DOCTORAL GENERAL EXAMINATION
PART 1
Study Guide

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Part I of the general examination in physics at MIT is totally based on core undergraduate material: mechanics, electricity and magnetism, statistical physics and thermodynamics, waves and quantum mechanics. The exam does not require detailed knowledge of particular subfields, such as nuclear and particle physics, astrophysics, condensed matter physics. Mathematical physics is also not emphasized.

The exam serves two purposes. It ensures that all of our students have a firm understanding of the most basic concepts in physics. It also acts as a diagnostic for incoming students. Students who readily pass at the first attempt are ready to begin full-time graduate study, while those who fail need to spend time reviewing the relevant undergraduate material. The exam is organized into five groups of four questions each. Each question is meant to be a straightforward application of undergraduate concepts and techniques. Complicated integrations, series sums and algebraic manipulations are avoided. There are few trick problems, but problems in which symmetry can be exploited to avoid long calculation are certainly fair game.

Problems fall into three general categories. First are quantitative problems in which a physical system is analyzed mathematically. Second are qualitative problems which emphasize the general properties of systems. Finally there are observation problems in which the student is given the simple physical system and asked to describe its properties through observation. Typically most of the exam is evenly divided between the first two categories with one or two questions from the third.

1 Scope

The problems are drawn from material covered in the MIT undergraduate core physics curriculum:

8.012	freshman mechanics
8.022	freshman electricity and magnetism
8.03	waves and vibrations
8.033	special and general relativity
8.011, 8.05, 8.06	quantum mechanics I, II, and III
8.044	statistical mechanics
8.13	laboratory 1
8.14	laboratory 2
8.07	advanced electricity and magnetism
8.08	advanced statistical mechanics (elective)
8.09	advanced mechanics (elective)

The syllabus for each course is given in Appendix 1.

2 Studying for the exam

Working problems is an important part of studying for the exam. However, each problem is designed to test some particular concept. Therefore, it is useful to organize your studying by concept. You may wish to make an outline of these concepts, writing just a few keywords and formulas under each section. Also, you may find it useful to make road maps of the derivations of key results. For Part I, you will not need to memorize such derivations, but you should know how the ideas and assumptions go into each important idea.

Many students spend the weeks or months before the exam studying full-time. This approach is very hard to sustain and can be quite frustrating. Instead, start three months before, spend one half hour per day, and ramp up to one or two hours a day toward the end of the study period. This makes the exam less onerous and leaves you plenty of time for research, courses and recreation. Try to be religious about the study time; make it a habit. Try to be realistic about the amount of time you can spend studying. Begin studying well in advance to accommodate a modest daily time.

There are a total of 20 questions on the Part I exam, which means on average you'll have 15 minutes per question. The Part I exam is written so that 15 minutes is plenty of time if you grasp the basic material. If you find yourself embarking on a long calculation, stop and think; you may be headed in the wrong direction.

$$
\begin{aligned}
N_A &= 6.02 \times 10^{23} \sim 6 \times 10^{23} \\
e &= 1.6 \times 10^{-19}\ \mathrm{C} \\
c &= 3 \times 10^{10} \mathrm{cm/s} \\
\hbar c &= 197\ \mathrm{MeV}\ \cdot\ \mathrm{fm} \\
&\approx 2 \cdot 10^{-5}\ \mathrm{eV} \cdot\ \mathrm{cm} \\
\frac{e^2}{\hbar c} &= \frac{1}{137} \sim 0.007 \\
k_B T &= \frac{1}{40}\ \mathrm{eV\ at\ room\ temperature} \\
m_p c^2 &= 0.938\ \mathrm{GeV/c^2} \sim 1\ \mathrm{GeV/c^2} \\
m_e c^2 &= 511\ \mathrm{keV/c^2} \sim 500\ \mathrm{keV/c^2} \\
g &= 981\ \mathrm{cm/s^2} \sim 1000\ \mathrm{cm/s^2} \\
R_{\mathrm{Earth}} &= 6400\,\mathrm{km} \\
\rho_{\mathrm{Earth}} &= 5\,\mathrm{g/cm^3}
\end{aligned}
$$

Table 1: Combinations of constants you should know.

Finally remember you have been a physicist for four years and have learned in practice most of the material on the Part I exam. You would not have been admitted to MIT if we did not think you could pass Part I. If you have any suggestions about improving Part I or these notes, please let us know.

3 Some important topics

3.1 Numerical evaluation

The final numerical result is an essential part of physics, and we will occasionally ask you to give one. This means you will need to know the fundamental constants. There is an easy way to do this: rather than remembering $\hbar, G, k_B, ...$ is is easier to remember physically significant combinations. Table 1 contains the most important ones.

Using these numbers, you can get almost anything you need to know. Suppose you want the energy, in electron volts, of an electron in two dimensions confined to a strip of width 1 cm. From quantum mechanics, the energy is

$$E = \frac{\hbar^2 \pi^2}{2 m_e a^2}$$

where $a = 1$ cm is the width. Then

$$\begin{aligned} E &= \frac{(\hbar c)^2 \pi^2}{2 m c^2 a^2} = \frac{4 \cdot 10^{-10}\ \text{eV}^2\ \text{cm}^2 \cdot (3.1)^2}{2 \cdot 5.1 \cdot 10^5\ \text{eV} \cdot 1\ \text{cm}^2} \\ &\approx 4 \times 10^{-15} \text{eV}. \end{aligned}$$

Now suppose instead you were asked for the answer in ergs instead of eV. Recall 1 gram of hydrogen is a mole of protons:

$$\begin{aligned} (6 \times 10^{23})(1 \times 10^9 \text{GeV}) &= 1\text{g} \cdot (3 \times 10^{10} \text{cm/s})^2 \\ 6 \times 10^{32} \text{eV} &= 9 \times 10^{20}\ \text{ergs} \\ \rightarrow 1\text{eV} &= 1.5 \times 10^{-12}\ \text{ergs}. \end{aligned}$$

Other examples: if you need k_B, use

$$k_B T_{\text{room}} = \frac{1}{40}\ \text{eV} \qquad \Longrightarrow \qquad k_B = \frac{1}{12,000}\ \text{eV K}^{-1}.$$

If you need G_N, use $g = G_N M_{\text{Earth}} / R_{\text{Earth}^2}$ and $M_{\text{Earth}} = 4\pi R^3_{\text{Earth}} \rho_{\text{Earth}}/3$. Bear in mind that you will seldom need G_N by itself.

Generally, Part I avoids much arithmetic; however, as a practicing physicist, you should begin to carry out numerical estimates quickly. Being able to do so will save you a great deal of work.

3.2 Checking Units

Checking units is the single most important diagnostic during an exam. Checking units during a calculation should become second nature; you should automatically check each step as you carry it out. Doing so will expose 90% of your mistakes.

For an algebraic expression, it is easy. for example, the answer to Problem IV-1 on the Fall 2002 exam is

$$h = \left(\frac{l^2}{g m^2 \tan^2 \alpha} \right)^{1/3}$$

(l is an angular momentum) and checking units gives

$$
\begin{aligned}
[h] &= \left[\frac{\mathrm{g}^2 \cdot\ \mathrm{cm}^4 \cdot\ \mathrm{s}^{-2}}{\mathrm{cm} \cdot\ \mathrm{s}^{-2} \cdot\ \mathrm{g}^2}\right]^{1/3} \\
&= \left[\mathrm{cm}^3\right]^{1/3} = \left[\mathrm{cm}\right] \qquad \surd
\end{aligned}
$$

When integrating, the differential carries the units. For example, the wavefunctions in 3D are normalized so that

$$\int \psi^* \psi d^3x = 1 \rightarrow [\psi]^2[d^3x] = 1 \qquad \Longrightarrow \qquad [\psi] = \mathrm{cm}^{-3/2}.$$

Similarly, differentiation carries the reciprocal of the infinitismal quantity:

$$p = \frac{\partial L}{\partial \dot{x}} \rightarrow [L] = [p][\dot{x}] = \mathrm{ergs}.$$

Delta functions are a bit tricky. The definition is

$$
\begin{aligned}
\int_a^b \delta(x) dx &= 1 \text{ if } a < 0 < b \\
&= 0 \,\text{otherwise}.
\end{aligned}
$$

Then,

$$1 = [\delta(x)][dx] \rightarrow [\delta(x)] = 1/\mathrm{cm}.$$

So for Problem I-1 on the Fall 2002 exam (note we are in one dimension here)

$$
\begin{aligned}
E_o^1 &= \int \psi^* W \delta(x) \psi dx \\
\rightarrow [W] &= \frac{[E_o^1]}{[\int \{|\psi|^2\, dx\}] \cdot [\delta(x)]} \\
&= \frac{[\mathrm{eV}]}{[\mathrm{dimensionless}] \cdot [\mathrm{cm}]^{-1}} = [\mathrm{eV} \cdot\ \mathrm{cm}].
\end{aligned}
$$

Logarithm $(\exp, \ln, ...)$ and trigonometric $(\cos, \sin, \tan, ...)$ functions must have dimensionless arguments. A subtlety comes in for logs, since $\ln ab = \ln a + \ln b$. The argument may have units, but they must be compensated by an additive term which casts the argument as dimensionless.

4 Coordinate systems

You should have a ready grasp of how to use cartesian, cylindrical and polar coordinates and be able to convert between them. Differential vector operations are more complicated. In general, Part I avoids the explicit evaluation of, for example, $\nabla \times \vec{A}$ in polar coordinates. However, you may need to work out differential vector relations for a component or two. These are straightforward to derive if you recall the definitions of the vector operators.

For example, suppose you need $\vec{\nabla}\phi$ in cylindrical coordinate. If $\phi = \phi(\rho, \theta, z)$, then

$$d\phi = \frac{\partial \phi}{\partial \rho} d\rho + \frac{\partial \phi}{\partial \theta} d\theta + \frac{\partial \phi}{\partial z} dz. \tag{1}$$

The gradient operator is defined as

$$d\phi = \vec{\nabla}\phi \cdot d\vec{r}.$$

The differential line element is $d\vec{r} = d\rho\hat{\rho} + \rho d\theta\hat{\theta} + dz\hat{z}$ and comparing with Eq. 1 gives

$$\vec{\nabla}\phi = \frac{\partial \phi}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial \phi}{\partial \theta}\hat{\theta} + \frac{\partial \phi}{\partial z}\hat{z}.$$

The other definitions of the vector operators are

$$\begin{aligned} \text{div}\vec{K} &= \lim_{\mathcal{V}\to 0} \frac{\int_{\mathcal{S}} \vec{K} \cdot d\vec{a}}{V} \\ \text{curl}\vec{K} &= \lim_{\mathcal{A}\to 0} \frac{\oint_{\mathcal{C}} \vec{K} \cdot d\vec{s}}{\mathcal{A}} \end{aligned} \tag{2}$$

where $\mathcal{V}$ is a volume enclosed by surface $\mathcal{S}$ and $\mathcal{A}$ is a surface enclosed by curve $\mathcal{C}$.

As a second example, suppose you need the $\hat{r}$ component of curl$\vec{A}$. Then, consider a little loop $\mathcal{C}$ normal to $\hat{r}$. Then

$$\begin{aligned} \oint_{\mathcal{C}} \vec{A} \cdot d\vec{s} &= -A_\theta(r, \theta + d\theta/2, \phi) r d\theta \\ &\quad -A\phi(r, \theta + d\theta, \phi + d\phi/2) r \sin(\theta + d\theta) d\phi \\ &\quad +A_\theta(r, \theta + d\theta/2, \phi + d\phi) r d\theta \\ &\quad +A_\phi(r, \theta, \phi + d\phi/2) r \sin\theta d\theta. \end{aligned}$$

Applying Eq. 2 gives

$$\begin{aligned}
\lim_{\mathcal{A}\to 0}\frac{\oint_{\mathcal{C}}\vec{A}\cdot d\vec{s}}{\mathcal{A}} = & \lim_{d\theta,d\phi\to 0}[\frac{(A_\theta(r,\theta+d\theta/2,\phi+d\phi)-A_\theta(r,\theta+d\theta/2,\phi))rd\theta}{r^2\sin\theta d\theta d\phi} \\
- & \frac{A_\phi(r,\theta+d\theta,\phi+\phi/2)r\sin(\theta+d\theta/2)d\theta - A_\phi(r,\theta,\phi+d\phi/2)r\sin\theta d\theta}{r^2\sin\theta d\theta d\phi}] \\
= & \frac{1}{\sin\theta}\frac{\partial A_\theta}{\partial\phi} - \frac{1}{r}\frac{\partial(A_\phi\sin\theta)}{\partial\theta}.
\end{aligned}$$

Similarly for the other components. When working a problem involving differential vector operatores, be sure to eliminate those terms which you can by using symmetry arguments; the reduction in work can be large.

4.1 Taylor expansions

Taylor expansions are a common technique for simplifying a problem, usually to first order. The Taylor expansion is given by

$$\begin{aligned}
f(x) &= \sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n \\
&= f(0) + \left.\frac{df}{dx}\right|_0 x
\end{aligned}$$

where the second line is first order and $x << 1$. There are several expansions you should just know:

$$\begin{aligned}
(1+x)^n &\sim 1+nx \\
\frac{1}{(1+x)^n} &\sim 1-nx \\
\sin x &\sim x \\
\cos x &\sim 1-\frac{x^2}{2}\sim 1 \\
\ln(1+x) &\sim x \\
e^x &\sim 1+x.
\end{aligned}$$

The expansions may be applied successively; for example, the redshift z can be shown to be approximately proportional to the source velocity:

$$\begin{aligned}
z &= \frac{\lambda'}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} \\
&= \sqrt{1+2\beta} = 1+\beta.
\end{aligned}$$

4.2 Scales

Part I assumes you have a rough idea of scales of energies, times and distances. An excellent way to study it to construct table for each and Table 2 shows the start of a distance scale table. Energies may be converted to times or distances using $\hbar$ and $\hbar c$, respectively: $[E/\hbar] = [\text{time}]^{-1}$, $[E/(\hbar c)] = [\text{length}]^{-1}$.

10^3 Mpc	Size of the universe
1 Mpc	typical distance between galaxies
10 kpc	size of a galaxy
3 pc	distance to nearest star
2×10^8 km	distance to the sun
6×10^3 km	radius of the earth
30 km	circumference of the largest accelerator
500 m	Wavelength of AM radio
10 m	diameter of largest optical telescopes
3 m	Wavelength of FM radio
1 m	laboratory scale
0.1 mm	smallest objects resolvable by unaided eye
1 μ m	wavelength of infrared radiation
0.5 μ m	wavelength of red light
0.1 μm	wavelength of ultraviolet light
0.2 nm	wavelength of X rays
200 fm	wavelength of γ rays
0.2 fm	nuclear radius
0.002 fm	range of weak force

Table 2: Distance scale.

5 How to study

As mentioned before, Part I is based on core undergraduate material. It is best to begin studying a few months before the exam, but 1/2-1 hour a day at the start is enough. Make studying a habit; try to do it at the same time and place each day. Studying is important, but do not let it dominate your life at any point up to the exam; by starting early, you can do less each day, which will make the whole think less onerous.

Here is a simple procedure for study. You may not wish to follow it, but it does contain all the important elements.

1. Begin by going through the MIT physics course outlines in this guide or the outlines from your courses. For each topic, make a short summary (a sentence or even a few words) and the relevant equations. The goal here is to see how the whole subject fits together, not memorize specific ideas or techniques. Identify the core and peripheral material.

2. Go back through the outline and think of examples of relevant physical systems. For example, in the central force section of mechanics, the planetary orbits are certainly one example, but the Bohr atom is another. You do not need to work through each system in detail.

3. At this point, work through any subject you think you are weak on. When working through, concentrate on understanding the key ideas, not the details of derivations. Also, make sure you understand the important examples.

4. Review the SI and CGS systems of units.

5. Construct tables for time, energy and distance scales.

6. After your review is complete, look over the sample Part I problems. Save one or two exams for a real practice and work the other problems. Review the material surrounding problems you cannot work and add that material to your notes. This part is good for a study group; you will learn a lot by comparing notes.

7. A week or so before the exam, sit down for five uninterrupted hours and work one of the practice exams in “battle conditions.” Remember basic exam strategy:

   - Go through the exam quickly at the start and mark each problem ”+” if you think you can do it with no trouble, ”0” is you think you can work it, but that it may take some time and ”-” if you do not have a good idea what to do.
   - Work the ”+” problems first. If you have trouble on ”+” problem, recatagorize it ”0” or ”-” and move on.
   - Work the ”0” problems next. Start with the one that seems the easiest.
   - Work the ”-” problems last. For the ”0” and ”-” problems, put down what you <u>do</u> know about the problem, even if it is not directly related.
   - Keep about 1/2 hour at the end for checking over work.
   - <u>Check units of your answers</u>. This is the best way to find mistakes.

- Work neatly and organize your work.

8. Once you have done a real practice, review any material you are still unsure of and add it to your notes.

9. If you ran out of time or had trouble organizing your thoughts during your practice, work on correcting your approach and take a second practice.

10. Do not study the day before the exam. Take the day off and do something fun. Get plenty of rest and eat properly the days before the exam.