# Massachusetts Institute of Technology Electromechanical Systems Group

# **Electronics First: RESONANCE**



## Materials:

Part	Quantity	Part Number	Vendor	Part	Quantity	Part Number	Vendor
Breadboard	1	377 - 2646 - ND	DigiKey	14-Pin DIP Socket	1	AE9989 – ND	DigiKey
Breadboard Power Supply Module	1	-	-	$R_0 \ 1 \ k\Omega$ Resistor	1	CF14JT1K00CT – ND	Digikey
Multimeter	1	MN35 – ND	DigiKey	$R_1 \ 1 \ k\Omega$ Potentiometer	1	3386P - 1 - 202TLF - ND	DigiKey
6-pin Header	1	609 - 3263 - ND	DigiKey	$R_2$ 10 k $\Omega$ Potentiometer	1	3386P - 103TLF - ND	DigiKey
2-pin Rail Connector Pins	4	952 - 2262 - ND	DigiKey	C <sub>0</sub> , C <sub>3</sub> , C <sub>7</sub> - 0.001 µF Capacitor	3	445 - 173586 - 1 - ND	DigiKey
Micro USB Power Cable	1	102 - 4123 - ND	DigiKey	$C_1, C_2 - 1 \ \mu F$ Capacitor	2	399 - 13923 - 1 - ND	DigiKey
Soldering Iron and Solder Wire	1 ea	T0052918199N - ND	DigiKey	SC <sub>4</sub> , $C_5$ - 0.015 $\mu$ F Capacitor	1	399 - 4217 - ND	DigiKey
74HC14 Hex Inverter	1	296 - 1577 - 5 - ND	DigiKey	C <sub>6</sub> - 0.0047 µF Capacitor	1	399 - 4243 - ND	DigiKey
74163 Counter	1	296 - 3661 - 5 - ND	DigiKey	C <sub>8</sub> - 220 pF Capacitor	1	BC2672CT - ND	DigiKey
16-Pin DIP Socket	1	AE9994 – ND	DigiKey	$L_0, L_1, L_2, L_3, L_4 - 1$ mH Inductor	5	M8158 - ND	Digikey

## The Build:

Many applications require a set of waveforms, each with a precisely controlled timing or frequency relationship with respect to the other waveforms. For example, computers might need several internal clocks with a precisely controlled relationship between the clock edges to synchronize the operation of different gates and logic functions. To create a properly tuned "electronic piano," we might need a collection of selectable sine waves in a precise frequency relationship. Medical systems like pacemakers might need several clocks synchronized with a fundamental clock or beat. In this lab, we will create four "phase locked" sine waves with frequencies related by powers of two and with a common time for the zero crossing of each sine wave. We will use these four sine waves to create aesthetically pleasing patterns, called "Lissajous figures," on an oscilloscope screen. To create these sine waves, we will exploit a critical phenomenon in engineering science called **resonance**. THE POET MAYA ANGELOU WROTE: "Everything in the universe has a rhythm, everything dances." This lovely observation finds expression in engineering science through a natural phenomenon called resonance. Resonance is a repetitive exchange of energy between kinetic and potential forms in a system. Two common examples of systems that exhibit a resonance include a clock pendulum and a playground swing. The oscillation of the pendulum or the swing involves a repetitive trading of energy between kinetic and potential forms. On a playground, the swing is replete with kinetic energy at the lowest point in its arc, when the rider and seat are moving fastest. This energy is completely traded to potential form, height in a gravity field, when the swing reaches the top of its arc. At this point, the swing pauses before swinging the other way, converting the energy back to kinetic form at the low point of the swing. As you know from the playground, a swing with a rider has a particular resonant frequency, that is, the frequency of the oscillations that trade energy back and forth between kinetic and potential forms. This resonant frequency is fixed by the physical paramters of the swing, i.e., the length of the swing and the acceleration due to gravity. One way to "measure" or compare the acceleration due to gravity on different planets would be to set up the same length swing on each planet and observe the different frequencies of oscillation.



The finished build.

#### Intuition...

A PLAYGROUND SWING IS A RESONANT SYSTEM driven by input pushes. Resonant systems respond fully only to inputs that are provided at or near the correct resonant frequency. Resonant systems can therefore be used as a "filter," that is, a device that can selectively respond to or "pass" input excitations in a specific frequency range. Pushing the swing "off resonance," i.e., failing to push the swing near the resonant frequency, results in a diminished ride and little or no swinging.

A resonant filter is more selective or choosy in the frequencies to which it responds when the filter has low losses. A well-oiled swing with smooth hinges and a flexible chain (low friction) will provide a "big" ride or swing when pushed at the resonant frequency, and smaller responses when pushed at other frequencies. We say that this low-loss filter has a high selectivity or "quality factor." On the other hand, a swing with a rusty chain and frozen hinges is largely unresponsive at essentially any frequency of pushing. This filter is lossy and not very selective. It has a low quality factor and exhibits no dramatically large response to any input frequency.

Resonant systems or filters can function as amplifiers. For example, we amplify sound when we blow over the top of a bottle to make a tone or musical note, e.g., as in a "jug band." The acoustic resonance of the jug is created by the interaction of the mass of air in the jug with the compressibility of the air in the jug, i.e., the air's "spring-i-ness." A mass and a spring can form a



Playground swing.



"Anchor" escapement mechanism as found in a grandfather clock. The escape gear is turned by a gradually falling weight (not shown). The escapement mechanism includes a pendulum and the two "pallets" that alternately receive a push from the escape gear or block the escape gear from moving. The timing of the pendulum swing determines when a push is received and for how long the escape gear is held. As the escape gear turns, the hands of the clock are moved by connected gears (not shown).



An unwanted resonance driven by wind caused the Tacoma Narrows Suspension Bridge in Washington state to collapse in 1940.

resonant system. The filtering provided by this jug of air selects and responds to a narrow part of the broad range of frequencies to be found in our blowing breath. The relative quiet of our breath sounds produces a considerably louder tone from the jug. As with a playground swing, the resonant response can be wonderfully large compared to what we might have expected from the input alone. Relatively few people could lift another person two feet in the air with one hand. However, most people can use a playground swing, with one hand pushing, to lift a person several feet in the air at the end of the swing's arc.

This selective amplification or gain provided by a resonator, e.g., a swing set or a jug, is not "free energy." To make the swing lift someone in the air, we work hard on the first several pushes to store energy in the swing, adding a little more with each push until the swing is "rocking." The resonator stores the energy, trading back and forth between kinetic and potential forms. Each additional push adds some energy to the resonator. If there are no losses, e.g., no air friction, no rusty chains or hinges, then the swing would get higher and higher as the stored energy increases with each push. In practical resonators, there is always some friction or other loss mechanism. We achieve a steady operation or steady swinging by first storing energy in the resonator and by then providing continued input or "pushes" to just overcome any losses. Our rider enjoys a steady swinging as long as we keep pushing to overcome the losses. If we stop pushing, the losses reduce the stored energy on each period of the resonance, e.g., the swing arc decreases until the swing finally stops. Similar losses affect a "Grandfather" clock. To compensate for losses, the stored energy of a gradually falling weight is transmitted through a gear arrangement called an "escapement" to keep pushing the clock pendulum with a carefully timed periodicity or resonance. The oscillations can be counted in order to track the progress of time. When the weight is fully lowered, the clock is out of stored potential energy necessary to overcome the losses on each swing, and we must lift the weight and chain again to keep the clock running.

For builders, resonance can be thrilling or vexing. Resonance can provide the highly selective filtering or "channel selection" that makes clocks and musical instruments and radios and televisions work. Unwanted resonances can plague systems like bridges and cars and computer and power cables, all of which can oscillate sickeningly or even destructively if driven at a resonant frequency. A mastery of the ability to predict and control resonant phenomenon is a hallmark of a great designer.

#### ...And A Little Math

WE CAN FIND THE NATURAL RESONANT FREQUENCY for the simplest resonant systems with some idealizations and a little math. A common way for engineers to describe a system is in terms of "push" and "flow" variables. The product of the push and the flow variable in an engineering discipline is called *power*.

We can create an idealized resonant system in many engineering disciplines by connecting two components and ignoring losses like friction or resistance. To create a resonant system, one component should respond to the derivative of a push or flow variable, and the other component should respond to the integral of the same push or flow variable. One example is a mass connected to a spring. The push and flow variables in this mechanical system are force and velocity. The product of force and velocity is power. A spring with one fixed end behaves according to the position of the movable spring end. This position is the integral of velocity. The mass responds to acceleration, which is the derivative of velocity. The mass obeys Newton's Second Law of Motion:  $F = ma = m \frac{dv}{dt}$ . The spring obeys Hooke's law, which says that the spring force is a function of the position x, that is:  $F = kx = k \int v dt$ . If the mass m shown in the figure is moving right to increase x, then the spring will stretch and pull the mass back to the left. The equation describing this interaction balances the force associated with the acceleration of the mass with the force of the spring:

or

$$m\frac{dv}{dt} + k\int vdt = 0$$

ma = -kx

We can clean up this equation by taking the derivative of both sides with respect to time and dividing by m to find a resonance equation:

$$\frac{d^2v}{dt^2} + \left(\frac{k}{m}\right)v = 0$$

A solution to this resonance equation for the velocity of the mass is  $v = V_i cos(\omega t)$ , where  $V_i$  is the initial velocity of the mass at time t = 0 and where  $\omega = \sqrt{\frac{k}{m}}$ . Try it! Plug the solution back into the differential resonance equation and check that it solves the equation. No other choice for  $\omega$  will work. This solution describes a back-and-forth oscillation or resonance, with a radian frequency of  $\sqrt{\frac{k}{m}}$  radians-per-second, or a resonant frequency of  $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$  hertz.

We can perform a similar analysis in other engineering disciplines. For example, an LC resonant circuit can be made by connecting an inductor to a capacitor. In this case, the "push" and "flow" variables are voltage and current, respectively. The product of voltage and current is power. The voltage across an inductor is determined by the time derivative of current though the inductor:  $v_l = L \frac{di_l}{dt}$ . The voltage across a capacitor is determined by the time integral of current through the capacitor:  $v_c = \int \frac{i_c}{C} dt$ . In the parallel LC circuit shown in the figure, the parallel connection forces the voltage across the inductor to equal the voltage across the capacitor by Kirchoff's voltage law. And we can also see that current entering the capacitor must be the negative of current entering the inductor, that is,  $i_c = -i_l$ , by Kirchoff's current law. The equation that describes this resonant circuit is therefore:

$$v_l = L\frac{di_l}{dt} = \int \frac{i_c}{C} dt = v_c$$

Newton's Second Law and Hooke's law are examples of *constitutive laws*. A constitutive law is a mathematical relationship between a push and a flow variable that describes how a component behaves.



A frictionless rolling mass *m* connected to a spring with spring constant *k*. The arrow indicates the direction for increasing position *x*.



Parallel Resonant LC circuit.

Since  $i_c = -i_l$ , we can eliminate  $i_c$  from the equation and write:

$$L\frac{di_l}{dt} + \int \frac{i_l}{C}dt = 0$$

We can again clean up the equation by taking the time derivative of both sides and dividing through by *L* to uncover the resonance equation:

$$\frac{d^2 i_l}{dt^2} + \left(\frac{1}{LC}\right) i_l = 0$$

This equation is structurally identical to the resonance equation for the massspring system. The solution will again be an oscillation, this time with a resonant frequency of  $\omega = \sqrt{\frac{1}{LC}}$  radians-per-second, or  $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$  Hertz.

Most engineering disciplines have components that can be interconnected to create a simple resonant system. If two components interconnect to yield a resonance equation in the same form that we found for the mass-spring and the inductor-capacitor systems, i.e., of the form:

$$\frac{d^2x}{dt^2} + \omega_o^2 x = 0$$

then we can "read" the resonant frequency  $\omega_o$  right from the equation. More complicated systems may be described by more complicated equations with multiple resonances at different frequencies.

Our build will use a series resonant connection of an inductor and capacitor, which turns out to have the same resonant frequency as the parallel connection of an inductor and capacitor. Please:

A1.) Write down a resonance equation for the series resonant LC circuit.

*A*2.) Explain why the resonant frequency is again  $\omega = \sqrt{\frac{1}{LC}}$  for the series resonant LC circuit.

## **Theory of Operation and Predictions**

THIS BUILD CONSISTS OF A CLOCK, A COUNTER, AND FOUR SERIES LC FILTER CIRCUITS. We will use the carefully tuned LC resonant filter circuits to create four sine waves, each with a frequency double that of the previous. The sine waves can be used to create "Lissajous figures" on an oscillscope screen.

The oscillating square wave produced by the clock will drive a 4-bit binary counter to produce four square waves at frequencies related by powers of two. Each square wave drives or "pushes" on an LC resonant circuit, or, more formally stated, each square wave is filtered by an LC resonant circuit, to produce a sine wave output. A separate "delay" circuit can be adjusted with a variable resistor to create phase shifts between component frequencies.

A notable exception occurs for thermal systems. Thermal systems exhibit thermal "capacitance" or heat storage. However, there is no known analog in thermal systems for an "inductor." This may fit with your everyday intuition about thermal systems. Heat diffuses away, i.e., out of a coffee cup. It never oscillates back and forth in natural thermal systems.



Series Resonant LC circuit.

#### The Clock

We will use a 74*H*C14 hex inverter to construct an RC relaxation oscillator which will produce an oscillating square wave. The pulses of this "base clock" will trigger the counter to count. The frequency of the relaxation oscillator is determined by a resistor-capacitor (RC) network with a variable resistor that permits you to control the output frequency.

#### The Counter

This build uses a 74*L*S163 counter, an integrated circuit (IC) with a single clock input and a 4-bit binary output. The "count" on this output increases with every rising edge of the clock square wave. This means that the frequency of the clock determines the speed of the count from the counter, which starts from 0*d* (0000*b*) and counts in binary to 15*d* (1111*b*). At 15*d*, the count resets back to the start 0*d*, and the cycle is repeated. The binary counting process causes the counter to create four waveforms at frequency multiples of 1, 2, 4, and 8. Each counter output bit (QA, QB, QC, and QD) is one of four separate output signals individually wired to LC circuits.

#### LC Resonant Filters

Resonant series-connected LC filters produce the final output sine waves. The square waves from the 74LS163 counter outputs are analogous to the "pushing hand" on each of these four LC filter "playground swings." The square waves pass through a 74HC14 buffer to provide adequate current to drive each filter. Each LC filter is carefully tuned to respond to the "pushing" to produce a sine wave at the same frequency as the input square wave.

This technique for producing a sine wave exploits the filtering capability of a high quality factor LC circuit. We can see the filtering behavior if we think mathematically about the square wave input to each LC filter. A square wave of period T may be represented as a sum of sinusoidal waves in a Fourier series:

$$\frac{4}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \sin(\frac{2\pi nt}{T})$$

Each inductor-capacitor series circuit is resonant at only one frequency. Therefore, to a good approximation, at most only one of the sine wave components of the input square wave will "pass" through the filter. Each filter have been selected to be resonant at the "fundamental" (n = 1) or lowest frequency of the series that describes the input square wave; this fundmantal component has the largest amplitude of all of the square wave components. Note that this means that we will have four different sets of L and C values to tune each of the four filters to extract a sine wave from each of the four counter square waves.

Specifically, the first four LC filters in the build have equivalent inductors (1 mH), but their capacitors vary in size. These specific capacitances were selected



010101010101010101 QA (x8) 0011001100110011 QB (x4) 000001111000111 QB (x4) 00000011111000111 QC (x2) 0000000001111111111 QC (x1) Counter Output Waveforms



LC Filter and Delay Circuit



Lissajous figure with A = B, identical horizontal and vertical frequencies, and no phase offset.



Lissajous figure with A = B, identical horizontal and vertical frequencies, and a non-zero phase offset.



Lissajous figure with A=B and differing horizontal and vertical frequencies.

to achieve resonant frequencies that are specific multiples of each other. For instance, the fourth LC filter (with an input from QD) has a resonant frequency that passes our slowest frequency, while the next filter (driven from input QC) has a resonant frequency set to pass an input frequency twice the baseline, and the next two filters (driven by QB and QA) produce signals with 4x and 8x the baseline frequency, respectively.

A fifth LC filter also operates at the baseline frequency, but includes a *delay* circuit that can adjust the phase of the output with respect to the other four outputs. Specifically, the QD square wave is also connected to a variable resistor and a capacitor that form an RC network. Adjusting this variable resistor allows you to add a "delay" or *phase shift* to the output sine wave. We will see that this option for adding a phase shift creates additional interesting opportunities for creating Lissajous figures.

#### **Lissajous Curves**

Once built, you can use an oscilloscope to see the four different output sine waves produced by our board, and also the fifth sine wave at baseline frequency with variable phase. You can also put the oscilloscope into its "XY" mode and plot one sine wave on the horizontal x-axis (oscilloscope channel 1) versus another sine wave on the vertical y axis (oscilloscope channel 2). An XY plot of two of our sine waves will have a mathematical description:

$$x(t) = A\sin(2\pi a t)$$
$$y(t) = B\sin(2\pi(bt + \phi))$$

Examples of Lissajous figures made with different parameters in the equations above can be found to the left.

Based on what you have learned about the components in this build, let's make some predictions:

1.) Assume the board is constructed as the schematic directs. You connect the *x*-axis oscilloscope lead to test point *TP*5 and the *y*-axis oscilloscope lead to test point *TP*4. Sketch what you predict the figure to look like.

2.) Now assume the *x*-axis oscilloscope is connected to test point *TP*6 and the *y*-axis lead to test point *TP*4. Sketch what you predict the figure to look like.



Prediction 1

Prediction 2

## Assembly

THIS BUILD CONTAINS TWO INTEGRATED CIRCUITS AND A BANK OF LC RESO-NANT FILTERS. The board has been divided into separate areas, referenced below. A diagram of the board areas can be found on the cover page.

#### **DIP Socket Installation**

Install sockets but do NOT insert the integrated circuits yet. Install the 14-pin DIP socket for the 74*H*C14 hex inverter in **AREA C**. Orientation matters - ensure the semi-circular notch at the top of the socket lines up with the corresponding notch illustrated on **U0** of the board. The pads for **U0** are arranged in two rows of 7. Press the 14 socket pins through the corresponding pads on the board. Make good solder connection between each of the 14 socket pins and the metal board contacts located on the underside of the circuit board. Repeat this process for the 74*L*S163 counter 16-pin DIP socket in **AREA B**. Align, insert and solder this socket into **U1**.

#### **Pin Installation**

Install one 6-pin header into **J4** at the top of your board. Insert the pins into the contacts and make soldered joints on the underside of the board. Then, install 2-pin rail connectors into **J0,J1,J2**, and **J3**. These are installed in the reverse direction, with the base of the pin on the underside of the board, and the location of the soldered joint on the top of the board. *BEFORE SOLDERING*, insert the bottom of the pins into your breadboard, and *then* make your soldered connections on the circuit board to ensure proper alignment.

#### **Resistor Installation**

Add the delay circuit potentiometer in **AREA A**. Install the blue, rotating-knob potentiometer into  $R_2$ . Ensure its three triangularly-positioned legs are aligned with the three corresponding points of contact. Make soldered connections at the proper locations on the opposite side of the board. Repeat this process for the clock variable resistor,  $R_1$  in **AREA C**. Add the clock fixed resistor in **AREA C**. This provides a minimum resistance for the clock RC circuit. Install and solder the 1 k $\Omega$  resistor into  $R_0$ . Remember to save your trimmed pieces of wire - you will use them to construct test points later.

#### **Capacitor Installation**

Add the 74*H*C14 RC network timing capacitor in **AREA C**. Together with the variable resistor, this capacitor sets the frequency of the clock. Properly install and solder one 0.001  $\mu$ F capacitor into  $C_0$ . Pull the leads through until the ca-



74HC14 Hex Inverter Socket Location



74LS163 Counter Socket Location



Header Pin Installation



Rail Connector Pin Installation



Delay Circuit Variable Resistor  $R_2$ Clock Variable Resistor  $R_1$ 



Clock Fixed Resistor R<sub>0</sub> Location

pacitor is raised just off the surface of the board. Solder in the proper locations beneath the board, and when your joints have cooled, trim any excess wire.

Repeat this process for the 1  $\mu$ F 74*H*C14 bypass capacitor at *C*<sub>1</sub>. This capacitor effectively "smooths out" the power supply, ensuring stable operation of the 74*H*C14 IC. Also add the 74*L*S163 bypass capacitor in **AREA B**. Properly install and solder one 1  $\mu$ F capacitor at *C*<sub>2</sub>. Add the capacitor for the delay circuit RC network in **AREA A**. Install and solder one 0.001  $\mu$ F capacitor into *C*<sub>3</sub>.

Add the LC resonant circuit capacitors in **AREA D**. Together with the inductors, these capacitors set the resonant frequency of each LC network. Install and solder one 0.015  $\mu$ F capacitor into  $C_4$  and another into  $C_5$ . Add a 0.0047  $\mu$ F capacitor into  $C_6$ , and put a 0.001  $\mu$ F capacitor into  $C_7$ . Finally, install the 220 pF capacitor into  $C_8$ . Ensure that you made good soldered joints at the pads on the underside of the board, and trim your leads.



74HC14 RC Network Timing Capacitor  $C_0$ 74HC14 Bypass Capacitor  $C_1$ 



74LS163 Bypass Capacitor  $C_2$ Delay Circuit RC Network Capacitor  $C_3$ 



LC Resonant Circuit Capacitors C4 - C8



LC Resonant Circuit Inductors L0 - L4

#### **Inductor Installation**

Add the five 1 mH inductors to **AREA D**. Together with the capacitors, these inductors form resonant LC networks that transform the output square waves into sine waves. Properly install and solder inductors into  $L_0$ ,  $L_1$ ,  $L_2$ ,  $L_3$ , and  $L_4$  *Again, save your wire cuttings to make ground and test clips later.* 

#### **Test Points**

Construct the ground clip in **AREA D**. Take one of the capacitor wire cuttings you saved earlier and bend it into an arc, inserting each end into the contact points at **TP0**. Trim the length of the wire as necessary. Make soldered joints beneath the board. This is a ground clip. Repeat this process at **TP1** in **AREA C**. This is your clock test point. Do this again at **TP2** in **AREA B** to construct your counter test point.

Move to **AREA D** and construct similar test points at **TP3**, **TP4**, **TP5**, **TP6**, and **TP7**. These testpoints allow you to connect to the output of the LC filters.



Ground Clip Soldering Location

### **Integrated Circuit Installation**

Once all of your soldered joints are completely cooled, carefully install the 74HC14 hex inverter and the 74LS163 counter into their respective DIP sockets. Be sure to put the right IC into the right socket! Orientation matters! Be careful to make sure tha every pin goes in the correct socket hole. Ensure that the notch on the topside of each IC body aligns with the notch on the corresponding DIP socket.

Clock and Counter Test Point Location

LC Filter Test Point Locations

# Testing

## TESTING INVOLVES THE USE OF AN OSCILLOSCOPE. PROCEED ONLY IF YOU HAVE ACCESS TO THIS PIECE OF EQUIPMENT. FOR THE TESTS BE-LOW, USE THE OSCILLOSCOPE IN XY MODE. ASK YOUR LAB INSTRUC-TOR FOR HELP WITH CONFIGURING AND USING THE OSCILLOSCOPE AS NEEDED.

Carefully install your circuit board on a breadboard in-line with the power supply module. Ensure that an appropriate power cord is properly fitted on the power supply module. Also, ensure that the pins of both boards are fully inserted into the breadboard contact holes. If all is correct, power up your power supply and check that the indicator light is illuminated.

## **Oscilloscope Testing**

1.) Connect the x-axis oscilloscope lead to test point TP5 and the y-axis oscilloscope lead to test point TP4. Sketch what you see on the screen. Does this concur with your prediction?

2.) Now connect the x-axis oscilloscope lead to test point TP6 and the y-axis oscilloscope lead to test point TP4. Sketch what you see on the screen. Does this concur with your prediction?

3.) With the x-axis lead still in place, connect your y-axis oscilloscope lead to TP3. While observing the screen, turn the knob of the delay circuit potentiometer all the way in the counter-clockwise direction, and then back all the way in the clockwise direction. How did the figure change? Describe what variable you are altering, and how it affects the governing equations.







Test 1 - Check against Prediction 1



Test 2 - Check against Prediction 2

