SIDE VARIES AS

$$\psi(t)\,\psi^*(t) \propto \cos^2 \frac{Bt}{\hbar}$$

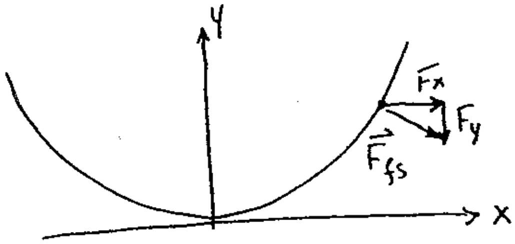
OSCILLATES BETWEEN 0 AND 1 IN A TIME

$$\frac{Bt}{\hbar} = \pi/2 \qquad t = \frac{\pi\hbar}{2B}$$

$t = \frac{1}{2}$ THE INVERSION PERIOD

THE LARGER $2B$, - THE ENERGY DIFFERENCE BETWEEN THE SYM. AND ANTI SYM STATES - THE HIGHER THE INVERSION FREQUENCY

IV - 1



Condition on surface: surface is normal to total force on volume element at surface, mass m.

$$\vec{F}_{\substack{\text{fluid}\\ \text{surface}}} = \vec{F}_{fs} = \hat{x}\,F_x + \hat{y}\,F_y$$

$$F_x = m x \omega_0^2 \qquad F_y = -mg$$

Surface is $\perp$ to $\vec{F}_{fs}$: its slope (in x-y plane) $= \dfrac{F_x}{-F_y}$

So $\dfrac{dy}{dx} = \dfrac{m x \omega_0^2}{+mg} = \dfrac{\omega_0^2}{g} x$

Integrate $\quad y = \dfrac{\omega_0^2}{2 \cdot g} x^2 + \text{const.} \quad \Rightarrow$ Parabolic

IV - 2

The relativistic velocity transformation for this symmetric case is:

$$\beta_{final} = \frac{2\beta}{1+\beta^2} \qquad \text{and} \quad \beta^2 = 1 - \frac{1}{\gamma^2} = \frac{\gamma^2 - 1}{\gamma^2}$$

so $\beta_f^2 = \dfrac{4\beta^2}{(1+\beta^2)^2} \xrightarrow{\text{algebra}} 1 - \beta_f^2 = \left(\dfrac{1-\beta^2}{1+\beta^2}\right)^2$

so $\gamma_{final} = \left(\dfrac{1+\beta^2}{1-\beta^2}\right)$ and $m = \gamma_{final} m_o$

for $\gamma = 10$ $\quad m = 199\, m_o$

IV-3

a) $\frac{1}{2} m v^2 = m g h \Rightarrow h = \frac{v^2}{2g}$

b) NOTE THAT NEITHER h NOR v IS NEGATIVE, AND h IS A SINGLE VALUED FUNCTION OF v.

$$p_h(h)\,dh = p_v(v)\,dv$$

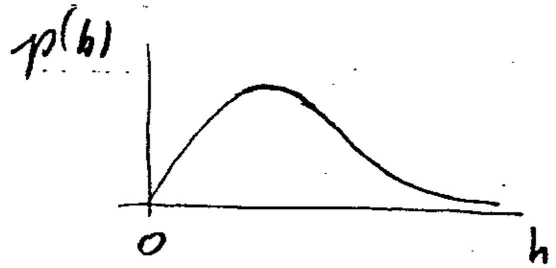
$$p_h(h) = p_v\big(v(h)\big)\ \frac{dv(h)}{dh}$$

$$dh = \frac{1}{g} v\,dv \Rightarrow \frac{dv}{dh} = \frac{g}{v}$$

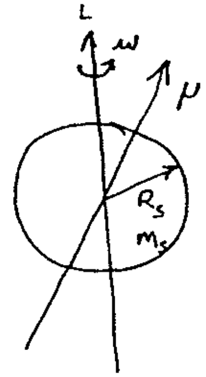
$$p(h) = \frac{2v^2}{v_0^4} e^{-v^2/v_0^2} \left(v \frac{dv}{dh}\right)$$

$$= \frac{4g}{v_0^4}\ h e^{-\frac{2gh}{v_0^2}}\ g \qquad \text{DEFINE } \underline{h_0 \equiv \frac{v_0^2}{2g}}$$

$$\underline{= \frac{1}{h_0^2}\ h e^{-h/h_0} \qquad h \geq 0}$$



PULSAR ENERGY LOSS



ENERGY STORED IN THE ROTATION OF THE NEUTRON STAR

$$E_{ROT} = \frac{1}{2} I \omega^2 = \frac{1}{5} M_s R_s^2 \omega^2$$

ASSUME STAR IS "STIFF" ENOUGH THAT THE MOMENT OF INERTIA IS CONSTANT, INDEPENDENT OF THE ROTATION FREQUENCY

RELATING A CHANGE IN ENERGY TO A CHANGE IN ROTATION FREQUENCY

$$\frac{dE_{ROT}}{dt} = I \omega \frac{d\omega}{dt}$$

a) MAGNETIC DIPOLE RADIATION LOSSES ENERGY INTO THE RADIATION FIELD AS

$$\left.\frac{dE}{dt}\right|_{\substack{\text{MAGNETIC}\\ \text{DIPOLE RADIATION}}} = c\,\omega^4$$

THE PULSAR ROTATION ENERGY IS REDUCED

$$\frac{dE}{dt}_{ROT} = -\left.\frac{dE}{dt}\right|_{\substack{\text{MAGNETIC}\\ \text{DIPOLE RAD}}} = -c\,\omega^4 = \omega I \frac{d\omega}{dt}$$

$$\Rightarrow \underline{\underline{n = 3}}$$

b) IF GRAVITATIONAL RADIATION DOMINATES - QUADRUPOLE RADIATION

$$\left.\frac{dE}{dt}\right|_{\substack{\text{GRAV}\\ \text{RAD}}} = c\,\omega^6$$

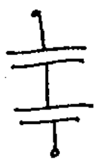
SAME AS IN QUADRUPOLE RADIATION IN E+M

$$\left.\frac{dE}{dt}\right|_{ROT} = -\left.\frac{dE}{dt}\right|_{\substack{\text{GRAV}\\ \text{RAD}}} = -c\,\omega^6 = \omega I \frac{d\omega}{dt} \Rightarrow \underline{\underline{n=5}}$$

V-1

(a) $C_0 = \frac{A}{d}$ (in cm) in Gaussian units

(b) Treat II as two capacitors in series

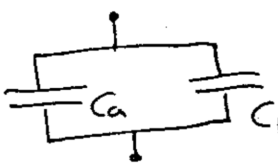


$C_a = 2C_0$

$C_b = 2KC_0$

$$C_{II} = \frac{C_a C_b}{C_a + C_b} = \frac{2K}{1+K} C_0$$

Treat III as a pair of parallel capacitors



$C_a = C_0/2$

$C_b = K C_0/2$

$$C_{III} = C_a + C_b = \frac{1+K}{2} C_0$$

(c) Now $E_I = \frac{1}{2}\frac{Q_0^2}{C_0}$: energy in C_0

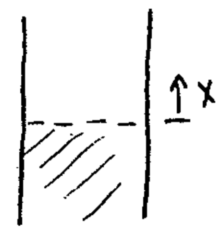
So $E_{II} = \frac{1}{2}\frac{Q_0^2}{C_{II}} = \frac{K+1}{2K} E_I$, $\left.\text{Work}\right]_{II} = E_{II} - E_I = \frac{1-K}{2K} E_I$

and $E_{III} = \frac{2}{K+1} E_I$

$$\left.\text{Work}\right]_{III} = \frac{1-K}{1+K} E_I$$

and this is negative, for $K > 1$, so get work out, for both cases

V-2



$P_0 A = mg$

$dF = A\,dP$

$$dP = \left.\frac{\partial P}{\partial V}\right|_S dV \qquad dV = A\,dx$$

$$\left.\frac{\partial P}{\partial V}\right|_S = -\frac{1}{V}\left(\frac{1}{-\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_S}\right) = -\frac{1}{V}\left(\frac{1}{\kappa_S}\right) = -\frac{1}{V}\left(\frac{\gamma}{\kappa_T}\right)$$

SINCE $\kappa_T / \kappa_S = c_p / c_v \equiv \gamma$

BUT $\kappa_T \equiv \frac{-1}{V}\left.\frac{\partial P}{\partial V}\right|_T = -\frac{1}{V}\left(\frac{-V}{P}\right) = \frac{1}{P_0}$

$$\text{so } dF = A\left(-\frac{1}{V_0} P_0 \gamma\right) A\,dx = -\frac{\gamma m g A}{V_0}\,dx = m\frac{d^2x}{dt^2}$$

$$\ddot{x} + \left(\frac{\gamma g A}{V_0}\right)x = 0 \Rightarrow \underline{\underline{\omega = \sqrt{\frac{\gamma g A}{V_0}}}} \qquad \text{WHERE } A = \frac{\pi d^2}{4}$$

21, 22 are at end
of Part II answers
which are before
Part III question

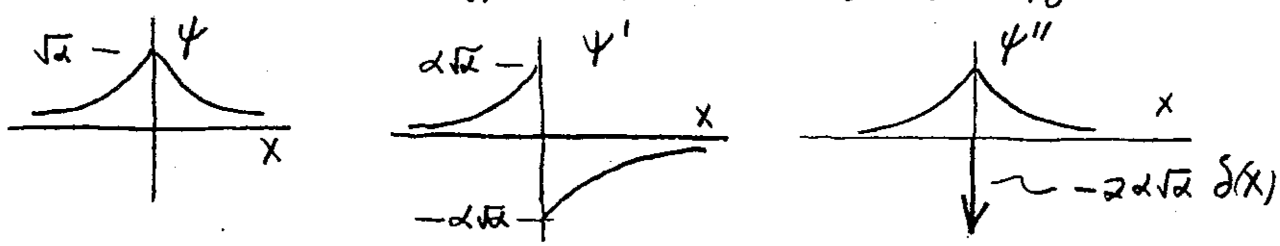
V-3

$$\Psi(t=0+) = \sqrt{\alpha}\, e^{-\alpha|x|} = \underbrace{c_0 \sqrt{\alpha'}\, e^{-\alpha'|x|}}_{\text{ONE BOUND STATE}} + \underbrace{\int_0^\infty C(E)\,\psi_E(x)\, dE}_{\text{CONTINUUM OF UNBOUND STATES}}$$

$$p(\text{STILL BOUND}) = |c_0|^2$$

$$c_0 = 2\sqrt{\alpha}\sqrt{\alpha'} \int_0^\infty e^{-(\alpha+\alpha')x}\, dx = \frac{2\sqrt{\alpha}\sqrt{\alpha'}}{\alpha+\alpha'} \qquad \text{BY ORTHOGANALITY}$$

USE THE WAVE EQ. TO RELATE α TO V_0



$$\underbrace{-\frac{\hbar^2}{2m}\psi''(x)}_{\text{CONTAINS}} + \underbrace{U(x)\,\psi(x)}_{-V_0\,\delta(x)\sqrt{\alpha}} = E\psi(x)$$

$-\frac{\hbar^2}{2m}\left(-2\alpha\sqrt{\alpha}\,\delta(x)\right)$

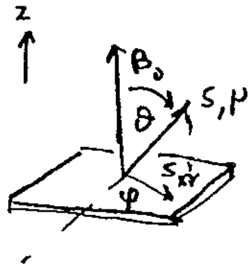
$$\Rightarrow \alpha = \frac{m}{\hbar^2} V_0$$

SO $V_0 \to f V_0 \Rightarrow \alpha \to f\alpha \equiv \alpha'$

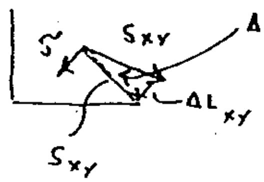
$$p(\text{STILL BOUND}) = \left(\frac{2\sqrt{f}}{1+f}\right)^2 = \underline{\underline{\frac{4f}{(1+f)^2}}}$$

V-4

CLASSICAL PRECESSION OF MAGNETIC DIPOLES



IN XY PLANE



$$\tau = \vec{\mu} \times \vec{B}_o = \mu B_o \sin\theta$$

$$S_{xy} = S \sin\theta$$

$$\frac{\Delta L_{xy}}{S_{xy}} = \Delta\varphi = \frac{\tau \Delta t}{S_{xy}} = \frac{\mu B_o \Delta t}{S}$$

a)

$$\frac{\Delta\varphi}{\Delta t} = \omega_{PREC} = \frac{\mu B_o}{S}$$

INDEPENDENT OF θ

ENERGY OF DIPOLE IN B_o

$$E = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

b) APPLYING B_{RF} CIRCULARLY POLARIZED IN THE XY PLANE WITH FREQUENCY ω_{PREC} AND ROTATION SENSE THE SAME AS THE PRECESSION, LARMOR'S THEOREM APPLIED IN ROTATING FRAME ELIMINATES B_o AND B_{RF} APPEARS STEADY AND IN FIXED RELATION TO μ. μ PRECESSES ABOUT B_{RF}. THEREBY CHANGING θ AND ENERGY OF μ IN B_o IN LAB IS CHANGED. "IN" THE ROTATING FRAME, THE PRECESSION AROUND THE B_{RF}

$$\omega_{RABI} = \frac{\mu B_{RF}}{S}$$

THE TIME NEEDED TO CHANGE THE ENERGY MAXIMALLY

$$\pi = \omega_{RABI}\, t \qquad t = \frac{\pi S}{\mu B_{RF}}$$