PHYSICAL-ACOUSTICAL DATA ASSIMILATION AND CHARACTERIZATION AND TRANSFER OF UNCERTAINTIES TO THE SONAR: ERROR SUBSPACE STATISTICAL ESTIMATION

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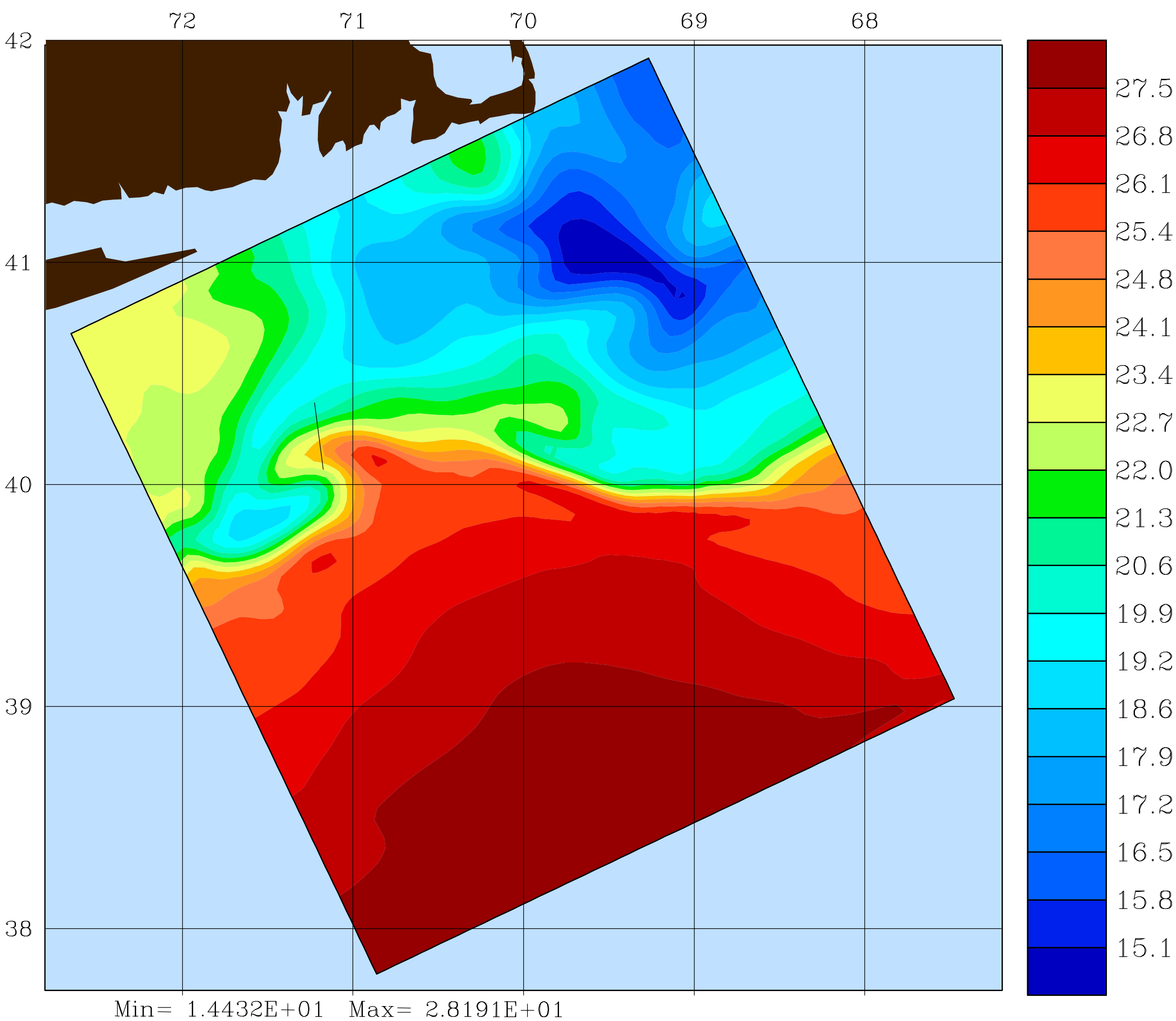


Temperature (°C) at Level 1

COUPLED PHYSICAL-ACOUSTICAL DYNAMICAL MODELS

• Physical model: Primitive-Equation (PDE, x, y, z, t: HOPS)

Horiz. Momen.	$\frac{D\mathbf{u_h}}{Dt} + f\,\mathbf{e}_3 \wedge \mathbf{u_h} = -\frac{1}{\rho_0}\nabla_h\, p_w + \nabla \cdot (A\nabla \mathbf{u}_h)$	(1-2)
Vert. Momen.	$\rho g + \frac{\partial p_w}{\partial z} = 0$	(3)
Thermal energy	$\frac{DT}{Dt} = \nabla \cdot (K\nabla T)$	(4)
Cons. of salt	$\frac{DS}{Dt} = \nabla \cdot (K^s \nabla S)$	(5)
Cons. of mass	$\nabla \cdot \mathbf{u} = 0$	(6)
Eqn. of state	$\rho(\mathbf{r}, z, t) = \rho(T, S, p_w)$	(7)
Sound speed eqn.	$c(\mathbf{r}, z, t) = C(T, S, p_w)$	(8)

• Acoustical model: Coupled Normal-Mode model (PDE, f, r, z, t: NPS)

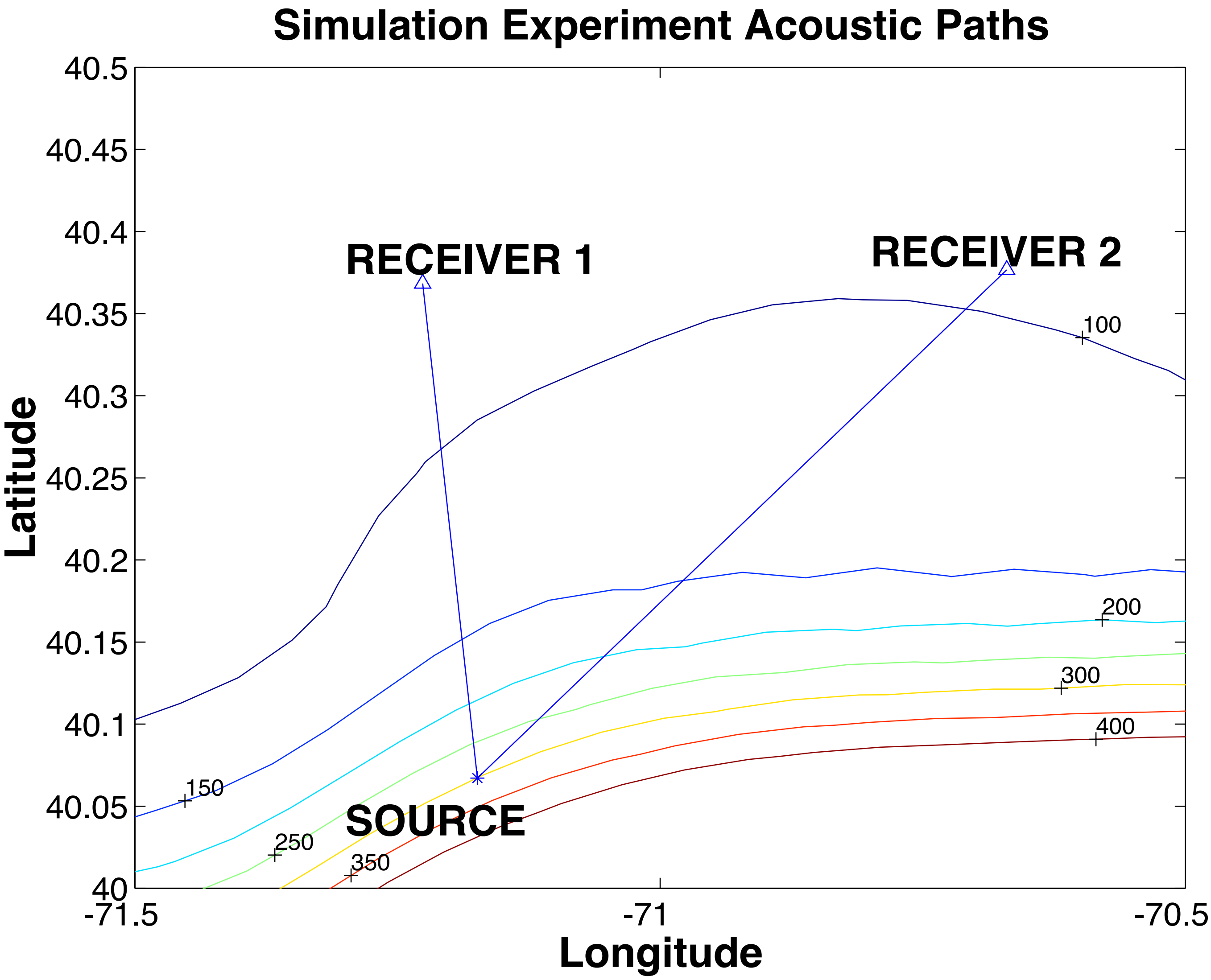
Wave eqn.
$$\rho\, c^2(\mathbf{r}, z, t)\, \nabla \cdot \left(\frac{1}{\rho}\nabla p_s(\mathbf{r}, z, t)\right) = \frac{\partial^2 p_s(\mathbf{r},z,t)}{\partial t^2}$$

p_s transfer fct.
$$\nabla^2 P_s - \frac{1}{\rho}\nabla\rho \cdot \nabla P_s + k^2 P_s = -2\frac{r_0}{r}\,\delta(r-r_0)(z-z_0)$$
$$\text{where} \quad k \doteq 2\pi f / c(\mathbf{r}, z, t) \qquad (9)$$

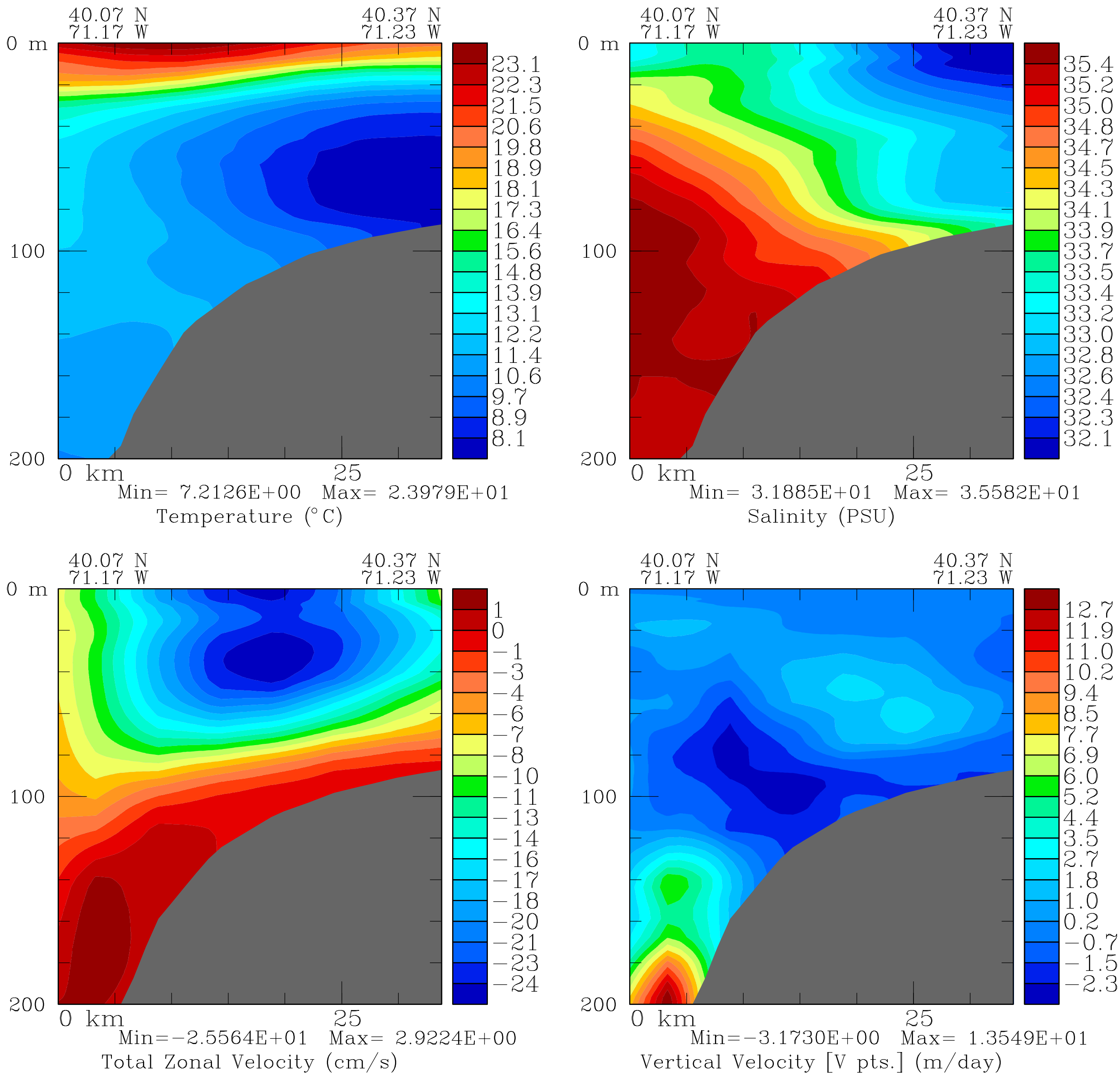
Coupled
$$\text{With} \quad P_s(r, z; f) \doteq \sum_n \frac{r_0}{\sqrt{r}} P_n(r; f) Z_n(z; r, f) \qquad (10)$$

Normal-modes
$$\left\{\frac{\partial^2}{\partial z^2} - \frac{1}{\rho(r,z)}\frac{\partial\rho(r,z)}{\partial z}\frac{\partial}{\partial z} + (k(r,z)^2 - k_n(r;f)^2)\right\} \times Z_n(z; r, f) = 0 \qquad (11)$$

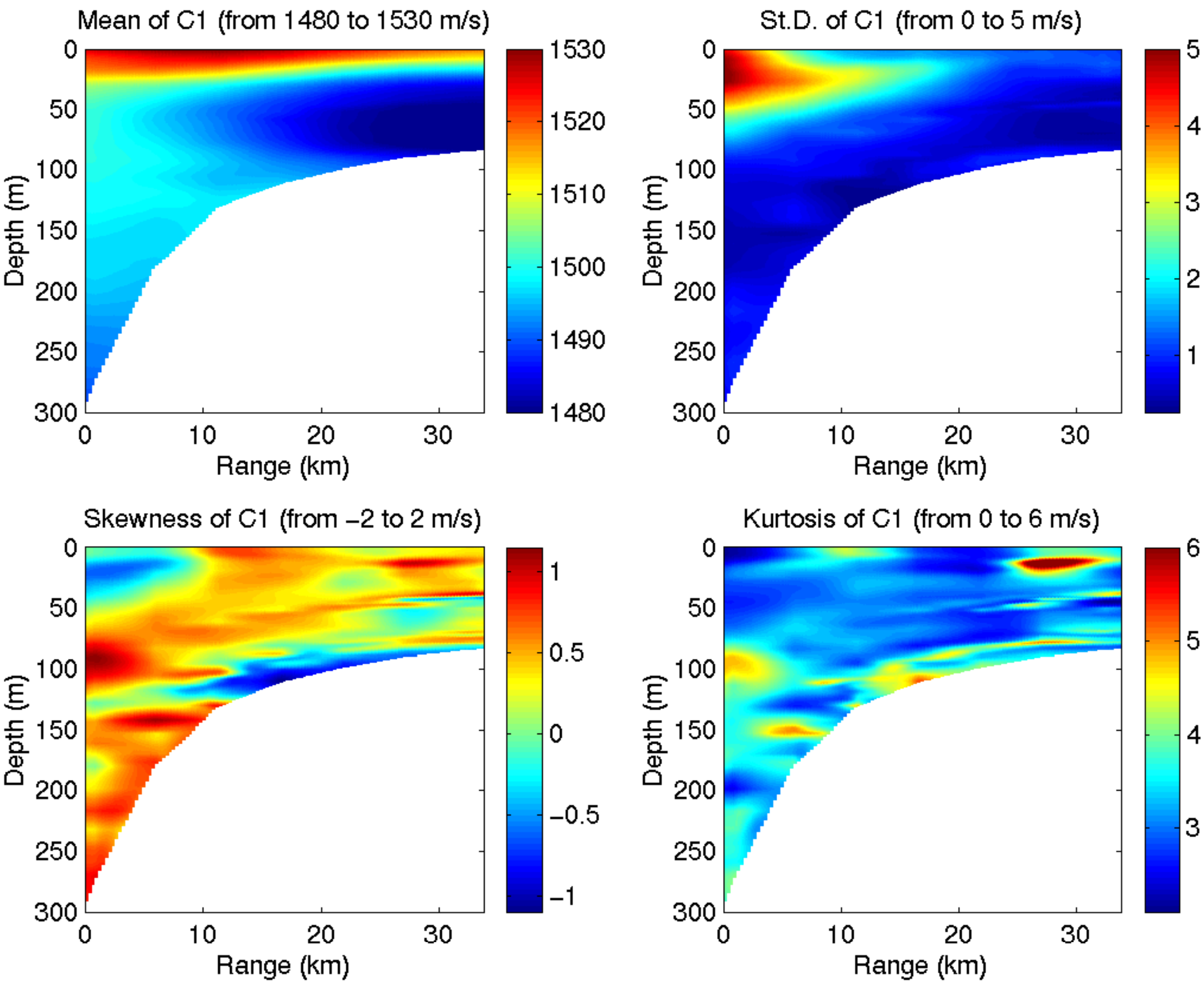
Modal amplit.
$$\left(\frac{d^2}{dr^2} + k_n^2\right) P_n = -\sum_m \left(\gamma_{mn}\frac{d}{dr} + C_{mn}\right) P_m \qquad (12)$$



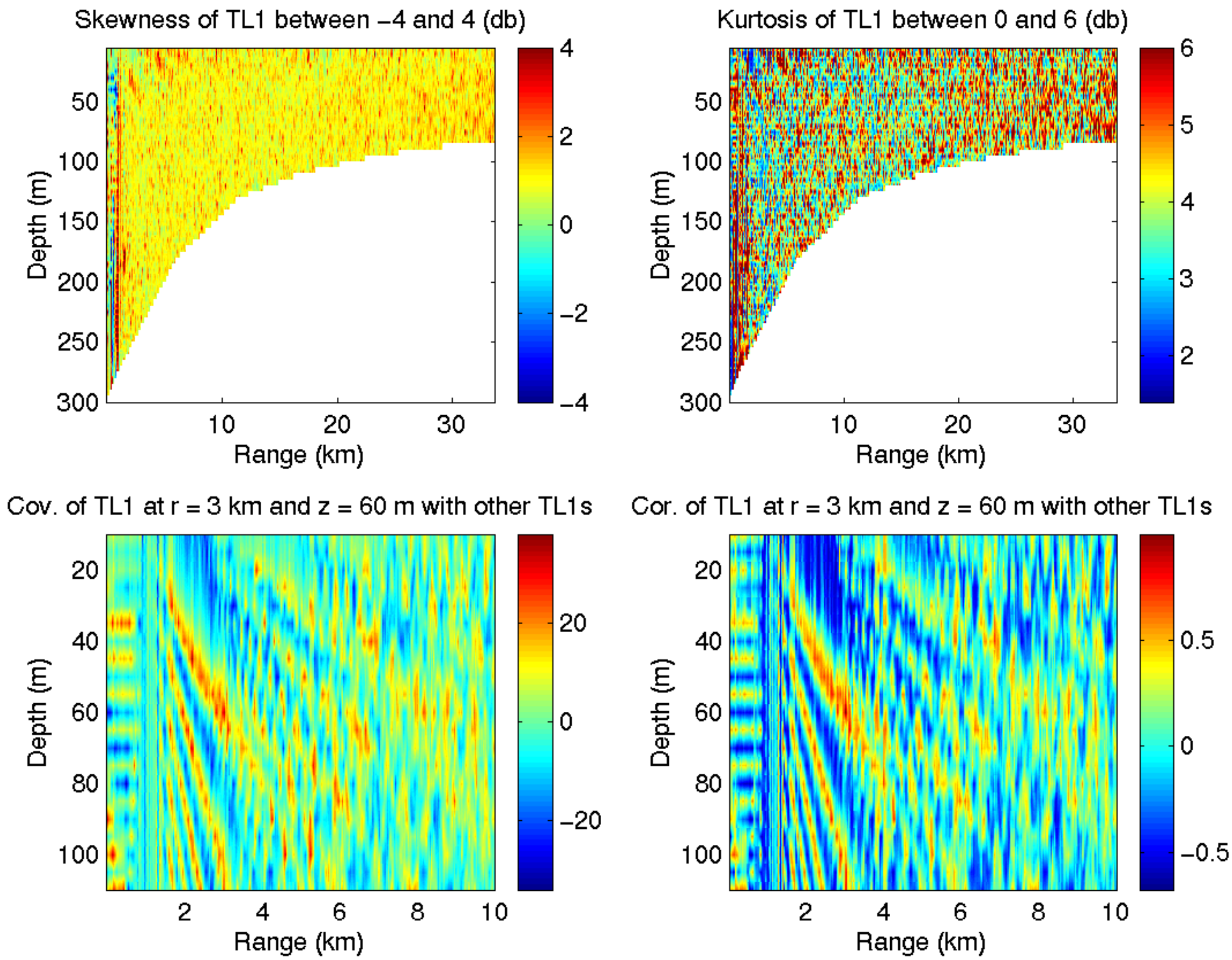
Acoustic paths considered (as in Shelfbreak-PRIMER), overlaid on bathymetry.



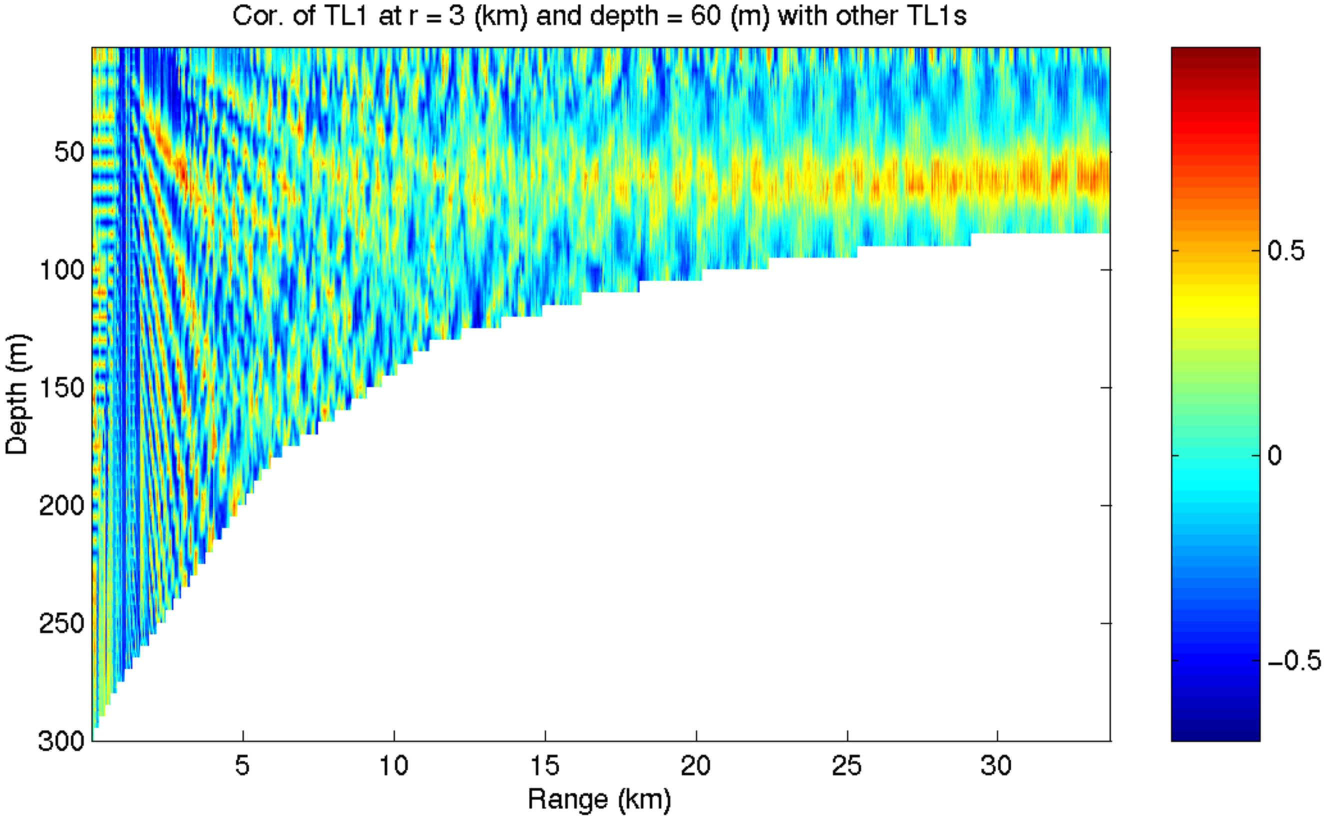
40.07 N
71.17 W
40.37 N
71.23 W
0 m
100
200
0 km
25
Min= 7.2126E+00 Max= 2.3979E+01
Temperature (°C)
23.1
22.3
21.5
20.6
19.8
18.9
18.1
17.3
16.4
15.6
14.8
13.9
13.1
12.2
11.4
10.6
9.7
8.9
8.1
Min= 3.1885E+01 Max= 3.5582E+01
Salinity (PSU)
35.4
35.2
35.0
34.8
34.7
34.5
34.3
34.1
33.9
33.7
33.5
33.4
33.2
33.0
32.8
32.6
32.4
32.3
32.1
Min=−2.5564E+01 Max= 2.9224E+00
Total Zonal Velocity (cm/s)
1
0
−1
−3
−4
−6
−7
−8
−10
−11
−13
−14
−16
−17
−18
−20
−21
−23
−24
Min=−3.1730E+00 Max= 1.3549E+01
Vertical Velocity [V pts.] (m/day)
12.7
11.9
11.0
10.2
9.4
8.5
7.7
6.9
6.0
5.2
4.4
3.5
2.7
1.8
1.0
0.2
−0.7
−1.5
−2.3

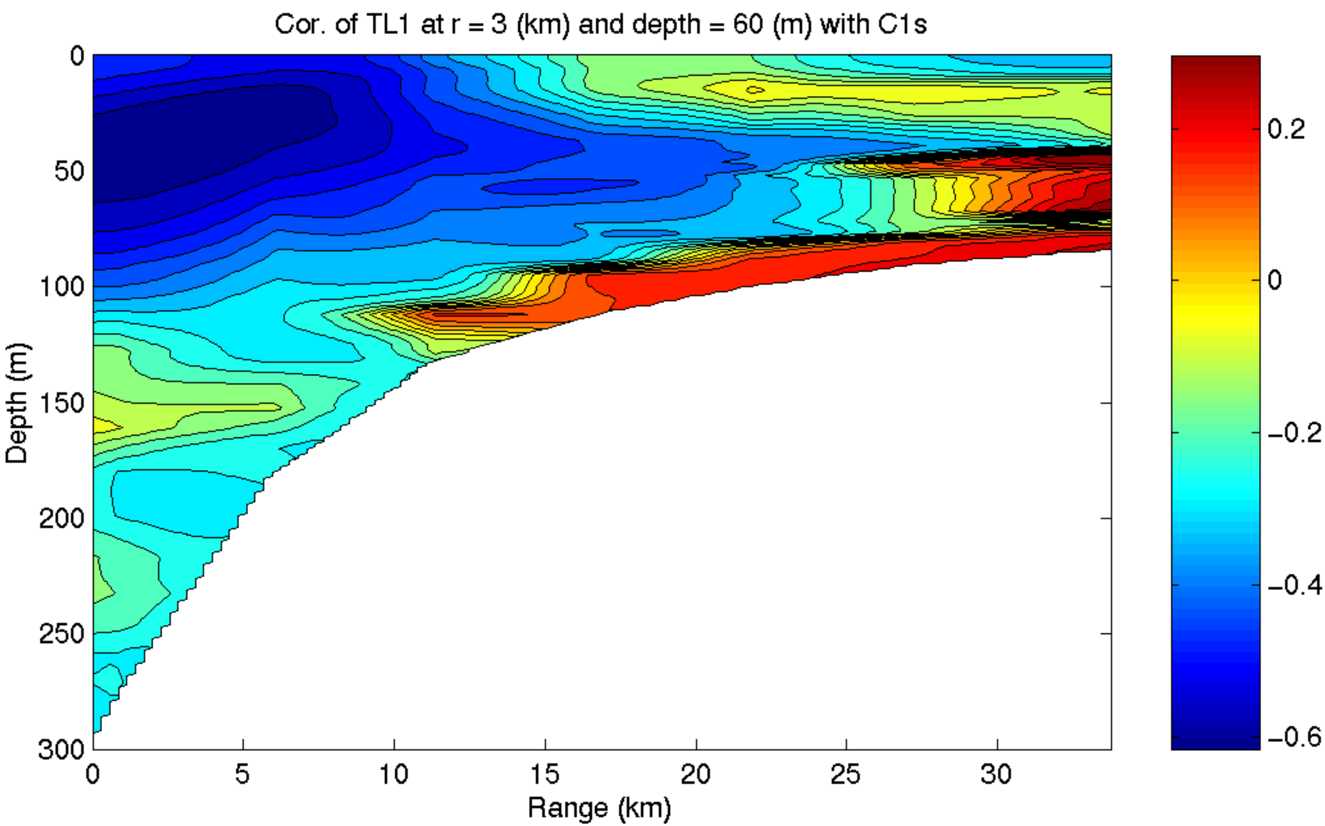


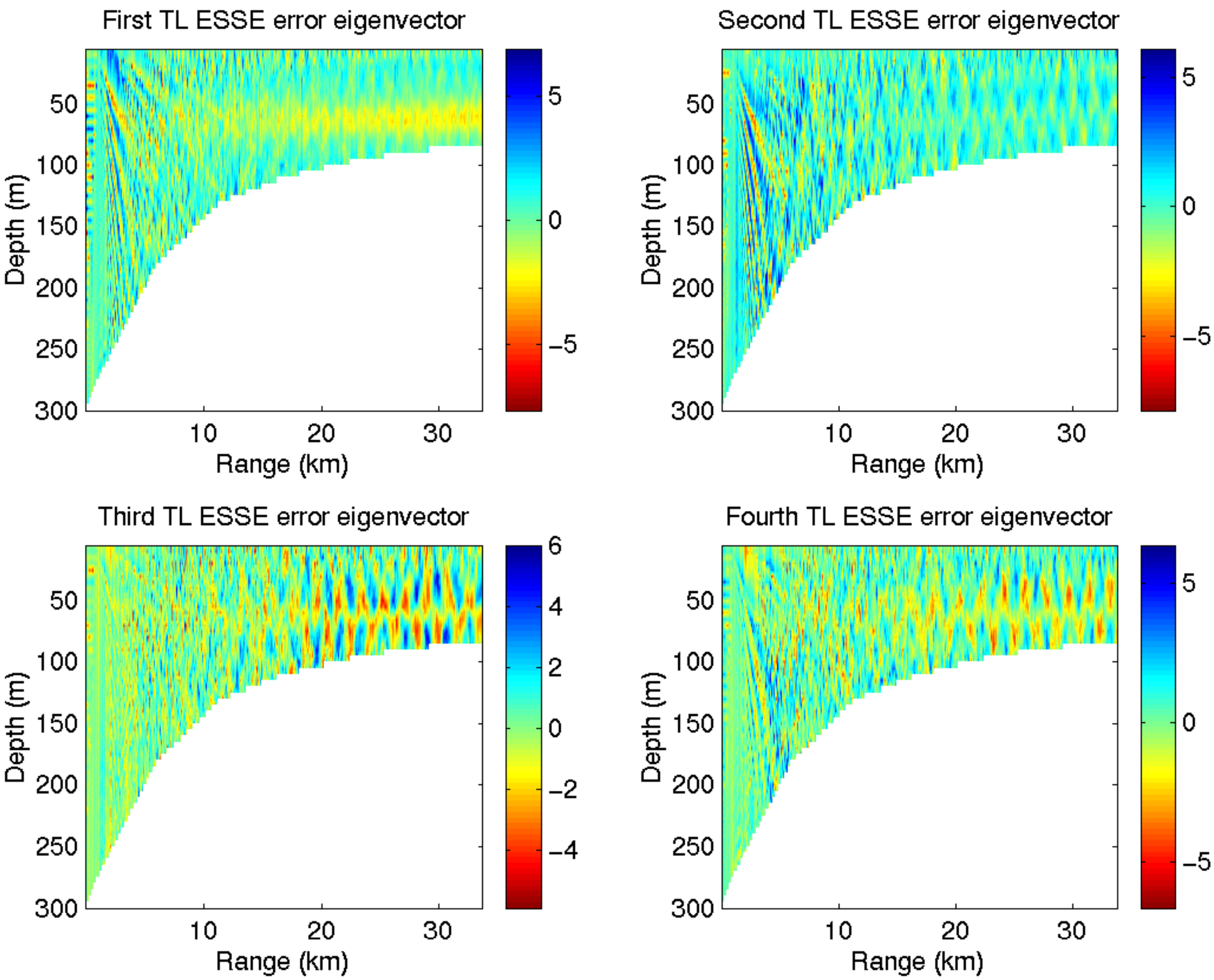
Mean of C1 and statistics of error estimate for C1



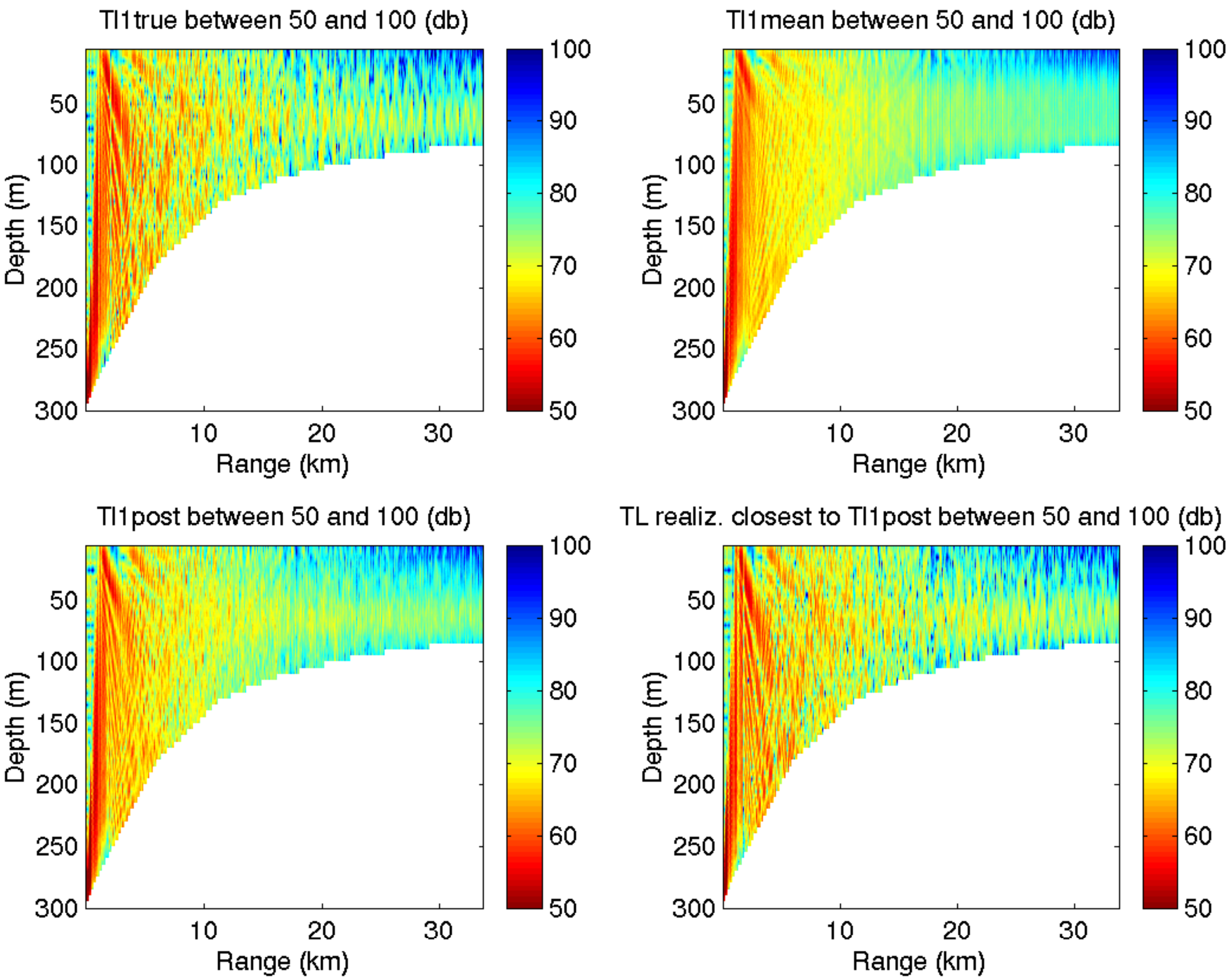
TL error statistics: sample skewness, kurtosis and zoom on a covariance/correlation field.


Cor. of TL1 at r = 3 (km) and depth = 60 (m) with other TL1s
Depth (m)
Range (km)
50
100
150
200
250
300
5
10
15
20
25
30
0.5
0
−0.5

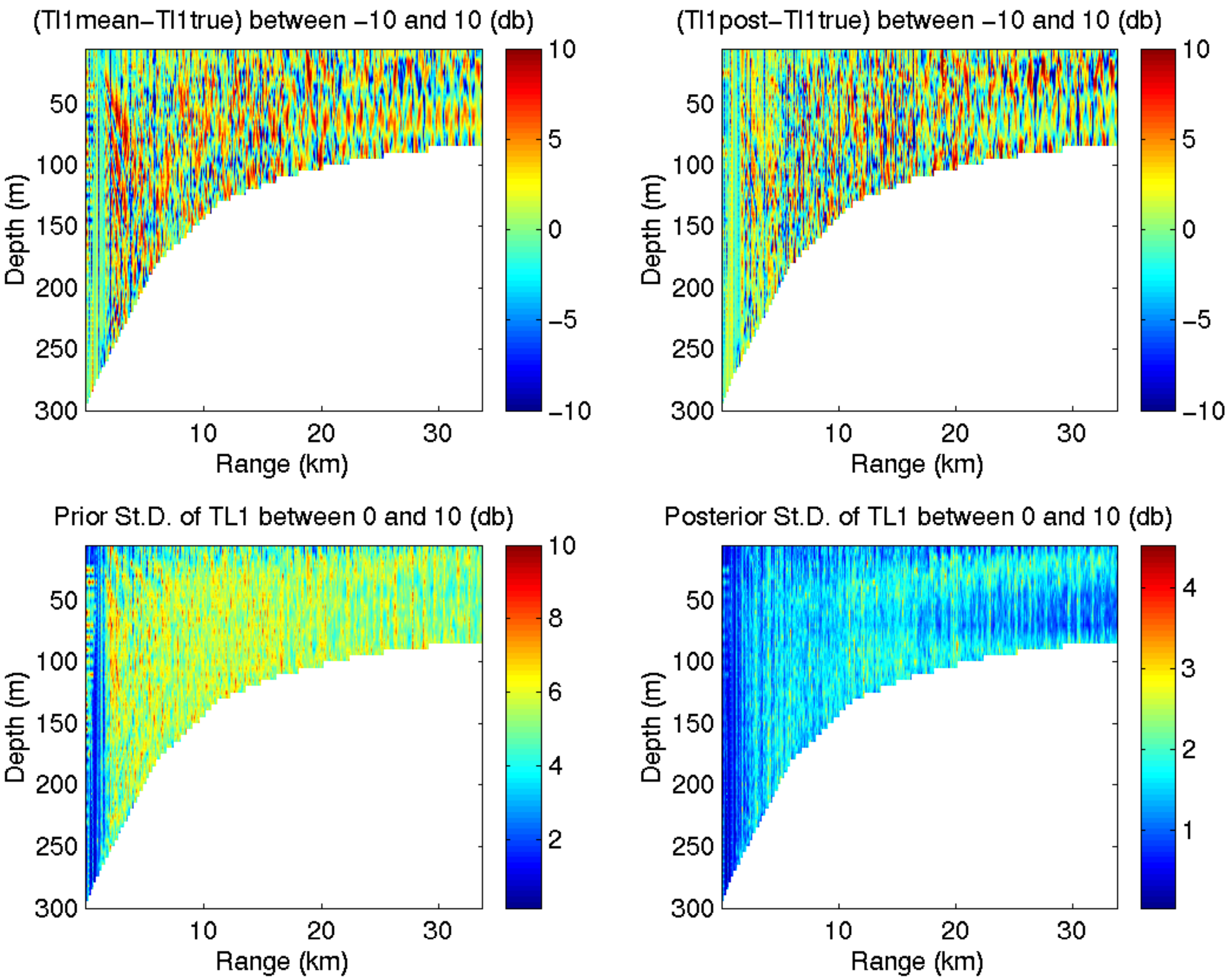

Cor. of TL1 at r = 3 (km) and depth = 60 (m) with C1s
Depth (m)
Range (km)
0
50
100
150
200
250
300
0
5
10
15
20
25
30
0.2
0
−0.2
−0.4
−0.6



First four ESSE error eigenvectors for TL along section 1: they explain 12.4%, 6%, 4.2% and 3.6% of the total TL error variance, respectively

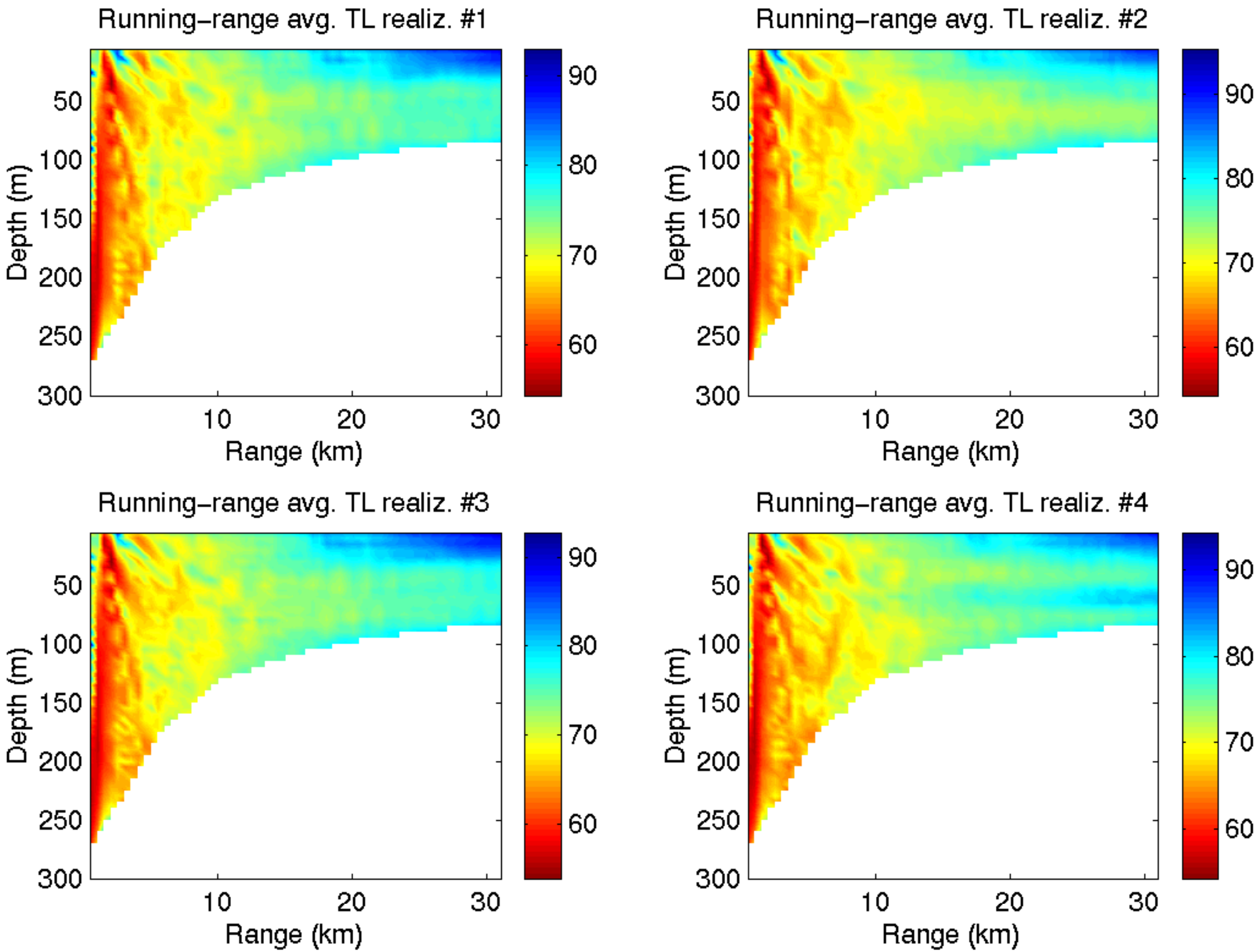


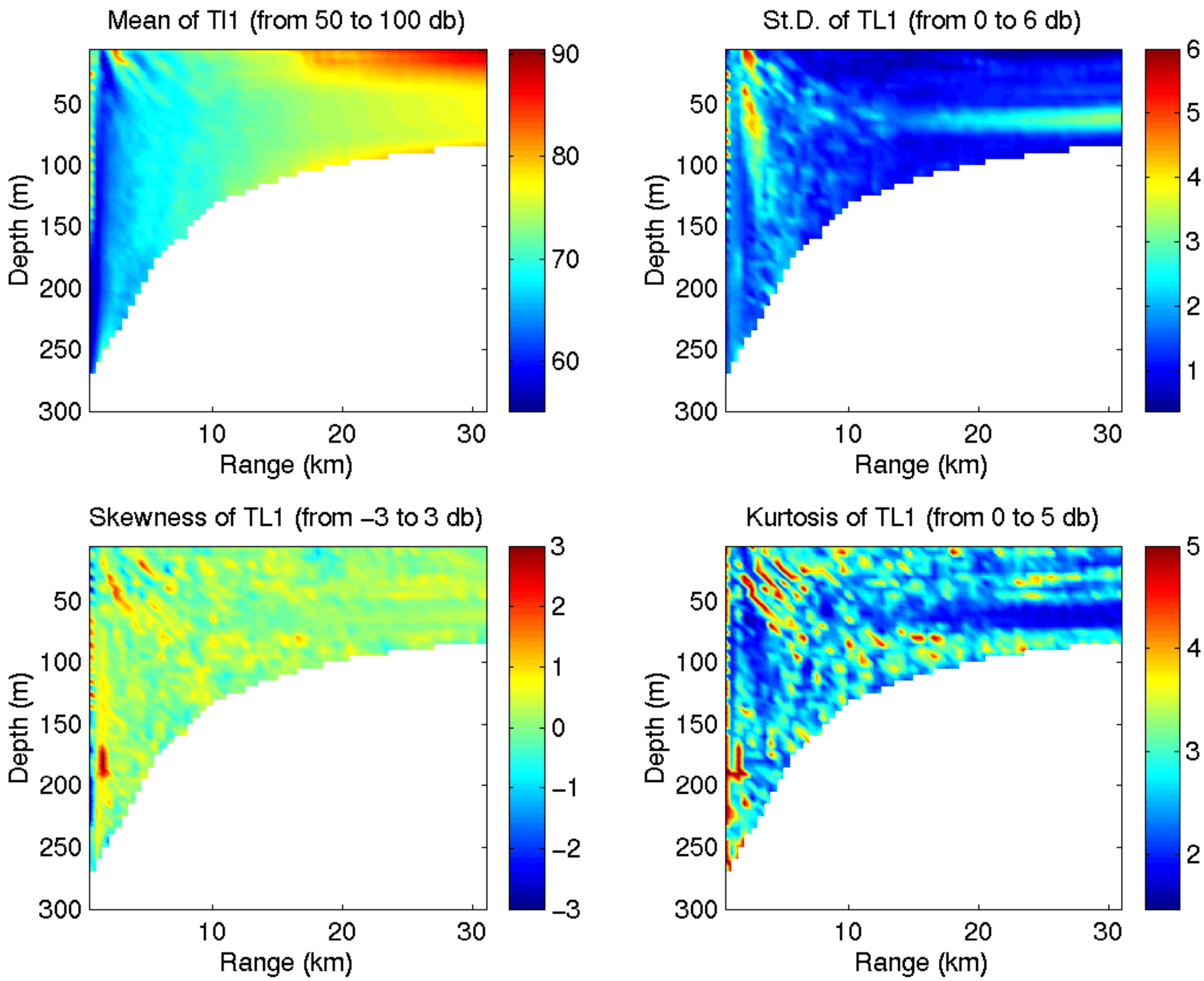
ESSE assimilation results (Twin Experiment):
"True" TL (from which towed-receiver data are sub-sampled),
a priori **TL (ensemble mean forecast),**
a posteriori **TL (after data assimilation) and**
TL realization closest to *a posteriori* **TL.**



***A priori* (before assi.) and *a posteriori* (after assi.) residuals, and *a priori* and *a posteriori* error Std. dev. for TL along section 1.**

Var.-width (32Hz/224Hz) running-range avg. TL realiz. #1-4 (from 50 to 100 db)





Mean and statistics of error estimate for the var.-width (32Hz/224Hz) running-range avg. TL1

Uncertainty (error PDF) of variable−width (32Hz/224Hz) running−range avg. TL