District Compactness and Electoral Volatility in Canada and the United States

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Introduction

Compactness is the degree to which the spatial area of an object is related to its center. Compactness is frequently considered by courts, perhaps because of its simplicity: Any layperson can appreciate that a square or circle is more compact than an elongated or sinuous shape. Schwartzberg (1966, p. 444) notes, "Present and proposed legislative definitions of compactness are generally nothing more than definitions of fairness." He notes that a legislative committee early in the reapportionment revolution defined compactness as the absence of any attempt:

1. To divide (a territorial unit) into election districts in an unnatural and unfair way with the purpose of giving one political party an electoral majority in a large number of districts while concentrating the voting strength of the opposition in as few districts as possible.
2. To divide (an area) into political units in an unnatural and unfair way with the purpose of giving special advantages to one group.[1]

This corresponds closely to what most people mean by *gerrymandering*, but it bears no relationship to the mathematical measures that scientists have devised in the past 200 years. Backstrom, Robins & Eller (1978, p. 1122) state that gerrymandering has come to be known as "the excessive manipulation of the shape of legislative districts." The earliest known measure of compactness was proposed in 1822 by Ritter for evaluating the shape of grains of sand. Nagel posited a measure of compactness using only the perimeter and area of the objects as inputs. This has been adopted by scholars in such diverse fields as geography, mineral engineering, and psychology. Schwartzberg introduced Nagel's index of compactness to political science by proposing that it be used for purposes of evaluating electoral districts. He stated that the value of a compactness measure is to "restrict the latitude for manipulation of district boundaries toward [gerrymandering] and reduce the number and magnitude of abuses." (p. 448)

Scholars have often debated the merits of various measures of compactness. Other methods of determining compactness that have been developed include the ratio of the area to the area of the smallest circumscribing circle (Ehrenberg 1892); the ratio of the area to the largest

[1] H.R. Report No. 140, 89th Congress, 1st Session 2 (1965), cited in Schwartzberg 1966, 444.

inscribing circle; the diameter of the largest inscribing circle divided by the diameter of the smallest circumscribing circle (Haggett 1966); the diameter of a circle of equal area divided by the diameter of the circumscribing circle (Schumm 1956); the area of interection of the object and circle of equal area divided by the area of union of the object and a circle of equal area (Lee & Sallee 1970); the ratio of the longest axis to the shortest axis; the variance in the length of radials extending outward from the object's center (Boyce & Clark 1964); the dispersion of unit of area around the center (Blair & Biss 1967). None commands a consensus of users.

The area-perimeter method has as its primary advantage its simplicity. It uses only the area and perimeter of the districts. These measurements are available as soon as a districting scheme is proposed and geographic data (e.g., map boundary files) are made available in digital form, or they can be easily calculated by digitizing a paper map or by aggregating existing units (towns, census tracts, precincts, etc.) into the desired district form.

As political scientists, we are often called upon to pass judgment on redistricting proposals or even to create those proposals ourselves.   Compactness, in general terms, is readily distinguishable to the human eye. The scientist's role is to add quantification in order to establish that what is believed to be compact or non-compact is really so. While various scientists can argue for one or other measure of compactness to be most reliable, none can dispute the speed at which compactness ratios can be produced from a large number of districts by the area-perimeter method. Thus, the area-perimeter method by its ability to contribute to a speedy analysis of a pending districting scheme more than makes up for any deficiencies alleged against it.

According to MacEachren, "compactness can probably be considered the single most important aspect of geographic shapes." (1985 p. 65). Pounds (1972, pp. 54-55) declared compactness second only to size in significance when it comes to evaluating countries. He notes that compactness affects ease of travel, communication, and the homogeneity of the population. Over the decades since redistricting plans came to be evaluated by courts, political scientists and geographers have proposed multiple measures of compactness. Some of the most readily accessible take into account only the area and perimeter of the district. These measures relate the object to a circle, maintaining that a circle is the most compact figure. Therefore, an object that has the same area as a circle whose circumference is the same as the object's perimeter (i.e., it is a circle) would receive a perfect score on such a measure, whereas a polygon that has more than a circle of equal perimeter would receive a lower score. (Some of these measures are inverted, so that a lower score is higher, or vice versa.) One such measure is the Nagel ratio, which is two times pi times the district's perimeter divided by the square root of the district's area divided by pi. Simply stated, it is the ratio of the circle formed from the district's perimeter to the circle formed from the district's area.

The area-perimeter measures can be divided into two groups: Those that are consistent with Cox and those that are consistent with the square root of Cox. All of the Cox-consistent area-perimeter measures work the same in terms of how they rank the same set of districts, and all of the square root based measures do the same among themselves. Many of the variants employ an inverted form whereby a low ratio means greater compactness.

The chief failing of the existing measures of the area-perimeter ratio is that they are based on circles. There is no electoral district in the world based on a circle. Therefore, all districts will fail to meet the ideal represented by the circle. Meanwhile, actual square districts exist but are underrecognized for their innate compactness due to the use of pi to create existing compactness measures. The arbitrariness of the circle ideal is reflected in the scales used for the area-perimeter measure. For example, the nearly perfect square of the 5th congressional district of Texas from 1932 into the 1960s gets a Cox ratio of 0.783; an Attneave and Arnoult ratio of 0.466; and a Nagel ratio of 1.13. None of these measures immediately communicates to the reader that this is a square district.

The author proposes a new measure, the Hill ratio, one that relates the district to a square as the ideal rather than a circle. The Hill ratio is the district's perimeter divided by four, divided by the square root of the district's area. Simply stated, it is the ratio of the square formed from the district's perimeter to the square formed from the district's area. Since squares and circles based on the same measures are definitively proportional, the Hill ratio can be expressed as the Nagel ratio times

0.886227. The 5th district of Texas mentioned above has a Hill ratio of 1.001. A scale that incorporates 1 as a perfect square and larger numbers being proportionately less compact is much more useful to a consumer than a scale that is purely relative, especially if the scale is inverted so that a higher number indicates greater compactness.

The most compact district in the U.S. as of 2009 is the state of Wyoming. Compared to a circle, its Nagel ratio is 1.137. Compared to a square, its Hill ratio is 1.008. It is much more useful to know how square Wyoming is as opposed to how round it is. The most uncompact district in U.S. congressional history, the 29th district of Texas created for the 1992 election, has a Hill ratio of 10.498. Morrill says that districts with Nagel ratios of 2 or greater ought to be considered suspect, and justification for them should be demanded. A Nagel ratio of 2 corresponds to a Hill ratio of 1.77. As of the 2006 election, 192 Congressional districts have Hill ratios of 1.77 or less, and 244 exceed this threshold. In other words, more districts fail Morrill's suspect test than pass it. Schwartzberg's standard of Hill 1.48 (Nagel 1.67) would cause even fewer districts to pass scrutiny.

It was not always so. As of the 1960 election, the average Hill ratio of all districts for which data are available was 1.526. As of 1982, it was up to 1.676; then to 2.146 in 1992, and since then the average has declined slightly to 2.103. However, the decline from the 1990s to the 2000s was largely due to a reduction of the excesses of 1990s racial redistricting in four states: North Carolina, Georgia, Louisiana, and Texas. If these states and Maryland are excluded, the change from 1992 to 2006 is negligible.

There is considerable variation among states in the compactness ratio. The eight districts of Minnesota have an average compactness ratio of 1.594. The eight districts of Maryland have an alarming average of 4.659. (Even excluding three complicated districts in eastern Maryland for which redistricters should possibly be given some latitude, the average is still above 3.) Neither is this a function of the composite units of the states with low compactness averages being relatively square. The averages of square-based Indiana (1.774) and Oklahoma (1.779) are practically indistinguishable from that of metes-and-bounds-based Connecticut (1.781) and haphazard Kentucky (1.759). The aggressiveness of redistricters in various states is aptly reflected in the compactness ratios. Oddly enough, many states that were outliers as early as 1922 (North Carolina, Maryland, California) continue to pursue extremes in redistricting today.

Canada has not yet seen the proliferation of partisan and racial redistricting that the United States has experienced. The national average Hill ratio for all ridings except excluded[2] was 1.399 in 1997 and 1.400 in 2004. This is lower than the national average for the United States has been in the entire period studied herein, 1922 to 2006. At its low point before the reapportionment revolution began, the U.S. national average was 1.526.

In the current 2004 remap of Canada, New Brunswick has the highest ratio, 1.547. Quebec is second at 1.501. Thus, the two Canadian provinces with the least compact federal ridings are approximately on a par with the U.S. state with the fifth most compact congressional districts. Newfoundland and Labrador (although Labrador riding is excluded from this analysis) has the most compact ridings in Canada, with a ratio of 1.238. Disaggregating the ridings further shows that the 18 ridings of Montreal Island have an average ratio of 1.315; the 23 ridings of the city of Toronto have an average of 1.260, illustrating that urban ridings in Canada tend to be more compact. The averages for Winnipeg (1.400), Calgary (1.236), Edmonton (1.347), and the British Columbia Lower Mainland (Greater Vancouver, 1.222) confirm this. The nine ridings of northern Ontario, by contrast, have an average ratio of 1.552.

One spot in Canada where redistricting has been contentious in the last two rounds is New Brunswick. As stated, the province has the least compact federal ridings in Canada, on average. The province's ratio increased from 1997 to 2004 by 0.126, the largest increase in Canada. Another province where redistricting was controversial in 1997 and 2004 is Saskatchewan, but the change in this time is negligible, and in fact the province's ridings overall are more compact than either of its provincial neighbors, Manitoba and Alberta, where federal redistricting has been reasonably non-controversial.

[2] The territories, Labrador, the Sable Island portion of Halifax riding, and discontiguous portions of two Quebec ridings.

The small change in averages from 1997 to 2004 (0.001 nationally) suggests that redistricting is not viewed by Canadian politicians as nearly as great a political opportunity as is the case in the United States. Although these data only cover two maps and one round of redistricting, the low ratios point to no past tradition of gerrymandering in Canada.

The same cannot be said for the United States, where a district created under the leadership of Gov. Elbridge Gerry of Massachusetts led to such notoriety that the entire practice of manipulating districts has been named for him. Most have never seen an accurate representation of the district that led to this situation. A cartoon image of the district drawn in the likeness of a salamander is familiar to many students of redistricting. However, an examination of the map reveals that much of the salamanderlike shape of the district came not from gerrymandering but from natural land forms, such as the state boundary and the coastline. The district is essentially a backward seven, and would be largely unremarkable if fomented on a modern election map. Although extreme for its time, the gerrymander district has a lower Hill ratio than 32 congressional districts currently in use in the United States, in more than half a dozen states.

In summary, compactness has declined precipitously in most of the United States over the period of time in which redistricting has been a regular happening. The same has not happened in Canada, with the Canadian average being below what the U.S. average was even in 1922. Canadian districts are also more uniform in their compactness ratios from sea to sea than American districts. American redistricters continue to create non-compact districts, often to extremes, apparently subject only to the limits that courts will accept. The tendency to aggressively seek to increase vote share through redistricting has not come to Canada except in rare instances.

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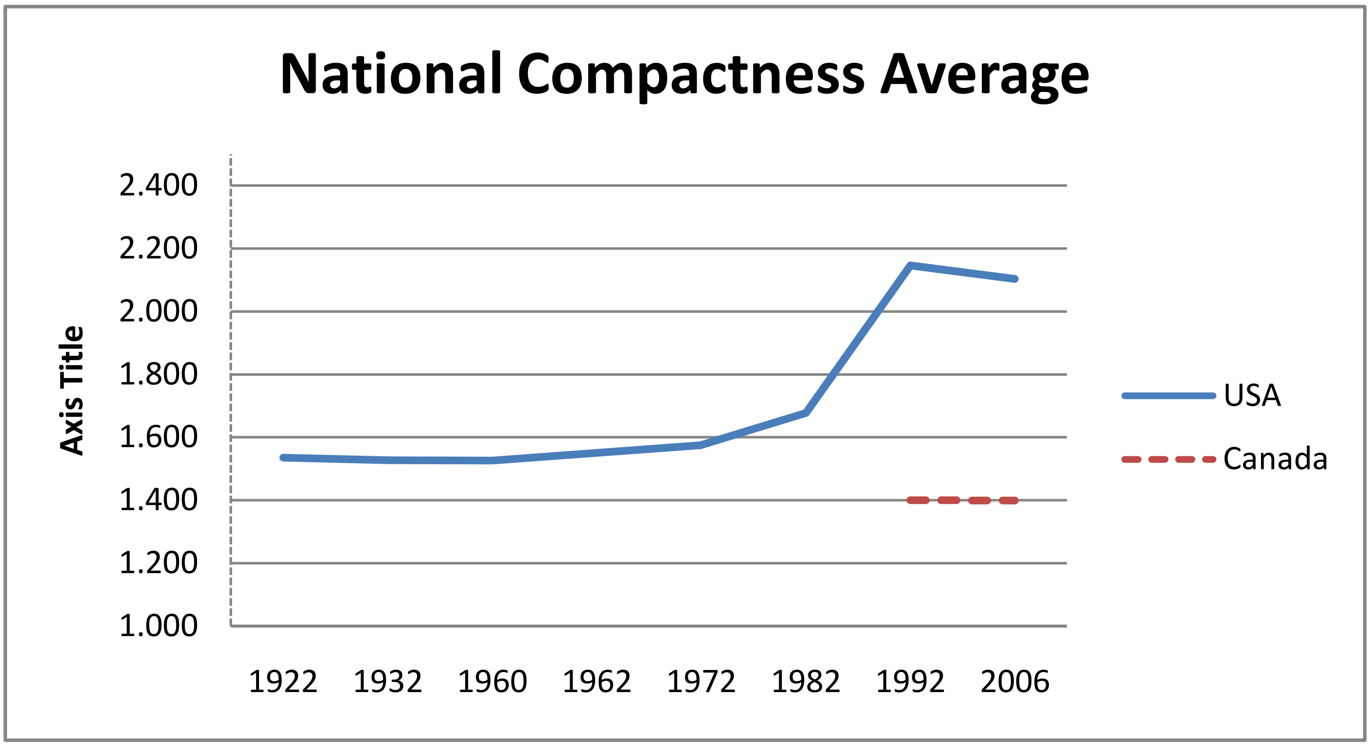


National Compactness Average
Axis Title
2.400
2.200
2.000
1.800
1.600
1.400
1.200
1.000
1922
1932
1960
1962
1972
1982
1992
2006
USA
Canada

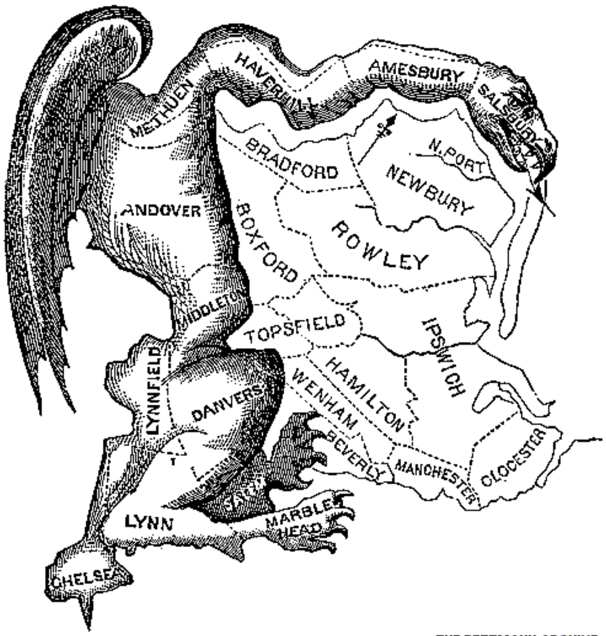
COMPACTNESS OF CONGRESSIONAL DISTRICTS 1922-2006

State	CD 1922	CD 1932	CD 1960	CD 1962	CD 1972	CD 1982	CD 1992	CD 2006	Change 1992-2006	Change 1960-2006	Change 1922-2006
Alabama	1.676		1.694		1.630	1.545	2.136	2.221	0.085	0.527	0.544
Arizona			1.414	1.414			1.878	1.743	-0.136	0.328	
Arkansas	1.877		1.648	1.604	1.758	1.735	1.768	1.775	0.007	0.127	-0.101
California	1.605	1.516	1.569	1.501			1.687	2.288	0.601	0.720	0.683
Colorado			1.765				1.910	1.756	-0.154	-0.010	
Connecticut			1.378				1.657	1.781	0.124	0.403	
Florida	1.711		1.549	1.596			2.596	2.419	-0.177	0.870	0.708
Georgia	1.504		1.517		1.524	1.717	2.349	1.779	-0.570	0.262	0.275
Idaho			1.564				1.776	1.711	-0.065	0.147	
Illinois	1.500		1.403	1.366			2.011	2.195	0.184	0.793	0.695
Indiana	1.384	1.449	1.438				1.817	1.774	-0.043	0.336	0.390
Iowa	1.336	1.299	1.282	1.288	1.392	1.459	1.396	1.497	0.102	0.215	0.161
Kansas	1.301	1.250	1.244	1.279		1.236	1.442	1.495	0.053	0.251	0.194
Kentucky	1.716	1.643	1.766	1.811		1.754	1.890	1.759	-0.131	-0.006	0.043
Louisiana			1.676				3.636	2.102	-1.533	0.427	
Maine	1.984		1.850	1.958			2.164	2.110	-0.055	0.260	0.126
Maryland	1.999		2.099	2.038			2.212	4.659	2.447	2.561	2.660
Massachusetts			2.290	2.240	2.200	2.340	2.488	2.213	-0.276	-0.077	
Michigan	1.507	1.425	1.425				1.530	1.651	0.121	0.226	0.144
Minnesota	1.442		1.310	1.424			1.545	1.594	0.049	0.284	0.152
Mississippi	1.615	1.598	1.668	1.580			2.015	1.896	-0.119	0.228	0.281
Missouri	1.467	1.512	1.498	1.655	1.440		1.616	1.748	0.133	0.250	0.282
Montana			1.373								
Nebraska	1.320	1.352	1.378	1.384	1.506		1.453	1.464	0.011	0.086	0.144
Nevada							1.549	1.797	0.247		
New Hampshire							1.991	1.959	-0.032		
New Jersey	1.557		1.446	1.412			2.255	2.322	0.067	0.876	0.764
New Mexico					1.292	1.535	1.531	1.481	-0.050		
New York	1.323		1.528	1.505			2.407	2.010	-0.397	0.482	0.687
North Carolina	1.807	1.825	1.848	1.828	1.628	1.797	3.814	2.721	-1.092	0.873	0.914
North Dakota	1.351			1.172							
Ohio	1.293		1.355				1.862	1.927	0.065	0.572	0.634
Oklahoma	1.477		1.640				1.930	1.779	-0.150	0.140	0.303
Oregon	1.423		1.424				1.775	1.774	-0.001	0.350	0.351
Pennsylvania		1.413	1.395	1.373	1.283		1.822	2.390	0.568	0.995	
Rhode Island							1.979	1.949	-0.030		
South Carolina	1.581		1.496		1.605	1.654	2.406	1.904	-0.502	0.408	0.323
South Dakota	1.250		1.288		1.399						
Tennessee			1.442		1.675	1.971	2.202	2.215	0.013	0.773	
Texas	1.561	1.463	1.462				3.490	2.143	-1.348	0.680	0.581
Utah			1.415		1.760		1.493	1.491	-0.002	0.075	
Virginia	1.871	1.898	1.811				2.407	2.219	-0.188	0.408	0.347
Washington	1.432	1.464	1.464				1.787	1.721	-0.066	0.258	0.289
West Virginia	1.998	2.016	2.036	2.180	2.035	2.018	2.352	2.303	-0.049	0.267	0.305
Wisconsin	1.429		1.377				1.649	1.726	0.076	0.348	0.297
Grand	1.535	1.527	1.526	1.551	1.575	1.676	2.146	2.103		0.577	0.568

Source: Tony L. Hill, District Compactness and Electoral Volatility
in Canada and the United States, presented at MPSA 2009 tlh@alum.mit.edu

Average Compactness by Province and Region of Canada

Province	1997	2004	diff	notes
Newf	1.314	1.238	-0.076	excl Labrador
PEI	1.247	1.266	0.019	
NS	1.317	1.438	0.121	excl Sable Island
NB	1.421	1.547	0.126	
Que	1.499	1.501	0.002	excl Iles-de-la-Madeleine, non-contig parts of Manic
MI	1.327	1.315	-0.012	
Ont	1.335	1.333	-0.002	
N.Ont	1.400	1.552	0.152	
ROO	1.286	1.313	0.027	
Tor	1.315	1.260	-0.055	
Subn Tor	1.251	1.231	-0.020	
Man	1.415	1.367	-0.048	
Wpg	1.418	1.400	-0.018	
Sask	1.258	1.264	0.006	
Alta	1.357	1.385	0.028	
Calg	1.214	1.236	0.022	
Ed	1.330	1.347	0.017	
BC	1.501	1.447	-0.054	
LM	1.285	1.222	-0.063	
Natl	1.399	1.400	0.001	excl excluded



THE BETTMANN ARCHIVE

GERRYMANDER, a fictional creature based on the shape of an electoral district of Massachusetts, as set up for political reasons.