Estimation of Cournot Games in the Presence of Indivisibilities and Heterogeneous Firms.[1]

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Abstract

This paper presents a general theoretical framework that allows estimation of equilibrium models of quantity competition when the unit of output is indivisible. Examples include a retailer deciding how many stores to build, franchises to license, suppliers to use, or products to produce. I focus upon the class of models, suitable for use with count data, in which heterogeneous players interact strategically, choosing their optimal integer strategy. The main theoretical result of the paper establishes weak, easily verifiable conditions under which market output is uniquely determined within the set of equilibria. The model's parameters may then be estimated by comparing predicted and observed market outputs in a cross section of markets. Using data from the supermarket industry, I estimate the model and quantify the magnitude of the effect of a change in the firm's financial structure on a supermarket chain's reaction function.

J.E.L. Classifications: D0, C1, C7, L0, L1

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1 Introduction

Limited dependent variable (LDV) models, such as the probit or logit models, allow researchers to use qualitative information about whether an event occurred to investigate the factors which caused it. For example, LDV models are used to determine the factors which make individuals statistically more likely to visit their doctors or accept job offers. LDV models are extremely useful and popular tools for empirical work primarily because qualitative information is more easily available than high quality quantitative data. This fact is at least as true for data derived from decisions made by corporations as those made by individuals. The availability of qualitative data about firms entry decisions, for example, motivated Bresnahan and Reiss (1991) and Berry (1992) to explicitly consider the game theoretic analogue of the simplest LDV model, the simultaneous move entry game. In the entry game, each participant simultaneously decides whether or not to enter a market, making a choice from the strategy set $S_i = \{0, 1\}$.

When detailed quantitative data is not available, researchers have successfully used rougher measures of scale to expand the set of questions beyond those which can be addressed using qualitative data alone. Ordered LDV models such as the ordered probit, the poission, and negative binomial models, enable count data to be used to analyze the factors which make an event likely to occur *and* the likely scale of an event. For example, a researcher can analyze the factors which determine the number of visits individuals make to their doctors, the number of job offers they receive, and so forth. In precisely the same vein, count data indicating the scale of activity of firms are frequently available, even when more detailed quantity data are not. However, to date, the econometric tools required to analyze count data arising from strategically interacting firms were not available.

The aim of this paper is to provide a set of theoretical results which can be used to estimate the game theoretic analogue of ordered LDV models. These results allow structural econometric analysis of models of quantity competition using count data. Examples might include retailers deciding the number of stores to build in a market, the number of franchises to license, suppliers to use, or products to produce. In each case, every potentially active agent makes a choice from the strategy set $S_i = \{0, 1, 2, ..., M\}$. I illustrate the methodology by estimating the model using count data from the supermarket industry.[1]

The economic importance of these results for policy analysis using structural econometric models is easy to illustrate since firms are able to build multiple (chain) stores in most retail markets of economic significance. Suppose a researcher wishes to draw inferences about the competitiveness of a type of retail market using data on the number of active stores. Following Bresnahan and Reiss (1990), one approach is to use the structure of the entry game to isolate the relationship between market size and the number of active firms. However, when data are observed from markets where chain stores could potentially operate, such an exercise would likely result in very imprecise estimates of the relationship of interest since the data would include markets of similar sizes with quite different numbers of active firms. Thus, while a considerable amount has been learned about entry into small retail markets from studies such as Bresnahan and Reiss (1990), it is important to develop tools which can be applied in larger retail markets, where most market participants operate chain stores.

Bresnahan and Reiss (1991) and Berry (1992) illustrate that the primary difficulty facing researchers who wish to estimate empirical games with discrete strategy spaces is the existence of multiple Nash equilibria. If a model has more than one equilibrium for a

[1] Throughout the paper I present the results for the case in which firms choose an integer level of output. In fact, the integer nature of the firms choices is *not* necessary for the main theoretical results, which apply equally to the continuous quantity choice game. I focus on the integer strategy sets because of the importance of count data for empirical work.

given set of parameters, the task of econometrically identifying the model's parameters is not trivial since identification relies upon a one to one mapping between the model's parameters and its prediction about the world. Until Bresnahan and Reiss (1990) and Berry (1992) suggested an alternative identification strategy, economists using the simultaneous LDV model had imposed a condition, known as the "coherency" or "principal" condition, which, as Schmidt (1981) observes, essentially rules out simultaneous decision making.[2]

The main theoretical result of this paper demonstrates that while the set of Nash equilibria of the discrete quantity game contains widely different individual output choices, it is possible to provide sufficient conditions under which *market* supply is uniquely determined within the set of equilibria. These sufficient conditions are much weaker than those necessary to ensure that there is a unique equilibrium which become more and more unreasonable as the number of players increases. (See Moulin (1984).) Unfortunately, it is *not* true that market output is uniquely determined within the equilibrium set for all plausible quantity games. In particular, non-convexities in costs may introduce equilibria with widely different aggregate supplies.

Theoretically, the approach is new to the literature on empirical oligopoly models, although the mathematical techniques I use are well known. Specifically, I draw on supermodular game theory. I combine the methods developed to analyze games of strategic complementarities with suitable assumptions, making these versatile tools applicable to the Cournot game, a game of strategic substitutes.[3]

[2]See also Blundell and Smith (1994) for a discussion of the existing econometric literature and most recently Tamer (1998) for an alternative approach to the estimation of the two player entry game.

[3]For an introduction to the theory of supermodular games see Fudenberg and Tirole (1991) or Vives (1990). For more on the mathematics of supermodular functions see Topkis (1978),Topkis (1979). Amir (1996) has independently noticed that supermodular game theory, developed to analyze games of strategic complementarities, could aid our understanding of games of strategic substitutes. He uses the implications of ordinal supermodularity (see Milgrom and Shannon (1994)) to generalize Novshek (1985)'s proof of existence of equilibrium in quantity games with an arbitrary number of players.

The paper is organized as follows. Section II introduces the model. Section III contains the main theoretical results in the paper, establishing sufficient conditions to ensure that market output is uniquely determined within the set of Nash equilibria. Section IV presents an application and empirical results, proposes an equilibrium computation procedure, and sketches the Generalized Method of Moments (GMM) estimation routine. Section V concludes.

2 The Model

A firm's strategy is an integer chosen from the strategy space $S_i = \{0, 1, 2, 3, ..., M\}$ where $M < \infty$. Each player considers her rivals' strategy s_{-i} fixed, and chooses her own strategy s_i to solve

$$\max_{s_i \in S_i} \quad \pi_i(s_i, s_{-i})$$

where $\pi_i(.,.)$ represents the payoff function of player i.

Let the best response correspondence, $r_i(s_{-i})$, denote the set of solutions to this problem for a given s_{-i}.

$$r_i(s_{-i}) = \{s_i \in S_i \mid \pi(s_i, s_{-i}) \geq \pi(t_i, s_{-i}) \text{ for all } t_i \in S_i\}$$

Let N denote the number of players, $i = 1, ..., N$. Define the Cartesian product, $S = X_{i=1}^{N} S_i$, and let

$$\mathcal{F} = \{s \in S \mid s_i \in r_i(s_{-i}) \text{ for all } i\}$$

be the set of all Nash equilibria of the game.

3 Characterizing the Set of Equilibria

3.1 The Firm's Decision Problem

First I characterize the solution to the firm's decision problem. Two weak assumptions about the profit functions of the players are sufficient to establish that best response functions are well defined non-increasing functions.

Assumption 1 (A1.) *For all i, $\pi_i(s_i, s_{-i})$ has the dual single crossing property in (s_i, s_{-i}). I.e., for all $(s_i, t_i) \in S_i^2$ and $(s_{-i}, t_{-i}) \in S_{-i}^2$ such that $s_i \geq t_i$ and $s_{-i} \geq t_{-i}$*

$$\pi_i(s_i, t_{-i}) - \pi_i(t_i, t_{-i}) \leq 0 \;\Rightarrow\; \pi_i(s_i, s_{-i}) - \pi_i(t_i, s_{-i}) \leq 0$$

This first assumption is a standard one of strategic substitutability. It states that *if* firm i finds increasing output unprofitable at the present level of output of her rivals, then she would also find an identical expansion unprofitable were her rivals producing more.[4] Milgrom and Shannon (1994) show that Assumption 1 implies that best response correspondences are non-increasing.

Lemma 1 (Milgrom and Shannon (1994)) *Under assumption 1, $r_i(s_{-i})$ is non-increasing.*

Proof: See Appendix.

The identification strategy being pursued in this paper is to provide sufficient conditions under which market output is uniquely defined within the set of Nash equilibria.

[4]Suppose $\pi_i(s_i, s_{-i}) = s_i P(s_i + s_{-i}) - C_i(s_i)$. Amir (1996) shows that if (i) $P(\cdot)$ is strictly decreasing and log concave, and (ii) $C_i(\cdot)$ is strictly increasing, then $\pi_i(s_i, s_{-i})$ satisfies the Dual single crossing property (upto regularity conditions not required here since all firms strategy sets are finite and bounded).

If a firm could obtain the same profits from different output choices, aggregate ouput may not be uniquely determined. Therefore I assume that a firm will never be indifferent between two action choices and, consequently, best response correspondences are single valued functions.

Assumption 2 (A2.) *For all i and $s_{-i} \in S_{-i}$, $r_i(s_{-i})$ is a singleton.*

Assumption 2 is attractive for empirical work since the addition of an action specific random component to profits, drawn from a continuous distribution of possible values, would make indifference between any two actions a zero probability event.[5]

3.2 Equilibrium in the Two Player Game

I first prove the results for the $N = 2$ player game and subsequently the N player results are proven by induction. Milgrom and Roberts (1990) observed that, with two players, a clever ordering on the strategy space (see Figure 1) places the Cournot game within the class of supermodular games. The idea is to turn the original best response mappings (each decreasing in rival's output) into best response functions which are *increasing* with respect to the new ordering. Tarski's (Tarski (1955)) existence result for increasing functions then applies directly.[6]

[5] To be concrete, if the existing profit function were replaced with a new, very similar, one so that $\tilde{\pi}_i(s, s_{-i}) = \pi_i(s, s_{-i}) + \epsilon_s$, where $\{\epsilon_s\}_{s=1}^{M}$ are scalar random variables drawn from a continuous distribution and known to the firm. Then, single valued best response functions are guaranteed even though the variance of the distribution of ϵ_s could be made arbitrarily small. When action spaces are continuous, ruling out indifference is somewhat more restrictive. In this case, the assumption corresponds to the usual requirement that firms profit functions are strictly quasi-concave in their own level of output.

[6] Definitions for the well known mathematical objects used in the text below are included for completeness in this footnote. Since the basic concepts are all I require, I refer the reader to any of the references on supermodularity for further discussion. Let $\geq$ be binary relation on a non-empty set S. The pair $(S, \geq)$ is a partially ordered set (POSET) if $\geq$ is reflexive, transitive and antisymmetric. (For completeness: the binary relation $\geq$ is antisymmetric if for $x, y \in S$, $x \geq y$ and $y \geq x$ implies that $x = y$.) A *lattice* is a POSET $(S, \geq)$ in which any two elements, $x, y \in S$ have a least upper bound

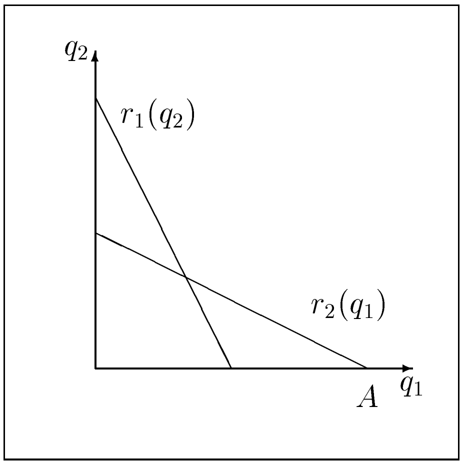

Figure 1: The figure shows the reaction functions of two firms engaged in a Cournot duopoly game, with their respective quantity choices shown on the axes. Under the standard ordering, represented by arrows on the axes, both reaction functions are decreasing. Now consider the point at which player two's reaction function cuts the q_1 axis, point A, and look left and up. From here, the reaction functions are both increasing! In mathematical terms, taking the new perspective in the picture can be represented by a new ordering on the strategy space, $S_1 \text{x} S_2$.

Theorem 1 (Existence of Equilibrium, Tarski (1955)) *If S is a non empty, compact sublattice of $\mathcal{R}^K$ and $f : S \to S$ is an increasing function, f has a fixed point in S.*

Specifically, I transform the problem from one analyzed on the poset $(S_1 \text{x} S_2, \geq\ )$ where $\geq$ is the usual vector ordering $(a, b) \geq (c, d)$ iff $a \geq c$ and $b \geq d$, to a problem analyzed on the poset $(S_1 \text{x} S_2, \geq_r)$ where $(a, b) \geq_r (c, d)$ iff $a \leq c$ and $b \geq d$. With respect to this new order, the vector function, $f(s) = (r_1(s_2), r_2(s_1))$, is an increasing function defined on a sublattice of $\mathcal{R}^2$. Tarski's theorem applies.

To characterize the set of equilibria, I begin by applying a theorem, due to Topkis (1979), to the function $f : S_1 \text{x} S_2 \to S_1 \text{x} S_2$.

Theorem 2 (Characterization Theorem, Topkis (1979)) *Let S be a non empty, compact sublattice of $\mathcal{R}^K$ with respect to the ordering $\geq_r$ and let $f : S \to S$ be an*

(sup) and a greatest lower bound (inf) in the set S. The strategy set $S_i = \{0, 1, ..., M\}$ is a lattice with respect to the standard orderings, $\geq$ or $\leq$. A lattice is *complete* if every non empty subset of S has a supremum and an infimum in S. A subset T of S, is a *sublattice* if for every pair of elements $x, y \in T$ the supremum and infimum is also in T. Let $(S, \geq)$ be a poset. A real valued function f with domain S is *increasing* if for all $x, y \in S$, $x \geq y$ implies that $f(x) \geq f(y)$.

increasing function. Then, the set of fixed points of the mapping f form a complete lattice with respect to the order $\geq_r$.

Topkis's theorem implies that the set of two player equilibria, $\mathcal{F}$, is a complete lattice with respect to the new order, $\geq_r$. By definition, in a complete lattice there are maximal, s^H, and minimal, s^L, elements in the set. i.e., there exists $s^L, s^H \in \mathcal{F}$ such that

$$s^L \leq_r s \;\; \forall\, s \in \mathcal{F}, \text{ and } s^H \geq_r s \;\forall\; s \in \mathcal{F}.$$

If the complete lattice were with respect to the usual vector ordering, $\geq$, it would be trivial to argue that the total output $s_1 + s_2$ is uniquely determined within the set of Nash equilibria[7]. However, in fact, the complete lattice is with respect to the new ordering $\geq_r$. Heuristically, the set of equilibria lie "around" a diagonal crossing from top left to bottom right and have a maximum and a minimum in this new ordering. Notice however, that at this level of generality, one equilibrium in the set might be (0,10) while another, for the same pair of profit functions, might be (100,0). Without further assumptions, it is trivial to construct examples in which wildly different total output can result across equilibria. Even in the two player game, an additional assumption is required to ensure that the set of Nash equilibria has a unique market output.

Assumption 3 (A3.) *For any pair of strategy profiles, $(s,t) \in (S_1\text{x}S_2)^2$ such that (i) $s_i + s_{-i} \;\geq\; t_i + t_{-i}$, and (ii) $s_i \geq t_i$, $\pi(.,.)$ satisfies*

$$\pi_i(t_i+1, t_{-i}) \leq \pi_i(t_i, t_{-i}) \;\Rightarrow\; \pi_i(s_i+1, s_{-i}) \leq \pi_i(s_i, s_{-i})$$

[7]Suppose not, then there is a $s \in \mathcal{F}$ such that $s \geq s^L$, strict for at least one player i. I.e., $s_i = r_i(s_{-i}) > r_i(s^L_{-i}) = s^L_i$. But this clearly violates $r_i(\cdot)$ non increasing. **QED.**

Assumption 3 requires that if it is unprofitable for firm i to produce the $(t_i + 1)^{st}$ unit of output, then it must also be unprofitable to produce the $(s_i + 1)^{st}$ unit, where own output and market output are both larger. Sufficient conditions for Assumption 3 include the requirement that the marginal profitability provided by an additional unit of output is non-increasing in total output *and* player $i's$ own output. To illustrate, consider the profit function for a firm engaged in Cournot competition, with the profit function $\pi_i(s_i, Q) = P(Q)s_i - C_i(s_i)$, where $Q = s_i + s_{-i}$, $P(\cdot)$ represents a homogeneous product demand function, and $C_i(s_i)$ is the firm's cost function. If marginal revenue is non-increasing in market output and own output, and marginal cost is non-decreasing in own output Assumption 3 is satisfied.[8] One important, but somewhat unfortunate, implication of Assumption 3 is that fixed costs are ruled out in all cases except the entry game. In general, quantity games with fixed costs have multiple equilibria with both market output and consumer welfare differing across the equilibria.

Example 1

Consider the game represented in the Figure 2. Each player chooses a number of stores to build in a town. For the purposes of this example I assume that each firm is allowed to build no more than two outlets so that $S_i = \{0, 1, 2\}$. Arrows on the vertices indicate the relative size of payoffs for the players. For example, when player II plays the strategy $s_2 = 2$, the horizontal arrows indicate that player I prefers playing 0 to 1. Similarly at $(1, 1)$, the vertical arrow (below) indicates that player II would prefer to play 0 over 1. Nash equilibria are nodes at which neither player would prefer to change her strategy,

[8] For the continuous case, replace A3 with the condition that $\frac{\partial \pi_i(Q,s)}{\partial s_i}$ is non-increasing in (Q, s). In that case, the sufficient conditions become $\frac{\partial^2 \pi_i(Q,s)}{\partial s_i^2} = P''(Q)s_i + 2P'(Q) - C''(s_i) \leq 0$ and $\frac{\partial^2 \pi_i(Q,s)}{\partial Q \partial s_i} = P''(Q)s_i + P'(Q) \leq 0$. The latter is clearly the more restrictive condition and is exactly the assumption required by Novshek (1985) to establish existence of equilibrium.

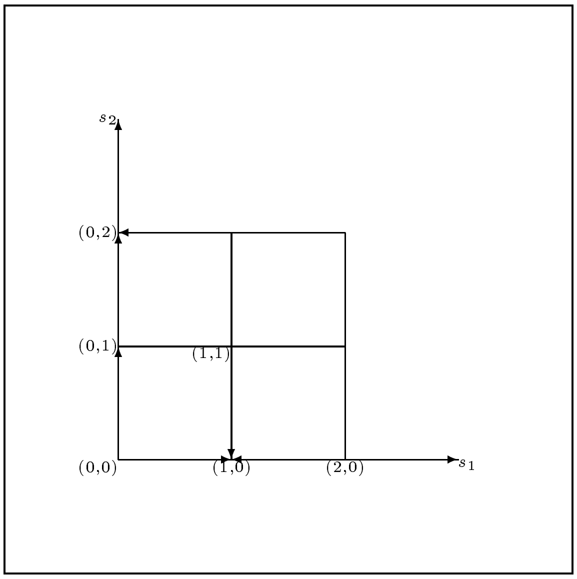

Figure 2: A two player game

given the strategy played by her rival.

Here the set of Nash equilibria is $\{(1,0),(0,2)\}$. Notice that total output differs across equilibria. To see why the assumptions rule out this case, compare the actions of player II at the nodes $(1,0)$ and $(0,1)$. Assumption 3 requires that if it is unprofitable for player II to produce a first unit of output when player I is producing one then it must be unprofitable for player II to produce a second unit when player I is producing none. Thus if moving from $(1,0)$ to $(1,1)$ is unprofitable, then it must also be unprofitable to move from $(0,1)$ to $(0,2)$, since total output is at least as large (it's the same) and player II's own output is higher. Clearly, this assumption rules out fixed costs in all cases but the one considered in the next example.$\square$

Example 2

This example is the entry game considered by Bresnahan and Reiss (1990) and Berry (1992). Assumption 3 states that if entry is unprofitable with a given number of active rivals, entry must also be unprofitable with a larger number of active rivals. With $S_i = \{0, 1\}$ and $\pi_i(0, \cdot) = 0 \; \forall \; i$, Assumption 3 becomes

$$\pi_i(1, t_{-i}) \leq 0 \Rightarrow \pi_i(1, s_{-i}) \leq 0$$

for all $s_{-i} \geq t_{-i}$. In this case, Assumption 3 reduces to the very natural condition that the profitability of entry is non-increasing in the number of rival active players. Note in particular that in this case alone, fixed costs of entry may differ arbitrarily across firms. □

Having developed Assumptions 1-3, I shall now turn to consider their implications. First I demonstrate that A1-A3 are sufficient to ensure that market output is uniquely determined within the set of Nash equilibria in the two player game. An analogous proof establishes the proposition for the N player game in the next subsection.

Proposition 1 *Under the assumptions A1–A3, the two player game has an equilibrium. Furthermore, market output is uniquely determined within the set of Nash equilibria,*

Proof Topkis(1979) establishes that the set of Nash equilibria is a complete lattice with respect to the order $\geq_r$. Thus there exists an $s^L = (s_1^L, s_2^L)$ such that $s^L \leq_r s$ for all $s \in \mathcal{F}$, i.e., $s_1^L \geq s_1$ and $s_2^L \leq s_2$. I prove that $s_1^L + s_2^L = s_1 + s_2$, using a proof by contradiction. There are two cases to consider.

Case (a) Suppose first that $s_1^L + s_2^L > s_1 + s_2$. In that case, $s_1^L > s_1$ since $s_2^L \leq s_2$ by definition of s_2^L. Consider the strategy profiles $(s_1^L - 1, s_2^L)$ and (s_1, s_2). Notice that the conditions of Assumption 3 are satisfied: $s_1^L + s_2^L - 1 \geq s_1 + s_2$ and $s_1^L - 1 \geq s_1$. Hence

$$\pi_1(s_1 + 1, s_2) \leq \pi_1(s_1, s_2) \;\Rightarrow\; \pi_1(s_1^L, s_2^L) \leq \pi_1(s_1^L - 1, s_2^L) \tag{1}$$

The first inequality follows since s_1 is a best response to s_2. However, the latter inequality contradicts the fact that s^L is a Nash equilibrium and best responses are single valued functions.

Case (b) Suppose $s_1^L + s_2^L < s_1 + s_2$. By definition, $s_1^L \geq s_1$, hence $s_2 > s_2^L$. Consider the strategy profiles $(s_1, s_2 - 1)$ and (s_1^L, s_2^L). By Assumption 3,

$$\pi_2(s_2^L + 1, s_1^L) \leq \pi_2(s_2^L, s_1^L) \Rightarrow \pi_2(s_2, s_1) \leq \pi_2(s_2 - 1, s_1).$$

Hence, s and s^L cannot both be equilibria. **Q.E.D.**

3.3 Equilibrium in the N Player Game

In this section I generalize the results to the N player game. Following Novshek (1985), I assume that firm i's optimal output choice depends only upon the total output produced by her rivals.

Assumption 4 (A4.) *For all i and for all s_{-i}, $r_i(s_{-i}) = r_i(\sum_{j \neq i} s_j)$.*

Proposition 2 (Results for the N player game.) *Under the assumptions A1–A4, the N player game has an equilibrium. Furthermore, market output is uniquely determined within the set of Nash equilibria,*

Proof See Appendix A.

The intuition for this result is the same as in the two player game. Consider any two equilibria in which total market output is strictly higher in one of them. At least one firm must be producing more in the high output equilibrium. In that case, she must want to produce more in the low market equilibrium because either her marginal revenue must, by Assumption 3, be higher and/or her marginal cost lower.

4 Competition in the Supermarket Industry

In this section I use the theoretical results obtained above to build a structural econometric model of competition in the supermarket industry. I model competition in the industry as a quantity game in which chain stores choose the number of stores to operate in a market. The discrete nature of the decision to build a supermarket, combined with the fact that count data is the best readily available measure of quantity choices motivates the decision to model competition among supermarkets as a discrete Cournot game.[9]

A great advantage of modeling competition using the quantity competition model developed in the previous sections of this paper is that we can test and quantify the impact of any variables hypothesized to shift a firm's reaction function in the well developed framework of Cournot competition. Moreover, the data requirements for the model are remarkably small.[10] Chevalier (1995) notes that the supermarket industry's dramatic increases in debt during the late 1980's provide a good opportunity to test and quantify theories of financial and product market interaction. In this section I use the model to investigate the impact of financial structure on product market competition.[11]

[9] The supermarket industry is of interest since it is a major component of the U.S. economy with annual sales larger than defense expenditure (approximately \$300 billion). The data used in this paper are from an annual industry publication, Progressive Grocer's *Market Scope*. A market area is defined as a Metropolitan Statistical Area (MSA) and Progressive Grocer defines a supermarket as a retail food store with annual sales of more than \$2 million and a sales area of at least 9000 square feet. See Chevalier (1995) for a full description of the data sources.

[10] Specifically, only market level data on the characteristics of the market and observations on the total number of stores operated in each of 85 market areas are required. While the assumption that supermarkets in each market are homogeneous products is an obvious abstraction from reality, a richer structure of the internal workings of the market when only market level data are available introduces enormous complexity with little obvious gain. See Davis (1998) for an approach to retail markets when store level data are available.

[11] Examples of the dramatic changes in the financial structure of supermarkets include the \$5 billion Safeway LBO, \$4.1 billion Kroger leveraged recapitalization, the \$1.8 billion Supermarkets General leveraged buyout (LBO), and the \$1.2 billion Stop and Shop LBO. Altogether, 19 of the 50 largest supermarket chains have undertaken LBO's, accounting for some \$72 billion sales in 1991.

Chevalier (1995) uses a reduced form model to test for significant interaction between the firm's financial structure and the intensity of product market competition. Here I test and quantify the impact of the firm's leveraged buyout (LBO) on its optimal quantity decision.

Theoretical analyses of the interaction between financial structure and product market competition delineate circumstances under which debt results in either a softening or a toughening of competition. However, the theoretical predictions depend on precise timing of the firm's output and financial structure decisions, about which there is little reason to hold a strong a priori position. For example, Brander and Lewis (1986) argue that increases in debt make firms more competitive. Making one small change to the model, allowing firms to change their level of output twice after the increase in debt levels, Glazer (1994) argues the opposite is true, that increases in debt soften competition. Maksimovic (1988), and Rotemberg and Scharfstein (1990), also argue that increases in debt toughen product market competition while Fudenberg and Tirole (1986), Bolton and Scharfstein (1990) and Phillips (1991) each argue the contrary position, that increases in debt softens competition.

4.1 Specification of the Profit Function

Firm $i's$ profit function is assumed to have the form

$$\pi_{ik}(s_{ik}, Q_k) = \max_{s_{ik} \in S_i} \; P_k(Q_k)s_{ik} - C_{ik}(s_{ik}),$$

where Q_k denotes total market demand and $k = 1, ..., K$ indexes markets. For simplicity of exposition, I assume that the market inverse demand curve is linear in market demand, so that

$$P(Q_k) = \beta_1^1 - \beta_2 Q_k + X_k' \gamma_1 + (Q_k X_k)' \gamma_2$$

where X_k represents a vector of market level variables that shift or rotate demand exogenously. Examples include market size, median income, the percentage of households with income below \$10,000, regional dummy variables and so forth.

The firms cost functions are assumed quadratic in own quantity with one source of heterogeneity across firms that is unobserved to the econometrician, but known to the firms and its rivals[12]

$$C_i(s_i) = (\beta_1^2 + \beta_3 shLBO_{ik} + \beta_4 LBO_i + \beta_5 s_i + \sigma_u u_{ik}) s_i.$$

Heterogeneity in marginal costs, u_{ik}, is assumed independent and identically distributed (*iid*) across players and markets, and is drawn from a standard Gaussian distribution. Heterogeneity in variable costs is important in an empirical specification because it allows observationally identical firms to own a different number of stores within a market. Other variables hypothesized to affect marginal costs are also included. For example, $shLBO_{ik}$ denotes the share of rivals in market k who have undergone leveraged buyouts, while the dummy variable LBO_i indicates whether or not firm i has undergone an LBO by 1991.

In a Cournot game, a firm's best response function is a decreasing function of the firm's own marginal cost of production. Thus, a variable which increases a firm's marginal cost will result in lower own output for every level of output of her rivals.

[12] While I motivate the form of the profit function in terms of demand and cost function specifications, it is actually more appropriate to think of the pair implying a specification for variable profitability since, for example, the model's prediction will only depend upon the value of $\beta_1 \equiv \beta_1^1 - \beta_1^2$, not each individual component. Similarly, the same profit function would result whether u_{ik} entered because it represents heterogeneity in marginal cost or in the firm's demand intercept. A full discussion of identification in the Cournot model is provided by Bresnahan (1982) and Lau (1982).

For example, if $\beta_4 > 0$, then undergoing a leveraged buyout "softens" product market competition since the firm's reaction function shifts inward. Similarly, if $\beta_3 > 0$, a larger share of rival's output provided by LBO stores leads to a lower level of firm i's output for every level of rival's output. As is usual in discrete choice models, the units of profits are not identified in the estimation procedure developed below. I impose the traditional constraint that the variance of the unobservable is unity, $\sigma_u \equiv 1$. With this specification, the model satisfies A1-A3 provided that β_5 is strictly positive.

4.2 Estimation

In this section, I outline the procedure used to estimate the parameters of the model.[13] For a given value of the parameters, the model has a prediction about an observable. Specifically, the model uniquely identifies the total market output provided A1-A3 are satisfied. Define the prediction error, ν_k, as the difference between the observed market equilibrium output and the model's conditional expectation of equilibrium market output[14]

$$\nu_k(Q_k^*, Z_{\cdot k}; \theta) \quad = \quad Q_k^* - E[Q_k^* | Z_{\cdot k}; \theta], \tag{2}$$

where $Z_{\cdot k} \equiv (\{X_{ik}\}_{i=1}^N, X_k)$ is the exogenous data for market k (i.e., the market and firm characteristics) and θ is the vector of model parameters $(\beta, \gamma)'$. By construction, the prediction error ν_k, is mean independent of the exogenous data when evaluated at the

[13] For clarity of exposition, I ignore several aspects of the actual estimation procedure in this section. The full details are provided in Appendix B.

[14] Note that, except in the entry game, I have *not* shown that the model will uniquely identify the number of active firms, and in general it will not be true. Hence I cannot impose a second moment condition which matches the number of observed players to the number of actual players.

true parameter value θ_o

$$E[\nu_k(\cdot, Z_{\cdot k}; \theta_o)|Z_{\cdot k}] = 0. \tag{3}$$

Condition 3, together with some standard regularity conditions, provides the basis for a generalized method of moments estimator (see Hansen (1982)) of θ_o using Equation 2. Unfortunately, $E[Q_k^*|Z_{\cdot k}; \theta]$ is difficult to compute exactly, as it requires computation of a high dimensional integral, over the unobserved (by the econometrician) heterogeneity. Fortunately, following Berry (1992), a simulation estimator is easy to construct. Given a fixed set of R random draws on each market, $(\{\{\hat{u}_{ik}^r\}_{i=1}^N\}_{r=1}^R)$, I calculate an unbiased estimate, $\hat{Q}_k$, of the expected market output, $E[Q_k^*|Z_{\cdot k}, \theta]$ as

$$\hat{Q}_k^R = \frac{1}{R}\sum_{r=1}^{R} Q_k(Z_{\cdot k}, \{\hat{u}_{ik}^r\}_{i=1}^N; \theta),$$

and calculate the simulated prediction errors and the corresponding simulated sample moment condition

$$\hat{\nu}_k = Q_k^* - \hat{Q}_k^R(\cdot;\theta), \quad G_K^R(\theta) = \frac{1}{K}\sum_{k=1}^{K} \hat{\nu}_k(\theta) f(Z_{\cdot k})$$

where $f(Z_{\cdot k})$ is an arbitrary (px1) function of the exogenous data.

The estimate of the parameter vector $\hat{\theta}$ is obtained as the solution to the extremum problem

$$\min_{\theta \in \Theta} \quad ||G_K^R(\theta)||_A^2 \tag{4}$$

where $||\cdot||_A$ is the Euclidean norm with respect to the weight matrix A.

Pakes (1989) provides regularity conditions,[15] that are satisfied here, under which the resulting estimator is consistent and asymptotically normal. Thus[16]

$$\sqrt{K}(\theta_K^{ns} - \theta_o) \sim N\left(0, (1+\frac{1}{R})(\Gamma' A \Gamma)^{-1}\Gamma' A V A' \Gamma (\Gamma' A \Gamma)^{-1}\right)$$

where

$$\begin{aligned} V &= E v_k(\theta_o)^2 f(Z_{\cdot k}) f(Z_{\cdot k})', \\ \Gamma &= E \frac{\partial v_k(\theta_o) f(Z_{\cdot k})}{\partial \theta}. \end{aligned}$$

The Nelder Meade derivative free minimization routine is used to solve Equation 4 because the GMM objective function is *not* smooth in finite samples. It does, however, become smooth as the number of markets becomes large because of the averaging across markets.

4.3 Equilibrium Computation: Cournot Tattonnement

The model developed in this paper has an extremely large strategy space. Each of N firms chooses a level of output from the set $\{0, 1, 2, 3...M\}$. Consequently, there are M^N points in the strategy space and finding an equilibrium is not a trivial computational task. Evidently, complete enumeration is not a feasible option except for *very* small M *and* N. In the supermarket data, $M = 163$ and $N = 48$.

To estimate the parameters of the model, the predicted equilibrium must be calcu-

[15]See also McFadden (1989). Pollard (1984) and Andrews (1994) provide details of the underlying empirical process theory.

[16]Notice that a single simulation draw provides an unbiased estimate of expected total market output. Consequently, a consistent estimate of θ is obtained with only one simulation draw. However, to reduce the variance in the estimator due to simulation I increase the number of simulation draws used. The variance due to simulation inflates the asymptotic covariance matrix of the estimates by the factor $1 + \frac{1}{R}$.

lated for every simulation draw in every market at each stage of the estimation procedure. Thus, if the non-linear optimization over the parameter space takes $1,000$ iterations to converge, and we use 100 simulation draws in each of 85 markets, estimation involves finding 8.5 million equilibria. Unfortunately, Cournot games such as this are not generally globally stable except under special conditions (See Moulin (1984)). Nonetheless, severe difficulty in finding *an* equilibrium is unlikely since (i) we know at least one equilibrium exists, usually many, and (ii) our task is only to find *one*: any equilibrium will suffice, since by Proposition 2 market output is the same at *every* point in the equilibrium set.

In practice, player by player iterated Cournot tattonnement always converged to an equilibrium without problems. That is, starting at the zero point, $s^0 = 0$, player 1 plays her best response to 0, moving to her monopoly output, $s^1 = (r_1(s^0), 0, 0, 0, .., 0)$. Next, player 2 picked his best response to s^1, and so on. Cycling through the players generally provided convergence within a reasonably small number of rounds.

4.4 Estimation Results

Tables 1 and 2 present the parameter estimates for a variety of specifications. In those tables, I consider two potential mechanisms (described below) which operate to link LBO activity to competition in the supermarket industry. Subsequently, I examine the robustness of the results to two types of changes: I allow the assumed set of potential entrants in each market to change, and I vary the set of market level regressors. In order to establish a baseline specification for the set of potential entrants, I use the results provided by Cotterill and Haller (1992) who find that new stores are primarily built by supermarket chains already active in a market.[17] All the estimates are computed using

[17]Table 1 presents a general specification, while Table 2 presents a more parsimonious set of results. The baseline specifications for each LBO mechanism are reported in columns one and two of the tables.

data from 1985 and 1991, before and after the leveraged buyout wave in the supermarket industry.[18]

First, notice that the estimates imply costs are convex in own output because the estimates of β_5 are positive. Therefore, we may apply the results in Proposition 2 and conclude that market output is uniquely determined in all equilibria and, consequently, that the model's parameters are econometrically identified. The exact value of β_5 is estimated precisely only when the set of potential entrants is assumed to be relatively small. This result is intuitive: ceteris paribus, the slope of the marginal cost function determines the size of an active firm. Additional uncertainty for the econometrician about the number of active entrants translates directly into uncertainty (for the econometrician) in the estimate of β_5.

Second, note that β_2 is estimated to be positive, which implies that the estimated demand function slopes downward in the total number of operated stores, a necessary condition for marginal revenue to decline in total output. In contrast to the estimates of β_5, the estimates of β_2 are precisely determined in most of the specifications and do not appear to rely quite as substantially on the assumption made regarding the set of potential entrants.

In the third and fourth columns I report estimates computed under the assumptions that (i) all chains which operate stores in the region are potential entrants , and (ii) all 48 chains are potential entrants. In the former case, regions are defined by dividing the country into four broad zones: Northeast, Center, Southeast, and West. Finally, to illustrate that the estimation results are not a quirk of either the structure imposed by the model or the over-identifying restrictions imposed to estimate the full model, a fifth set of results are presented in Table 1. Specifically, column five reports a reduced form regression of the total number of stores in each market during 1991 on market characteristics and the fraction of stores operated by firms which had undergone a leveraged buyout by 1991.

[18]The estimates presented here each assume that the static cournot game is played by market participants each year. Obviously, in reality the game played by supermarkets has rich and complex dynamics affected by both economic and political factors. Evidently, even a simple model of those dynamics is far beyond our current capabilities. However, since exit costs for a store are relatively small, each year every store can and does adjust the total number operating in a market. Consequently, the static simultaneous move game appears to be sufficiently close to reality to provide a useful representation of the data generating process.

Figure 3 displays the estimated profits as a function of a firm's and rivals' output, evaluated for a median market with $u_{ik} = 0$.[19] For a given level of output produced by firm i, her profitability is everywhere decreasing in the output of her rivals. For a given output of rivals, profits are hump shaped in the firm's own output. Thus, even if firms are identical and market output is the same, the distribution of profits across firms can vary widely across equilibria as the equilibrium supply profile varies. For example, a firm will always make zero profits in an equilibrium in which her best response is not to produce. However, there will typically be many other equilibria in which the firm produces positive quantities and makes profits in equilibrium.

Next, consider the parameter estimates for the variables which differ across markets, but are common across firms. The land area and the total number of households play an important role in determining the intercept of the inverse demand curve. These variables may be interpreted as determinants of market size, and explain the vast majority of the variance in the number of active stores across markets.[20] The remaining coefficients on market level variables are estimated imprecisely. In particular, no statistically significant role was found for either the median income or the median age of consumers in the market. The variables measuring inequality within the market, such as the fraction of the population with income below \$10,000 or above \$50,000, were also not found to play an important role. For this reason, I focus on the results in Table 2 for the remainder this section.

The results I present next, which focus on the leveraged buyout variables are related to those provided by Chevalier (1995). In that paper, Chevalier develops a reduced form

[19] Recall that the units of profit indicated on the vertical axis are only defined relative to the cross firm heterogeneity in variable profitability which is normalized to have a variance of one.

[20] Although the estimated relationship between these measures of market size and variable profitability is non-linear, the exact degree of the non-linearity is somewhat sensitive to the inclusion or exclusion of specific observations since there are very few markets as large as New York or LA.

model which attempts to determine whether supermarket chains that did not undertake an LBO between 1985 and 1991 were likely to expand when their rivals undertake LBOs. She concludes that when a firm with a 10 percent market share undertakes an LBO, it increases the probability that a given non-LBO firm will add stores to the market by approximately 6.5 percent.[21] The estimates obtained here are also consistent with the hypothesis that LBO's soften product market competition. However, the reduced form techniques used by Chevalier (1995) describe only the magnitude of the probabilistic event that a firm will add or remove stores. They do not indicate the magnitudes of the financial structure effects. In contrast, the approach advocated in this paper allows both qualitative and quantitative conclusions to be drawn. Moreover, the results are not subject to econometric concerns regarding the endogeneity of total market output, they allow for unobserved heterogeneity in variable profitability across firms,[22] and enable presentation of the results in terms of a direct response of the firms reaction functions.

First, I estimate a model (reported in column 1 of the tables) in which the marginal costs of LBO and non-LBO firms are allowed to differ systematically. This difference might arise from differences in the cost of capital to LBO and non-LBO firms. The effect is a direct strategic effect of debt since the firm's debt structure is allowed to change the firm's own reaction function. Specifically, a dummy variable in the cost function indicates whether firm i undertook an LBO by 1991. Since β_4 is estimated to be positive, the firm's own reaction function shifts leftward when the firm undertakes an LBO. Thus, own output falls and rivals' equilibrium output increases. If the increase in rivals' output is less than the reduction in own output, equilibrium total output decreases. Figure 4 reports the implied impact of changing the financial structure of the rivals of a non LBO

[21] This effect is measured when all market characteristics are held at their mean values.

[22] Unobserved heterogeneity provides the model with the flexibility of an alternative explanation for the observed differences in total market outcomes and therefore places a greater burden of proof on the data.

firm in a median sized market. Undertaking an LBO is estimated to reduce the optimal number of stores in median sized market by approximately ten stores, roughly a 10% reduction.

Second, I estimate a model in which LBO activity by one firm changes the reaction functions of all rival chains; there is an indirect strategic effect of debt. Specifically, I allow reaction functions to shift as the share of stores operated by rivals who have undergone a LBO by 1991 changes.[23]

The results for this specification are reported in the second column of each table. As the fraction of rival stores operated by LBO chains increases, the firm's reaction function is estimated to shift inward, softening competition. Figure 5 shows the effect as the percentage of rival stores operated by LBO chains increases from zero to one hundred percent. In practice, the highest observed share was 66% with the median chain facing 21% of stores operated by LBO rivals.

In principle, given enough data, the model could distinguish between the direct and indirect strategic effects of debt. In practice however, specifications of the structural model which allowed for both mechanisms did not precisely identify the magnitude of either. Moreover, only the model with a direct strategic impact of debt is consistent with the observed behavior of the two types of stores. Thus, the results indicate both the power and the dangers of estimating a structural model using only market level data. It is certainly possible to determine whether market level aggregates are shifted in the manner implied by the underlying model, but it is difficult to distinguish between the underlying mechanism when the actual pattern of firm level responses is observed but cannot, at least at the present time, be incorporated into the model. Perhaps the single

[23]The literature on the strategic effects of leverage usually results in indirect effects only through the equilibrium nature of competition rather than directly in each firm's cost function. Of course, allowing rival's debt levels to affect each firm's cost function is a strict generalization of the model the importance of which can be determined empirically.

most important task remaining for future research on multiple agent count data models is to develop methods capable of incorporating the firm level count data as well as the aggregate market data on quantities.

4.4.1 Further Remarks on Robustness, Precision, and Identification

The parameters which are estimated with precision remain remarkably similar in magnitude across the specifications. However, different assumptions about the set of potential entrants led to markedly different estimates of the precision of the parameters. Generally, the smaller the set of potential entrants is assumed to be, the higher the reported precision of the estimates.

A second issue which emerged during the estimation process is that some specifications where the set of potential entrants is sufficiently large suffer from a practical identification problem even though identification is assured asymptotically provided u_{ik} has unbounded support. In particular, if LBO_i enters the model, the set of potential entrants is large, and $\beta_4 >> 0$ (so that LBO firms are at a significant cost disadvantage) then none of the LBO firms are predicted to be active in equilibrium. If that is true for a large fraction of the simulation draws and observations, the objective function will be perfectly flat as β_4 increases beyond that point; increasing it further does not disrupt those equilibria, and therefore does not change their contribution to the value of the objective function.

Naturally, the fact that there is a flat area of the objective function only poses practical identification problems when the flat area occurs at, or very close to, the minimum of the function. Unfortunately, this happens in the models with $\beta_4 > 0$ when the set of potential entrants is expanded much beyond the set of incumbents. This identification problem is different in nature from those emphasized in the previous literature and arises because the decision to undertake an LBO is not modeled directly.

This type of identification issue may arise whenever a variable in the profit function affects the profitability of only a small subset of active firms.

5 Conclusions

In this paper I develop and implement a method to estimate structural econometric models of quantity setting oligopolistic competition allowing for indivisibilities and firm level heterogeneity in variable profitability. I focus on a class of models appropriate for use with count data, although the results easily extend to the quantity setting game when quantity choices are perfectly divisible. The results provided by the previous literature on entry models (Bresnahan and Reiss (1991), Berry (1992)) are important, but very special, cases of this class of models.

The theoretical contribution of this paper is to provide sufficient conditions to ensure that market output is the same in all the equilibria of a quantity setting game. In those circumstances, market output may be used to identify the parameters of the model by matching observed and predicted market output in a cross section of markets. Specifically, I show that if marginal revenue is declining in own and total output and marginal costs increase in own output, then market output is uniquely determined within the set of equilibria. Most standard Cournot demand and convex cost function specifications satisfy these conditions. However, non-convexities in costs introduce the potential for wide variation in market output across equilibria.

An extremely large number of potential applications are easily envisioned. In fact, this may prove a fruitful approach whenever an applied researcher needs to model strategically interacting agents and count data are available. Examples from industrial organization might include deciding how many franchises to license, suppliers to use, or products to produce. Other examples could include countries deciding how many nuclear war-heads, fighter aircraft, or tanks to build.[24]

Using data from the $300 Bn per annum supermarket industry, I estimate the model with data from 1985 and 1991; before and after a wave of leveraged buyouts changed the financial structure of many firms in the industry. The qualitative conclusions from the empirical results are consistent with those provided by Chevalier (1995), who found that LBOs soften product market competition. Moreover, these qualitative conclusions are found to be robust to the introduction of unobserved firm and market heterogeneity and the appropriate treatment of total market output as an endogenous variable. In addition to these qualitative conclusions, count data allows quantitative conclusions to be drawn. I found that undertaking an LBO reduced the optimal number of supermarkets operated in a median sized market by aproximately ten percent.

[24]As in every modelling process, single agent LDV models are useful abstractions from reality. For example, a study which determines the number of trips a patient makes to the doctor as a function of individual demographic characteristics makes the falacious but useful assumption that each trip can be treated equally. In reality each trip is a unique experience in which patients frequently use different doctors, each with their own skills and abilities, and the visits may last for different lengths of time. Since the aim of this paper is to provide a set of tools for the multi-agent environment which is analogous to the single agent LDV models, it is obviously subject to the same critique. Retail stores, for example, are certainly not all created equal. Despite this abstraction, single agent LDV models have proven to be an extremely powerful set of tools for researchers because they provide a logically consistent framework which is appropriate for the kind of data that is typically available to applied researchers. The multi-agent model is developed and used in that spirit.

6 APPENDIX A: Proofs

Lemma 1. *Assumption 1 implies $r_i(s_{-i})$ is non increasing in s_{-i}.*

Proof The proof of this lemma is a special case of Theorem 4 in Milgrom and Shannon (1994). Suppose that $s_i \in r_i(s_{-i})$ and $t_i \in r_i(t_{-i})$. Since t_i is a best response to t_{-i}, $\pi_i(x, t_{-i}) \leq \pi_i(t_i, t_{-i})$ for all $x \in S_i$. By the dual single crossing condition, $\pi_i(x, s_{-i}) \leq \pi_i(t_i, s_{-i})$ for all $x \geq t_i$. Hence $s_i \leq t_i$. **Q.E.D.**

Proposition 2. (Results for the N player game.) *Under assumptions A1–A4, the N player game has an equilibrium. Furthermore, within the set of Nash equilibria, market output is uniquely determined.*

Proof.(By induction) Without loss of generality, I consider only the cases where $N = 2^{m+1}$ for some integer m[25]. For $m = 0$, the 2 player game, Proposition 1 provides the result. Assume that for an arbitrary m there exists an equilibrium in the 2^m player game and that total output is unique within the equilibrium set. Now consider the 2^{m+1} player game.

Let $A = \{1, 2, ...2^m\}$, $B = \{2^m + 1, \cdots, N\}$ and denote total output by players in group A, $Q_A^s = \sum_{i \in A} s_i$ for some $s \in S$. Suppose, for the moment, that players in group A consider the choices of group B fixed. The induction hypothesis states that every equilibrium of the group A subgame involves the same total output (from group A.) Thus, for each Q_B, there is a unique Q_A consistent with all the players in group A

[25] If N is not a power of 2, then expand the game by inventing players who always play zero until the expanded game has the number of players which is a power of 2. Being able to add irrelevant players without changing the result is often considered an axiom in social choice problems, the axiom of *extendability*.

playing their best response, and vice versa. Denote these functions [26]

$$\begin{aligned} f_A(Q_B) &\equiv (s_1^* + \cdots + s_{\frac{N}{2}}^*)(s_{\frac{N}{2}+1} + \cdots + s_N) \\ f_B(Q_A) &\equiv (s_{\frac{N}{2}+1}^* + \cdots + s_N^*)(s_1 + \cdots + s_{\frac{N}{2}}). \end{aligned}$$

Evidently, a fixed point of the 2^{m+1} player game is also a fixed point of the function $f(Q_A, Q_B) = (f_A(Q_B), f_B(Q_A))$. By construction, each player is playing a best response against the other players actions, hence a fixed point of $f(\cdot)$ corresponds to at least one strategy profile $(s_1, ..., s_{2^{m+1}})'$ which is a Nash equilibrium of the 2^{m+1} player game.

Notice that $f_A(\cdot), f_B(\cdot)$ are each decreasing functions of the other group's total output. Therefore, $f(\cdot)$ is *increasing* on a sublattice of $\mathcal{R}^2$ with respect to the new order, $\geq_r$. Hence, by Tarski's theorem, there exists a fixed point. Applying Topkis(1979), the set of fixed points of $f(\cdot)$, $\mathcal{F}$, is a complete lattice with respect to the order $\geq_r$. Consequently, there exists a pair $(Q_A^L, Q_B^L) \in \mathcal{F}$ such that

$$(Q_A^L, Q_B^L) \leq_r (Q_A^s, Q_B^s), \quad i.e., \; Q_A^L \geq Q_A^s \text{ and } Q_B^L \leq Q_B^s, \text{for all } (Q_A^s, Q_B^s) \in \mathcal{F}.$$

Recall, we wish to show that $Q_A^s + Q_B^s = Q_A^L + Q_B^L$.

Case (a) Suppose instead, $Q_A^s + Q_B^s > Q_A^L + Q_B^L$. Then $Q_A^s \leq Q_A^L \;\Rightarrow\; Q_B^s > Q_B^L$. Hence, $\exists\, i \in B$ such that $s_i > s_i^L$. Consider the strategy profiles $(s_i - 1, s_{-i})$ and (s_i^L, s_{-i}^L). Notice that the conditions of Assumption 3 are satisfied: $s_i + s_{-i} - 1 \geq s_i^L + s_{-i}^L$ and

[26] Consider the four firm case and the set of points in $S_1 \mathrm{x} S_2$ consistent with players 1 and 2 playing best responses, $s_1 = r_1(s_2, s_3, s_4)$ and $s_2 = r_2(s_1, s_3, s_4)$. For any fixed s_3 and s_4 we may apply Proposition 1. Thus, there exists at least one fixed point $(s_1^*(s_3, s_4), s_2^*(s_3, s_4))$. Moreover, within the *set* of fixed points, $\{(s_1^*(s_3, s_4), s_2^*(s_3, s_4))\}$, the total output from players 1 and 2 is uniquely defined. Here $A = \{1, 2\}, B = \{3, 4\}$ and $f_A(\cdot) \equiv (s_1^* + s_2^*)(s_3, s_4)$ gives the uniquely defined output by players 1 and 2 consistent with both players playing a best response to the strategies of their rivals. $f_A(\cdot)$ is decreasing.

$s_i - 1 \geq s_i^L$. Hence

$$\pi_i(s_i^L + 1, s_{-i}^L) \leq \pi_i(s_i^L, s_{-i}^L) \;\Rightarrow\; \pi_i(s_i, s_{-i}) \leq \pi_i(s_i - 1, s_{-i})$$

However, since both s and s^L are Nash equilibria and best response functions are single valued, both inequalities cannot hold true. Hence, total output under s cannot be higher than total output under s^L.

Case (b) (Use the symmetric argument to case (a).) Suppose $Q_A^s + Q_B^s < Q_A^L + Q_B^L$. Then $Q_B^L \leq Q_B^s \;\Rightarrow\; Q_A^s < Q_A^L$. Hence $\exists\ i \in A$ for whom $s_i^L > s_i$. Following the same argument as before, consider the strategy profiles $(s_i^L - 1, s_{-i}^L)$ and (s_i, s_{-i}). Notice that the conditions of Assumption 3 are satisfied: $s_i^L + s_{-i}^L - 1 \geq s_i + s_{-i}$ and $s_i^L - 1 \geq s_i$. Hence

$$\pi_i(s_i + 1, s_{-i}) \leq \pi_i(s_i, s_{-i}) \;\Rightarrow\; \pi_i(s_i^L, s_{-i}^L) \leq \pi_i(s_i^L - 1, s_{-i}^L)$$

which once again provides a contradiction since s and s^L are both Nash equilibria and player i's best response function is single valued.

Thus, equilibrium exists in the 2^{m+1} player game and total market output is uniquely determined. **Q.E.D**.

7 APPENDIX B: Further Estimation Details

A two stage estimation procedure was found most effective. At the first stage, a consistent estimate of the parameters, θ were obtained using the moment conditions

$$G_K^R(\theta) = \frac{1}{K}\sum_{k=1}^{K}\begin{pmatrix} \hat{\nu}_k^1 Z_k^1 \\ \hat{\nu}_k^2 Z_k^2 \end{pmatrix}$$

where $\hat{\nu}_k^t$ denotes the simulated prediction error in market k from the data in period t. Since the data are observed for two periods, a more efficient estimator for θ is obtained by using data from both periods.[27]

At the second stage, I found that the best results were obtained using the simulated analogue of non-linear least squares (see Laffont, Ossard, and Vuong (1995), Gourieroux and Monfort (1990), and Gourieroux and Monfort (1993).) Specifically, I replaced the instruments Z_k^t constructed from the data by a simulated estimate of $\frac{\partial \nu_k^t(\theta)}{\partial \theta}$ evaluated at the consistent estimate obtained from the first stage. Since the estimate of θ is consistent, this function needs to be approximated only once before the second stage estimator is computed and, consequently, a very large number of simulation draws can be used. Naturally, to obtain a consistent second stage estimator, each of the simulation draws used to calculate the instruments must be independent of the R simulation draws used to calculate $\hat{\nu}_k^t(\theta)$.

$$G_K^R(\theta) = \frac{1}{K}\sum_{k=1}^{K}\begin{pmatrix} \hat{\nu}_k^1 \frac{\partial \widehat{\nu_k^1(\theta^1)}}{\partial \theta} \\ \hat{\nu}_k^2 \frac{\partial \widehat{\nu_k^2(\theta^1)}}{\partial \theta} \end{pmatrix}$$

Since data from both cross sections are utilized, the model is over identified at both

[27]Note that this estimator allows for arbitrary correlation over time between the within market prediction errors.

stages of the estimation procedure. At the first stage, the included market specific variables were treated as instruments. In addition, the average across active firms of the fraction of stores operated by rival LBO supermarkets in 1991 is included as an instrument in Z_k^2. Thus, in Table 1, 19 instruments are used to identify the twelve parameters, nine in Z_k^1 and ten in Z_k^2. [28] At the second stage, the number of over identifying restrictions is equal to twice the number of parameters, i.e., 2x12 = 24 moment conditions are used to construct Table 1.

[28] In fact, 18 instruments are used since median income and the percentage of households with incomes above $50,000 have a raw correlation of 0.98 in the second cross section.

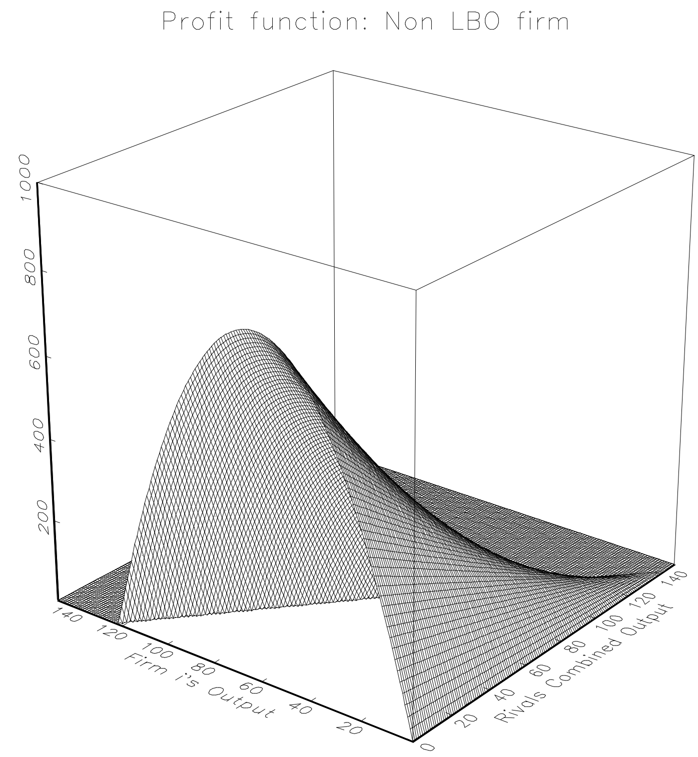

Figure 3: Estimated profit function for a Non-LBO firm facing no rivals who have undergone LBO's. The figure is computed using market variables at their median values in 1991.

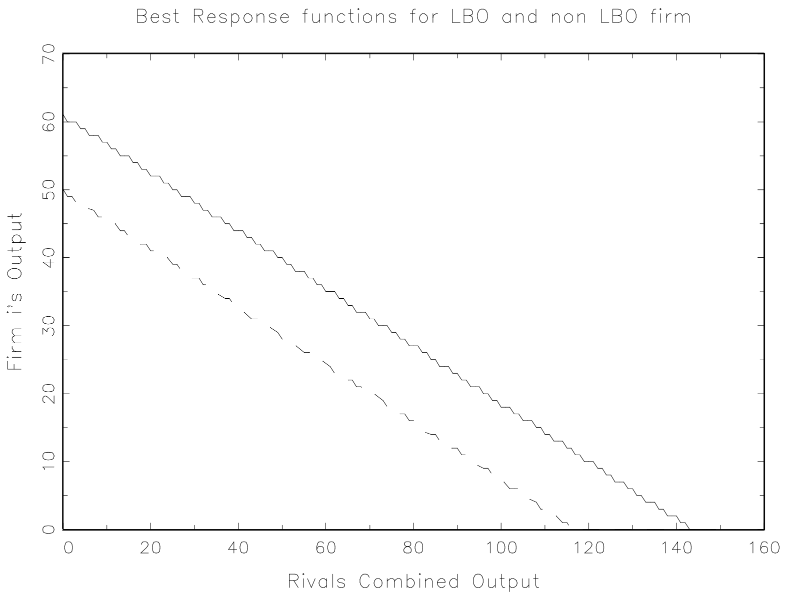

Figure 4: Estimated best response functions using the point estimates reported in column (1) of Table 2. These results are reported using all explanatory variables at their median values. Undergoing a leveraged buyout shifts the best response function inward for every level of output produced by rivals, making competition softer.

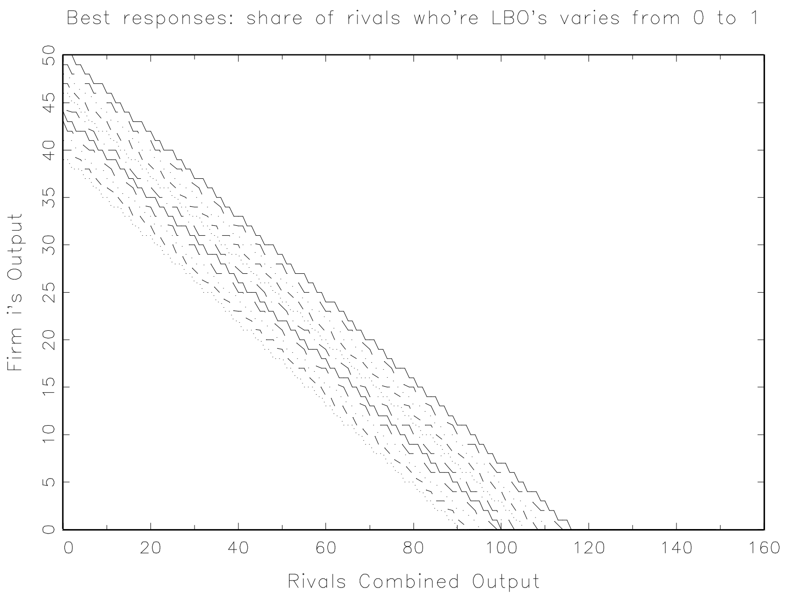

Figure 5: Estimated best response functions using the point estimates reported in column (2) of Table 2. These results are reported using all explanatory variables at their median values. In that specification, increasing the fraction of rivals who are LBO chains shifts the firm's best response function inward, making competition softer.

	Incumbents	Incumbents	Regional Chains	All Chains	OLS: 1991
	θ ($\lvert t_\theta \rvert$)	θ ($\lvert t_\theta \rvert$)	θ ($\lvert t_\theta \rvert$)	θ ($\lvert t_\theta \rvert$)	β^{OLS} ($\lvert t_{\beta_{OLS}} \rvert$)
median income (x1e4)	1.30 (0.45)	0.83 (0.45)	-0.76 (0.71)	0.04 (0.14)	3.39 (0.16)
percent income $<$ 10k	0.04 (0.38)	0.04 (0.44)	-0.03 (0.30)	-0.01 (0.11)	0.89 (0.64)
percent income $>$ 50k	-0.12 (0.52)	-0.13 (0.85)	0.05 (0.43)	-0.02 (0.37)	0.25 (0.14)
median age	0.04 (0.38)	0.12 (0.77)	-0.01 (0.09)	-0.07 (0.39)	-1.14 (1.12)
households (x1e5)	5.78 (2.78)	5.73 (4.74)	5.69 (2.40)	5.44 (2.74)	28.4 (19.6)
landarea (x1e5 sq. miles)	1.68 (2.05)	1.57 (3.05)	1.22 (2.67)	1.34 (1.56)	6.34 (3.08)
households^2	-0.04 (2.42)	-0.03 (2.53)	-0.05 (2.00)	-0.03 (2.74)	-0.24 (5.41)
land area^2	-0.05 (1.73)	-0.05 (2.71)	-0.04 (2.70)	-0.04 (1.63)	-0.20 (2.53)
β_1	2.94 (0.54)	1.66 (0.44)	2.87 (1.38)	3.97 (0.56)	
β_2	0.21 (2.59)	0.21 (5.06)	0.20 (2.50)	0.20 (2.58)	
β_3	-	4.03 (0.83)	8.79 (2.09)	3.46 (1.27)	
β_4	5.07 (1.64)				
β_5	0.03 (1.11)	0.07 (2.36)	0.02 (0.11)	0.03 (0.12)	
fraction *LBO*					-50.1 (3.17)
R^2	0.90	0.89	0.85	0.85	0.97

Table 1: Parameter estimates using data from 1985 and 1991 for the general specification. $\lvert t \rvert-$statistics are reported in parentheses. The columns titled "Incumbents" report results in which the firms which were actually present in 1985 and 1991 respectively are considered to be the set of potential entrants in those years. The column titled "Regional Chains" allowed all firms operating in the region that year to be considered as a potential entrant in the market. "All Chains" treats all 48 chains in the dataset as potential entrants. The dependent variable in the ordinary least squares regression reported in the final column is the number of stores operating in each market area in 1991.

	Incumbents	Incumbents	Regional Chains	All Chains
	θ ($\lvert t_\theta \rvert$)	θ ($\lvert t_\theta \rvert$)	θ ($\lvert t_\theta \rvert$)	θ ($\lvert t_\theta \rvert$)
households (x1e5)	5.32 (3.89)	3.91 (3.26)	5.51 (2.64)	5.61 (1.52)
landarea (x1e5 sq. miles)	1.27 (4.70)	1.39 (4.71)	1.32 (2.53)	1.18 (2.46)
households^2	-0.04 (3.23)	-0.03 (3.39)	-0.03 (2.01)	-0.04 (1.95)
land area^2	-0.04 (5.18)	-0.05 (5.64)	-0.04 (3.57)	-0.04 (2.34)
β_1	5.84 (3.59)	3.91 (3.25)	2.61 (1.54)	2.26 (0.41)
β_2	0.19 (4.02)	0.14 (6.34)	0.21 (2.68)	0.21 (1.44)
β_3		5.80 (2.47)	5.37 (1.89)	5.56 (0.93)
β_4	4.91 (3.13)			
β_5	0.03 (0.95)	0.04 (1.90)	0.03 (0.25)	0.04 (0.16)
R^2	0.90	0.89	0.86	0.85

Table 2: Parameter estimates using data from 1985 and 1991 for the parsimonious specification. $|t|-$statistics are reported in parentheses. The columns titled "Incumbents" report results in which the firms which were actually present in 1985 and 1991 respectively are considered to be the set of potential entrants in those years. The column titled "Regional Chains" allowed all firms operating in the region that year to be considered as a potential entrant in the market. "All Chains" treats all 48 chains in the dataset as potential entrants.

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