

1. Let  $G$  be a compact Lie group, and  $T$  a maximal torus. The inclusion of  $T$  into  $G$  induces a map on the cohomology of their classifying spaces  $f: H^*(BG) \rightarrow H^*(BT)$ . Since the action of the Weyl group  $W$  is homotopically trivial on  $BG$ , the image of  $f$  lies in the  $W$ -invariant subgroup of  $H^*(BT)$ . The question is, when does  $f$  give an isomorphism between  $H^*(BG)$  and  $H^*(BK)^W$ ? (ie, conditions on  $G$  and the coefficient group.)
2. Let  $T$  be a unitary operator on a complex Hilbert space  $H$  with no eigenvectors. Fix  $v \in H$ . Define a function  $f_n: S^1 \rightarrow H$  by

$$f_n(\theta) = \frac{1}{n} \sum_{k=1}^n e^{ik\theta} T^k(v).$$

Show that  $f_n$  converge uniformly to the zero function.

3. Let  $\{T_\alpha\}_{\alpha \in A}$  be a family of commuting diagonalisable endomorphisms of a vector space  $V$ . Can they be simultaneously diagonalised?
4. Let  $X$  be a Hausdorff topological space and suppose  $a$  and  $b$  are distinct points of  $X$ . Do there exist open neighbourhoods  $A$  and  $B$  of  $a$  and  $b$  respectively such that  $\overline{A} \cap \overline{B} = \emptyset$ .
5. For which topological spaces  $X$  is the Banach space of continuous functions on  $X$  separable?